NURTURE COURSE — ALL INDIA OPEN TEST # 01

TEST TYPE : MAJOR | PATTERN : JEE (Advanced)

TARGET : JEE (Advanced) 2016 | Date : 08 - 02 - 2015

PAPER-1

PART-1 : PHYSICS — **ANSWER KEY**

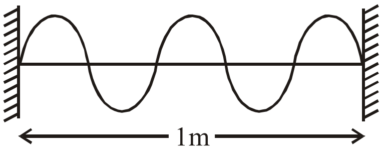
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C,D	C	A,C,D	A,C,D	A,C	B,D	A,B,C	A,D	B,D	A
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	6	2	1	3	3	2	2	5	3

SOLUTION

SECTION-I

1. **Ans. (A,B,C,D)**

Sol.



vibrating in 200th harmonic

$\frac{n\lambda}{2} = 1 \Rightarrow \frac{200\lambda}{2} = 1 \Rightarrow \lambda = \frac{1}{100}$

Distance of antinode from node $\Rightarrow \frac{\lambda}{4} = \frac{1}{400}$ m

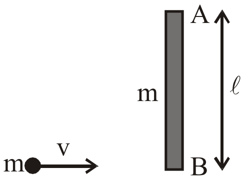
Now at t = 0 $\Rightarrow 0.1525 \Rightarrow \frac{61}{400}$ m

mean particle is as antinode condition

Mean at t = 0 string will be straight.

2. **Ans. (C)**

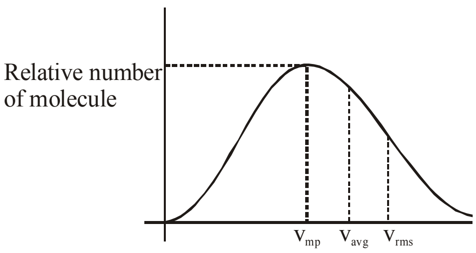
Sol.



If we observe motion of particle from point A as frame of A non inertial so angular momentum of rod as well as linear momentum will not conserved

3. **Ans. (A,C,D)**

Sol.



$V_{rms} = \sqrt{\frac{3RT}{M}}$ where temperature T is in k

$V_{mp} = \sqrt{\frac{2RT}{M}}$

$V_{avg} = \sqrt{\frac{8RT}{\pi M}} \Rightarrow V_{rms} = \sqrt{\frac{3R(273+T°C)}{M}}$

$V_{rms} = \sqrt{\frac{3R(273+4T°C)}{M}}$

4. **Ans. (A, C, D)**

5. **Ans. (A,C)**

Sol. Area of PV curve in process 1 > area of PV curve in process 2

Means $w_{process\ 1} > w_{process\ 2}$

As final volume in both process is same $P \propto T$

$T_{final\ process\ 2} < T_{final\ process\ 1}$

6. **Ans. (B,D)**

7. **Ans. (A,B,C)**

8. **Ans. (A,D)**

Sol. $(k\ell\theta)\ell > \frac{mg\ell}{2}\sin\theta$

$k\ell^2\theta > \frac{mg\ell}{2}\theta$

$k > \frac{mg}{2\ell}$

9. **Ans. (B, D)**

Sol. At this point particle will lie on the y-axis and moving in horizontal direction.

10. **Ans. (A)**

SECTION-IV

1. Ans. 3

Sol. VP^k = constant

50% heat as work.

dQ = dw + du

$du = dw = \frac{f}{2}nR\Delta T$

$dQ = \frac{f}{2}nR\Delta T + \frac{f}{2}nR\Delta T$

$dQ = fnR\Delta T = nC\Delta T$

C = FR = 3R

as gas is monoatomic

$$C = R\left[\frac{1}{(\gamma-1)} - \frac{1}{(x-1)}\right]$$

$$3R = R\left[\frac{1}{\left(\frac{5}{3}-1\right)} - \frac{1}{(x-1)}\right]$$

$$x = \frac{1}{3} \Rightarrow k = \frac{1}{x} = 3$$

2. Ans. 6

Sol. Let a be the length of the sting

Tension T at any point is given by

$$T = \frac{M}{a}\left[u^2 + 3ag\cos\theta - 2ag\right] \quad \text{... (i)}$$

Where M is the mass of the particle and as such

$M = M = \frac{W}{g}$.

$\therefore$ From Eq. (i), we get :

$$T = \frac{W}{ag}\left(u^2 + 3ag\cos\theta - 2ag\right) \quad \text{...(ii)}$$

At the highest point T = mW (given) and $\theta = \pi$

$\therefore$ From Eq. (ii)

$$mW = \frac{W}{ag}\left[u^2 - 3ag - 2ag\right]$$

$mag = u^2 - 5ag$ ... (iii)

At the lowest point T = nW (given) and $\theta = 0$

$\therefore$ From Eq. (ii)

$$nW = \frac{W}{ag}\left[u^2 + 3ag - 2ag\right]$$

$nag = u^2 + ag$ ...(iv)

Eq. (iii) – Eq. (iv) $\Rightarrow$ mag – nag = 5ag – ag

m + 6 = n

3. Ans. 2

Sol. Longitudinal speed = $\sqrt{\frac{y}{\rho}}$

Transverse speed = $\sqrt{\frac{T}{A\rho}}$

$$\sqrt{\frac{y}{\rho}} = 10\sqrt{\frac{T}{A\rho}}$$

$$\sqrt{20\times10^{10}} = 10\sqrt{\frac{T}{\pi\left(1\times10^{-3}\right)^2}}$$

$$20\times10^{10} = \frac{100\times T}{\pi\times10^{-6}}$$

$T = 2000 \times \pi = 3.14 \times 2000 = 6280$ m

4. Ans. 1

Sol. Net loss = k (temperature different between both sphere & surrounding)

$$ms\frac{d\theta}{dt} = -k\left[\theta - \theta s\right]$$

$$\int_{\theta_1=0}^{\theta_2=\frac{\theta s}{2}} \frac{Cd\theta}{(\theta-\theta s)} = \int_0^t kdt$$

$$= -C\left[\ell n(\theta - \theta s)\right]_0^{\frac{\theta s}{2}} = kt$$

$$-C\ell n\left[\frac{\frac{-\theta s}{2}}{-\theta s}\right] = kt$$

$$-C\ell n\left(\frac{1}{2}\right) = kt$$

$C\ell n(2) = kt$

$$t = \frac{C}{k}\ell n(2)$$

5. Ans. 3

Sol. When the source (the train) moves towards the observer, the apparent frequency is

$$f' = \frac{v}{v - v_s}f \quad \text{... (i)}$$

When the source is moving away, the apparent frequency is

$$f'' = \frac{v}{v + v_s}f \quad \text{... (ii)}$$

$$\therefore \quad \frac{f'}{f''} = \frac{v + v_s}{v - v_s}$$

or $\frac{100}{50}=\frac{2}{1}=\frac{\left(\frac{v}{v_s}+1\right)}{\left(\frac{v}{v_s}-1\right)}$

or $\frac{v}{v_s}=3$ ... (iii)

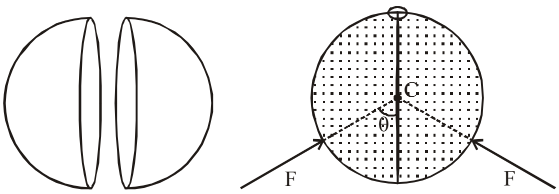
When the observer moves with the train, there is no relative motion between the source and the observer and hence he will listen the true frequency f. Substituting (iii) in (i), we get

$$100=\frac{3}{2}f$$

or $f=\frac{200}{3}=66.6\text{Hz}$

6. Ans. 3

Sol.



$2F\cos\theta = mg$

$F\cos\theta=\frac{mg}{2}$

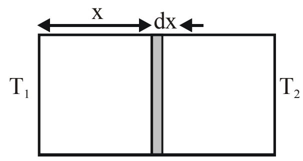
$F\sin\theta = (\rho gR)\,\pi R^2$

$$\tan\theta=\frac{2\rho gR^3\pi}{mg}\Rightarrow\frac{2\rho gR^3\pi}{\rho\times\frac{4}{3}\pi R^3\times g}=\frac{3}{2}$$

7. Ans. 2

8. Ans. 2

Sol.



$$\frac{dQ}{dt}=\frac{CAdT}{Tdx}$$

$$\frac{dQ}{dt}\int_0^x dx = CA\int_{T_1}^{T}\frac{dT}{T}$$

$$\frac{dQ}{dt}x = CA\ \ln\frac{T}{T_1}$$

$$\frac{dQ}{dt}\ell = CA\ \ln\frac{T_2}{T_1}$$

$$\frac{x}{\ell}=\frac{\ln T/T_1}{\ln T_2/T_1}$$

$$T=T_1\left(\frac{T_2}{T_1}\right)^{x/\ell}$$

9. Ans. 5

10. Ans. 3

Sol. Speed just before collision

$$v=\omega\sqrt{a^2-\left(\frac{a}{2}\right)^2}=\omega\frac{\sqrt{3}a}{2}$$

Speed just after collision

$$v'=\frac{v}{2}=\frac{\omega\sqrt{3}a}{4}=\sqrt{\frac{k}{m}}\frac{\sqrt{3}a}{4}=\sqrt{\frac{3k}{16m}}a$$

PART-2 : CHEMISTRY — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C,D	A,B,C	A,B	A,B,D	B,C,D	A	B	B,C,D	A,B,C,D	A,B,D
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	3	2	2	6	0	4	5	9	6

SOLUTION

SECTION-I

1. Ans.(A, B, C, D) ; **2. Ans. (A,B,C)**

3. Ans.(A, B) ; **4. Ans. (A, B, D)**

5. Ans. (B, C, D) ; **6. Ans. (A)**

7. Ans. (B) ; **8. Ans. (B, C, D)**

9. Ans. (A, B, C, D) ; **10. Ans. (A, B, D)**

SECTION-IV

1. Ans. 4

2. Ans. 3

$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$H_2(g)$	$I_2(g)$	$2HI(g)$	
1	3		
$1-\frac{x}{2}$	$3-\frac{x}{2}$	x	at equib.
$3-\frac{x}{2}$	$3-\frac{x}{2}$	x	after addition of H_2
$3-x$	$3-x$	2x	new equib.

$$K=\frac{x^2}{\left(1-\frac{x}{2}\right)\left(3-\frac{x}{2}\right)}=\frac{4x^2}{(3-x)^2}\Rightarrow 2x=3$$

3. Ans. 2

$\lambda \alpha \frac{1}{P}$

$\Rightarrow$ P increases 5 times then λ becomes $\frac{1}{5}$

$\Rightarrow$ 2 cm

4. Ans. 2
5. Ans. 6
6. Ans. 0
7. Ans. 4
8. Ans. 5
9. Ans. 9
10. Ans. 6

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B,C	A,B	A,B,C	C	A,C,D	B,C	A,B,C,D	A,C	A,D	B,D
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	8	2	7	5	5	3	4	5	4	4

SOLUTION

1. Ans. (B,C)

It is possible if $m\in\left(1,\frac{\pi}{2}\right]$

2. Ans. (A,B)

$f(0)=0=f(10)\Rightarrow f(10n)=0\ \forall\ n\in I$

$\therefore$ we get atleast 11 solutions in [–50,50] and also if we get p solution in (0,10) then we get also p solution in (10n,10n+10), n$\in$I

$\therefore$ we get always $10\lambda+11$, $\lambda\in W$ solutions.

3. Ans. (A,B,C)

$a+b+c=-a$ ...(i)

& $ab+bc+ca=b$ ...(ii)

& $abc=-c$ ...(iii)

on solving (i), (ii) & (iii) we get

$a=1$ & $b=-2$ & $c=0$; $a=0$ & $b=0$ & $c=0$; $a=1$ & $b=-1$ & $c=-1$ & one other solution

4. Ans. (C)

$V_n=2a_{n+2}+S_n$

$=2\left[-4+(n+1)\right]+\frac{n}{2}\left[-8+(n-1)\right]$

$=\frac{n^2-5n-12}{2}$

V_n is minimum at n = 2 or n = 3

$\Rightarrow$ Minimum value of V_n is –9.

5. Ans. (A,C,D)

$8^3x^9+9.8^2x^6+27.8x^3-x+219=0$

$(8x^3)^3+3.(8x^3)^2.3+3(8x^3).(3)^2+27=512x-192$

$[8x^3+3]^3=512x-192$

$8x^3+3=(512x-192)^{1/3}$

$x^3+\frac{3}{8}=\left(x-\frac{3}{8}\right)^{1/3}$

$\Rightarrow f(x)=f^{-1}(x)$

$\Rightarrow f(x)=x$ [$\because f(x)$ is continuous and $\uparrow$ fn]

$\Rightarrow x^3+\frac{3}{8}=x$

$\Rightarrow (2x-1)(4x^2+2x-3)=0$

$\Rightarrow x=\frac{1}{2},\frac{-1+\sqrt{13}}{4},\frac{-1-\sqrt{13}}{4}$

6. Ans. (B,C)

Given curve is circle

centre $\left(\frac{\tan^{-1}\alpha-\cot^{-1}\alpha}{2},\frac{\pi}{4}\right)$

length of line L = diameter

$=\sqrt{\left(\frac{\pi}{2}\right)^2+\left(2\tan^{-1}\alpha-\frac{\pi}{2}\right)^2}$

$L\in\left[\frac{\pi}{2},\frac{\sqrt{10}}{2}\pi\right]$

7. Ans. (A,B,C,D)

$$x^3+2x^2-3x+1=0\begin{cases}\alpha\\ \beta\\ \gamma\end{cases}$$

$$x^3-3x^2+2x+1=0\begin{cases}1/\alpha\\ 1/\beta\\ 1/\gamma\end{cases}$$

Given expression $\dfrac{1}{3-\frac{1}{\alpha}}+\dfrac{1}{3-\frac{1}{\beta}}+\dfrac{1}{3-\frac{1}{\gamma}}$

$$=\frac{\sum\left(3-\frac{1}{\alpha}\right)\left(3-\frac{1}{\beta}\right)}{\left(3-\frac{1}{\alpha}\right)\left(3-\frac{1}{\beta}\right)\left(3-\frac{1}{\gamma}\right)}$$

$$=\frac{27-6\left[\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right]+\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}}{\left(3-\frac{1}{\alpha}\right)\left(3-\frac{1}{\beta}\right)\left(3-\frac{1}{\gamma}\right)}=\frac{11}{7}$$

8. Ans. (A,C)

$S_1 \equiv x^2+y^2-2x-4y-4=0$

$S_2 \equiv x^2+y^2-8x-12y+36=0$

Intersection point of direct common tangent is $(-8,-10)$

equation of line AB is $T=0 \Rightarrow 3x+4y=8$

Let equation of $S_3=0$ is

$(x^2+y^2-2x-4y-4)+\lambda(3x+4y-8)=0$

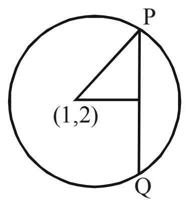
$\because$ AB is diameter $\Rightarrow \lambda=\dfrac{6}{25}$

required circle $25x^2+25y^2-32x-76y-148=0$

equation of common chord is $6x+8y-40=0$

$PQ=2\sqrt{9-\left(\dfrac{18}{10}\right)^2}$

$=\dfrac{24}{5}$



9. Ans. (A,D)

Given $f(f(x))=x$

$\Rightarrow \dfrac{af(x)+b}{cf(x)+d}=x$

$\Rightarrow f(x)=\dfrac{-dx+b}{cx-a}$ ......(1)

given $f(x)=\dfrac{ax+b}{cx+d}$ ......(2)

from (1) & (2) $a=-d$ ......(3)

$f(19)=19$

$\Rightarrow 38a+b=19\times 19c$ ....(4)

$f(97)=97$

$\Rightarrow 194a+b=97\times 97c$ ....(5)

from (5) – (4)

$a=58c$

$\therefore \alpha=58=2^1 . 29^1$

10. Ans. (B,D)

$f(a_1)\le f(a_2)\le f(a_3)\le.....\le f(a_{100})$

it is equivalent to distribution of 100 objects to 50 persons such that each person get atleast one object.

$\therefore$ number of such mapings = ${}^{99}C_{49}={}^{99}C_{50}$

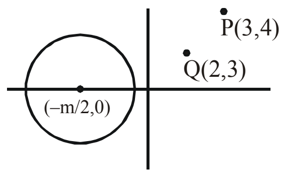
SECTION-IV

1. Ans. 8

$f^{-1}(g(5))=\sin^{-1}(\sin 5)$

$=\sin^{-1}(\sin(3\pi-5))=3\pi-5$

2. Ans. 2



Equation of line PQ is chord of circle

$\Rightarrow$ perpendicular distance $\le$ radius

$\Rightarrow \left|\dfrac{m-2}{2\sqrt{2}}\right|\le\sqrt{\dfrac{m^2}{4}-m}$

$\Rightarrow m^2-4m-4\ge 0$

$\therefore M=5$

3. Ans. 7

$x^2y^2+x^2+y^2+1+9-6x-6y=0$

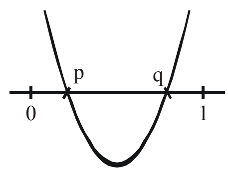
$\Rightarrow x^2y^2-2xy+1+x^2+y^2+9+2xy-6x-6y=0$

$\Rightarrow (xy-1)^2+(x+y-3)^2=0$

$\Rightarrow xy=1$ and $x+y=3$

$\Rightarrow x^2+y^2=(x+y)^2-2xy=7$

4. Ans. 5



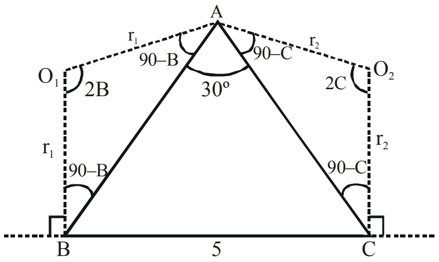
$0<\dfrac{b}{2a}<1$ ...(1)

& $b^2-4ac>0$ ...(2)

& $a-b+c>0$ ...(3)

from (1), (2) & (3) we get minimum value of a = 5

5. Ans. 5



in ΔAO_1B $\dfrac{AB}{\sin 2B} = \dfrac{r_1}{\cos B}$ ...(1)

in ΔAO_2B $\dfrac{AC}{\sin 2C} = \dfrac{r_2}{\cos C}$ ...(2)

from (1) & (2)

$$\frac{AB.AC}{\sin 2B.\sin 2C} = \frac{r_1 r_2}{\cos B \cos C}$$

$$\frac{2R\sin C.2R\sin B}{2\sin B.2\sin C} = r_1 r_2$$

$$r_1 r_2 = R^2 = \left(\frac{5}{2\sin 30°}\right)^2 = 25$$

6. Ans. 3

Let $x_i = \sin\alpha_i$

Given $\sum_{i=1}^{9} x_i^3 = 0$

$$\sum_{i=1}^{9}\left(\frac{3\sin\alpha_i - \sin 3\alpha_i}{4}\right) = 0$$

$$3\sum_{i=1}^{9}\sin\alpha_i = \sum_{i=1}^{9}\sin(3\alpha_i)$$

$$\sum_{i=1}^{9} x_i = \frac{\sum_{i=1}^{9}\sin(3\alpha_i)}{3} \Rightarrow \left(\sum_{i=1}^{9} x_i\right)_{max} = 3$$

7. Ans. 4

$2(\tan x - \sin x) + 3(\cot x - \cos x) + 5 = 0$

$$\frac{2(\sin x - \sin x\cos x + \cos x)}{\cos x} + \frac{3(\cos x - \sin x\cos x + \sin x)}{\sin x} = 0$$

$$(\cos x + \sin x - \sin x\cos x)\left(\frac{2}{\cos x} + \frac{3}{\sin x}\right) = 0$$

$\Rightarrow$ sinx + cosx = sinx cosx

or $\tan x = -\dfrac{3}{2}$

Case-I :

$\tan x = -\dfrac{3}{2} \Rightarrow$ 2 principal solutions

Case-II :

sinx + cosx = sinx cosx

$2(\sin x + \cos x) = (\sin x + \cos x)^2 - 1$

$\therefore$ $\sin x + \cos x = \sqrt{2} + 1$ (rej.) or $\sqrt{2} - 1$

$\Rightarrow$ $\sin x + \cos x = \sqrt{2} - 1$

$\Rightarrow$ $\sin\left(x + \dfrac{\pi}{4}\right) = 1 - \dfrac{1}{\sqrt{2}} \Rightarrow$ 2 principal sol.

$\therefore$ Total 4 solutions

8. Ans. 5

On subtract given equations

$[(\tan x - 1)^3 - (1 - \cot y)^3] + 2015$

$[(\tan x - 1) - (1 - \cot y)] = 0$

$\Rightarrow [\tan x + \cot y - 2]\ [(\tan x - 1)^2 + (1 - \cot y)^2 + (\tan x - 1)(1 - \cot y) + 2015] = 0$

it is possible if tanx + coty = 2

$\therefore \sin z + \cos z = 2 \Rightarrow z = -2\pi, -\dfrac{3\pi}{2}, 0, \dfrac{\pi}{2}, 2\pi$

9. Ans. 4

$${}^4C_3\left[D(3).{}^3C_1 + {}^3C_2.(D(2).2)\right]$$

$= 4[6 + 3.2] = 48$

D(n) means dearrangement of n objects.

10. Ans. 4

Let r = p + 1

$\therefore T_r = {}^nc_p 2^{n-p} x^{n-p-mp}$

$\therefore P = \dfrac{n}{1+m}$ ...(1)

$T_r = {}^mC_p.(x^{-4})^{m-p}(-2)^p(x^{n-6})^p$

$P = \dfrac{4m}{n-2}$ ...(2)

from (1) & (2)

$n = 2(m+1) \Rightarrow r = 3$

$\therefore$ m = 1 reject

$\Rightarrow$ for minimum value m = 2, then n = 6

NURTURE COURSE — ALL INDIA OPEN TEST # 01

TEST TYPE : MAJOR — PATTERN : JEE (Advanced)

TARGET : JEE (Advanced) 2016 — Date : 08 - 02 - 2015

PAPER-2

PART-1 : PHYSICS — **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	A	B	B	A	A	D	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	A	D	C	B	A	C	B	B

SOLUTION

SECTION-I

1. Ans. (C)

Sol. $\frac{\lambda}{2}=1 \Rightarrow \lambda = 2 \Rightarrow v=\frac{\omega}{k} \Rightarrow \frac{\lambda}{T}=\frac{2}{2} \Rightarrow 1\,cm/sec$

$v_p = \omega\sqrt{A^2-x^2} \quad = \frac{2\pi}{T}\sqrt{4^2-\left(2\sqrt{3}\right)^2}$

$=\frac{2\pi}{2}\sqrt{16-12}$

$\frac{2\pi}{2}\times 2 = 2\pi\,cm/sec$

2. Ans. (C)

Sol. By mechanical energy conservation

$\frac{1}{2}mv^2 - mg\ell = \frac{1}{2}\frac{mv^2}{4} - mg\ell\cos\theta$

$\frac{v^2}{2}\left[1-\frac{1}{4}\right] = g\ell\left[1-\cos\theta\right]$

$\frac{3v^2}{8} = g\ell(1-\cos\theta)$

$\left[v=\sqrt{\frac{8g\ell(1-\cos\theta)}{3}}\right]$ ... (i)

At point A $\Rightarrow T - mg = \frac{mv^2}{\ell}$

$5\,mg - mg = \frac{mv^2}{\ell} \Rightarrow v=\sqrt{4g\ell}$ ... (ii)

By solving equation (i) & (ii)

$\sqrt{4g\ell} = \sqrt{\frac{8g\ell(1-\cos\theta)}{3}}$

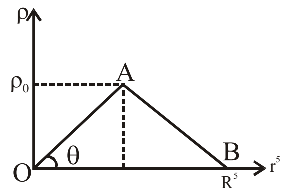
$4=\frac{8}{3}(1-\cos\theta) \Rightarrow \frac{12}{8} = 1-\cos\theta$

$\cos\theta = \frac{-1}{2} \Rightarrow \theta = 120°$

3. Ans. (A)

4. Ans. (B)

Sol.



$dm = \rho dV$

$dm = \rho 4\pi r^2 dr$

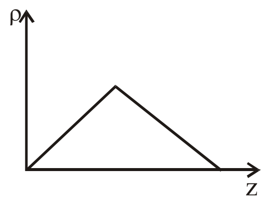
$I = \frac{2}{3}\int\left(\rho 4\pi r^2 dr\right)r^2 \; = \int \rho 4\pi r^4 dr$

$I = \frac{2}{3}(4\pi)\int \rho r^4 dr$ let $r^5 = z$

$5r^4\,dr = dz$

$r^4 dr = \frac{dz}{5} \; = \frac{2}{3}\frac{4\pi}{5}\int \rho dz$

Area under curve



$=\frac{2}{3}\frac{4\pi}{5}\times\left(\frac{1}{2}\times\rho_0 R^5\right)$

$I = \frac{2}{3}\times\frac{2}{5}\pi\rho_0 R^5 = \frac{4}{15}\pi\rho_0 r^5$

5. Ans. (B)

Sol. $\dfrac{I_R}{I_0} = \dfrac{A_R^2}{A_0^2} = \dfrac{(v_2 - v_1)^2}{(v_2 + v_1)^2}$

$= \left(\dfrac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}}\right) = \left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)^2$

$I_R = \left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right) I_0$

$x = 0 \qquad I_R = I_0$

$x \to \infty \qquad I_R = I_0$

(at $x = 1 \qquad I_R = I_0$

6. Ans. (A)

7. Ans. (A)

Sol. $F = \mu A \left(\dfrac{\Delta V}{\Delta x}\right)$

$F = \mu\omega L \left(\dfrac{V}{H/2}\right) \times 2 = \left(\dfrac{2\mu v \omega L}{H}\right) \times 2$

$F = \dfrac{4\mu v \omega L}{H}$

8. Ans. (D)

Sol. Here sources are incohesent so

$I_{resultant} = I_1 + I_2 + I_3 + ...$

$I = 200\, I_S$

$60 = 10 \log \left(\dfrac{200 I_S}{I_0}\right) \quad ....(i)$

$x = 10 \log \left(\dfrac{50 I_S}{I_0}\right) \quad ....(ii)$

Solve (i) & (ii)

$10^6 = 200 \dfrac{I_S}{I_0}$

$10^{x/10} = 50 \dfrac{I_S}{I_0}$

$10^{6 - \frac{x}{10}} = 4$

take log both side

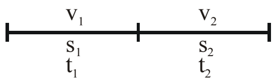
$6 - \dfrac{x}{10} = \log 4 = .6$

$6 - .6 = \dfrac{x}{10}$

$x = 54$ dB4

9. Ans. (A)

Sol.



$v_1 = \dfrac{s_1}{t_1} \qquad v_2 = \dfrac{s_2}{t_2}$

$t_1 + t_2 = \left(\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2}\right)$

$v_{avg} = \dfrac{s_1 + s_2}{t_1 + t_2} = \dfrac{s_1 + s_2}{\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2}} = \sqrt{v_1 v_2}$ (given)

$\Rightarrow$ divided by s_2 in left so

$\dfrac{\dfrac{s_1}{s_2} + 1}{\dfrac{s_1}{s_2 v_1} + \dfrac{1}{v_2}} = \sqrt{v_1 v_2}$

divided by v_1 in both side

$\dfrac{s_1}{s_2} = x$

$\dfrac{x+1}{x + \dfrac{v_1}{v_2}} = \sqrt{\dfrac{v_2}{v_1}}$

$\dfrac{v_1}{v_2} = y$

$\dfrac{x+1}{x+y} = \dfrac{1}{\sqrt{y}}$

$\dfrac{x+1-x-y}{x+y} = \dfrac{1-\sqrt{y}}{\sqrt{y}}$

$\dfrac{1-y}{x+y} = \dfrac{1-\sqrt{y}}{\sqrt{y}}$

$$\frac{x+y}{\sqrt{1-y}} = \frac{\sqrt{y}}{1-\sqrt{y}}$$

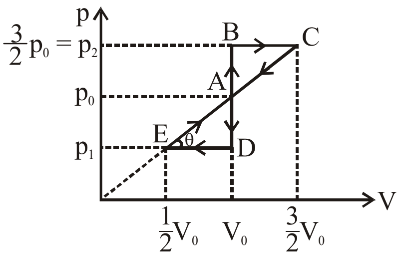
$$x = \sqrt{y} \qquad y = \sqrt{\frac{v_1}{v_2}}$$

$$x = \left(\frac{v_1}{v_2}\right)^{3/2}$$

$$x = \sqrt{\frac{v_1}{v_2}}$$

10. Ans. (B)

Sol.



$$\eta = \frac{w_{net}}{Q_{supplied}}$$

ABCA

$$(A \to B)\begin{pmatrix} P\uparrow \Rightarrow T\uparrow \\ v = const. \end{pmatrix}$$

$$\Rightarrow \Delta U > 0 \Rightarrow \Delta Q > 0$$

$$\tan\theta = \frac{P_0}{v_0} = const.$$

$$P_1 = \frac{P_0}{2} \qquad P_2 = \frac{3}{2}P_0$$

$$\omega D_{ABCA} = Area = \frac{1}{2}\times\frac{v_0}{2}\times\frac{P_0}{2} = \frac{-v_0P_0}{8}$$

$$wD_{ADEA} = Area = \frac{1}{2}\times\left(\frac{v_0}{2}\right)\frac{P_0}{2} = \frac{v_0P_0}{8}$$

$$At\ 'C' \Rightarrow PV = \left(\frac{3}{2}P_0\right)\times\frac{3}{2}V_0 = \frac{9}{4}P_0V_0$$

at A $PV = P_0V_0 \Rightarrow T_C > T_A \Rightarrow \Delta U_{C\to A}$

$$\Rightarrow (\Delta Q)_{CA} < 0 \qquad \Delta w_{C-A} < 0$$

$$\Rightarrow \underset{ABCA}{\eta_1} = Z\frac{P_0V_0/8}{\Delta Q_{AB} + \Delta Q_{BC}}$$

$$\eta_{ADEA} = \frac{P_0V_0/8}{Q_{EA}}$$

$$ndT = \left(\frac{P_fV_f - P_iV_i}{R}\right)$$

$$Q_{AB} = nC_VdT$$

$$= \frac{3R}{2}\times\left(\frac{\frac{3}{2}P_0V_0 - P_0V_0}{R}\right) = \frac{3}{2}\times\frac{1}{2}.P_0V_0$$

$$Q_{BC} = nC_PdT = \frac{5R}{2}\times\frac{\left(\frac{9}{4}P_0V_0 - \frac{3}{2}P_0V_0\right)}{R}$$

$$= \frac{5}{2}\times\frac{3}{4} = \left(\frac{15}{8}\right)_{P_0V}$$

$$Q_{EA} = \Delta U + \Delta\omega$$

$$= \frac{3R}{2}\times\left(\frac{P_0V_0}{nR} - \frac{P_0V_0}{4}\right)$$

$$+\left(\frac{P_0}{2}\times\frac{V_0}{2} + \frac{1}{2}\times\frac{V_0}{2}\times\frac{P_0}{2}\right)$$

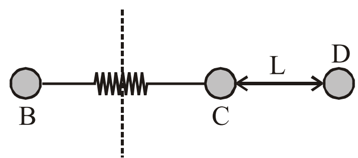
$$= P_0V_0\left(\frac{3}{2}\times\frac{3}{4} + \frac{1}{4} + \frac{1}{8}\right) = \frac{12}{8}P_0V_0$$

$$\frac{\eta_2}{\eta_1} = \frac{3/4 + 15/8}{12/8} = \frac{6+15}{12} = \frac{21}{12} = \frac{7}{4}$$

$$\frac{\eta_1}{\eta_2} = \frac{4}{7}$$

11. Ans. (C)

Sol.



Just after collision of first ball with dumb-bell

Velocity of COM of dumb-bell = $\frac{V}{2}$

Ball B & C performe SHM in COM frame of dumbell.

Velocity of ball D will be V if C coilied with velocity v.

Velocity of C will be v if it is moving towards right with $\frac{v}{2}$ in COM frame.

$\therefore$ Minimum time after which C will be moving with $\frac{v}{2}$ (right) in COM frame is

$$t = \frac{1}{2}\times 2\pi\sqrt{\frac{\mu}{k}} = \pi\sqrt{\frac{m}{2k}},$$

$$\therefore \quad L_{min} = \frac{v}{2}t = \frac{v}{2}\times\pi\sqrt{\frac{m}{2k}}$$

12. Ans. (A)

Sol. From momentum conservation, finally linear momentum of dumb-bell = 0

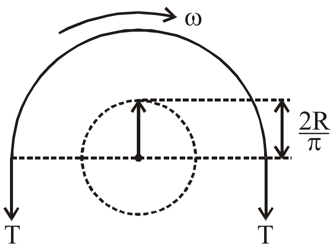
$\Rightarrow v_{cm} = 0$

13. Ans. (A)

14. Ans. (D)

15. Ans. (C)

Sol.



$$2T = \frac{m}{2}\left(\frac{2R}{\pi}\right)\omega^2$$

$$2T = \frac{mR}{\pi}\left(\frac{v}{R}\right)^2 = \frac{mv^2}{R\pi}$$

16. Ans. (B)

Sol. $y = \dfrac{\frac{T}{A}}{\left(\frac{\Delta\ell}{\ell}\right)} = \dfrac{\frac{T}{A}}{\left(\frac{2\pi\cdot\Delta R}{2\pi R}\right)}$

$$\Rightarrow \Delta R = \frac{T}{yA}\cdot R = \frac{mv^2}{2R\pi}\times\frac{R}{yA}$$

$$\Delta R = \frac{mv^2}{2\pi yA}$$

17. Ans. (A)

Sol. (1) $E = \dfrac{\mu A^2\omega^2\lambda}{2}$

$= \dfrac{\mu A^2\omega^2\ell}{2}$ $\quad(\omega = 2\pi f \;\&\; f = \frac{v}{\ell})$

(2) $E = \frac{1}{8}\mu A^2\omega\lambda \Rightarrow \frac{1}{8}\mu A^2\omega^2(2\ell)\;\frac{\lambda}{2} = \ell$

$\dfrac{\mu A^2\omega^2\ell}{4}$ $\quad(\omega = 2\pi r \;\&\; f = \frac{v}{2\ell})$

(3) $E = \frac{1}{8}\mu A^2\omega^2\lambda + \frac{1}{16}\mu^2A^2\omega^2\lambda$

$\frac{\lambda}{2}+\frac{\lambda}{4} = \frac{3\lambda}{4} = \ell$

$\lambda = \frac{4\ell}{3} = \frac{3}{16}\mu A^2\omega^2\lambda$

$= \frac{3}{16}\mu A^2\omega^2\left(\frac{4\ell}{3}\right) = \frac{\mu A^2\omega^2\ell}{4}$

$(\omega = 2\pi r \;\&\; f = \frac{3v}{4\ell})$

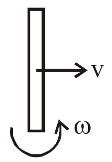
(4) $E = \frac{2}{8}\mu A^2\omega^2\lambda \Rightarrow \frac{1}{4}\mu A^2\omega^2\ell$

$(\omega = 2\pi r \;\&\; f = \frac{v}{\ell})$

$\lambda = \ell$

18. Ans. (C)

Sol. (P)



Angular momentum conservation

$$0 = \frac{mv\ell}{2} - \frac{m\ell^2}{12}\omega \Rightarrow v = \frac{\omega\ell}{6}$$

(Q) Angular momentum conservation

$$0 = Mv\ell - \frac{M\ell^2}{12}\omega \Rightarrow v = \frac{\omega\ell}{6}$$

(R) Angular momentum conservation

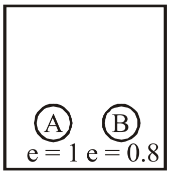
$$0 = Mv\ell - \frac{M\ell^2}{12}\omega \Rightarrow v = \frac{\omega\ell}{6}$$

(S) Angular momentum conservation

$$\frac{Mv\ell}{4} - \frac{M\ell^2}{12}\omega = 0 \Rightarrow v = \frac{\omega\ell}{3}$$

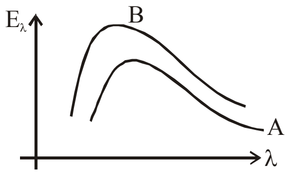
19. Ans. (B)

Sol. (P)



emissive power = $e\sigma T^4$
area of A & B is same, so heat energy radiated will be more for which have more value of e.

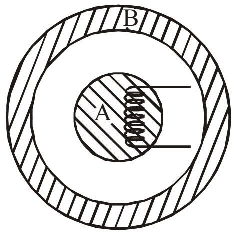
(Q)



on increasing temperature peak of graph shift ϕ left.
so $T_B > T_A$

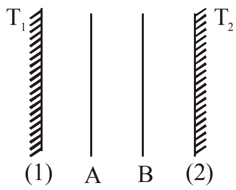
B have more area under curve so emissive power is more but heat energy of A & B cannot be predicted untiel area of A & B is known.

(R)



Direction & heat flow from A to B so $T_A > T_B$

(S)



At steady state $\frac{dQ}{dt}$ is same for all but direction of heat flow from T_1 to T_2 so $T_A > T_B$

20. Ans. (B)

Sol. $\rho_b < \rho_l \Rightarrow$ motion of ball is opposite to g_{eff}

PART-2 : CHEMISTRY — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	D	B	B	A	C	C	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	D	B	D	B	C	B	C	A	A

SOLUTION

SECTION-I

1. Ans. (C)

$$M(NaCl) = \frac{w/58.5}{V}; M(KCl) = \frac{\frac{w}{74.5}}{V}$$

M (NaCl) > M(KCl)

2. Ans. (C)

3. Ans. (D)

4. Ans. (B)

5. Ans. (B)

6. Ans. (A)

7. Ans. (C)

8. Ans. (C)

9. Ans. (B)

10. Ans. (A)

11. Ans. (B)

$\because En_2 > En_1 \Rightarrow n_2 > n_1$

$$\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2} = 6 \ \& \ \frac{n_2(n_2 - 1)}{2} = 15$$

$\Rightarrow n_2 = 6 \ \& \ n_1 = 3.$

12. Ans. (D)

13. Ans. (B)

14. Ans. (D)

15. Ans. (B)

16. Ans. (C)

17. Ans. (B)

18. Ans. (C)

19. Ans. (A)

20. Ans. (A)

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I										
	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	D	C	C	C	B	B	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	C	A	D	B	A	C	B	A

SOLUTION

SECTION-I

1. Ans. (A)

m/3	w/4	4m	3w
3	0	0	3
2	1	1	2
1	2	2	1
0	3	3	0

$({}^3C_3.{}^4C_0)^2 + ({}^3C_2.{}^4C_1)^2 + ({}^3C_1.{}^4C_2)^2 + ({}^3C_0.{}^4C_3)^2$

$1 + 144 + 324 + 16 = 485$

2. Ans. (A)

$y = 3\tan^2 x + \cot^6 x + 3$

$$\frac{\tan^2 x + \tan^2 x + \tan^2 x + \cot^6 x}{4} \ge 1$$

$y_{min} = 7$

3. Ans. (D)

$2[x] = x + \{x\}$

$[x] = 2\{x\}$

solving $x = \frac{3}{2}, 0$

4. Ans. (C)

$$f(x) = 3^k f\left(\frac{x}{3^k}\right) \Rightarrow f(2001) = 3^k f\left(\frac{2001}{3^k}\right)$$

$$k = \frac{2001}{3^k} \le 3 \Rightarrow k = 6$$

$$f(2001) = 729\left[1 - \left|\frac{2001}{729} - 2\right|\right] = 186$$

5. Ans. (C)

Let $\sin^{-1}x = \theta$

$$\Rightarrow -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\sin^{-1}\sin\left(\theta + \frac{\pi}{4}\right) = \frac{\pi}{4} + \theta$$

$$-\frac{\pi}{2} \le \frac{\pi}{4} + \theta \le \frac{\pi}{2} \Rightarrow \frac{-3\pi}{4} \le \theta \le \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} \le \theta \le \frac{\pi}{4}$$

$$-1 \le x \le \frac{1}{\sqrt{2}}$$

6. Ans. (C)

Let ω_3 be circumcirle of P,Q,R,S. Let r_i & O_i be the radius & centre respectively of circles ω_i, i = 1, 2, 3.

Now O_1 lies on radical axis of ω_2 & ω_3

and O_2 lies on radical axis of ω_1 & ω_3.

equating powers

$$O_1O_2^2 - r_2^2 = O_1O_3^2 - r_3^2$$

$$O_2O_1^2 - r_1^2 = O_2O_3^2 - r_3^3$$

subtracting

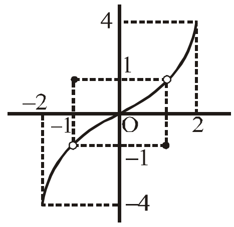
$$O_1O_3^2 - r_1^2 = O_2O_3^2 - r_2^2$$

$\Rightarrow$ Power of O_3 with respect to ω_1 & ω_2 same

$\Rightarrow$ Locus of O_3 is radical axis of ω_1 & ω_2 i.e. on $2x + 2y - 3 = 0$

$$\Rightarrow \tan^{-1}(\alpha + \beta) + \tan^{-1}(\alpha + \beta - 1) = \tan^{-1}8$$

7. Ans. (B)



$f(x) = f^{-1}(x)$ has 3 solutions.

8. Ans. (B)

Obviously one-one into.

9. Ans. (C)

Required number of ways

$= 16 \times 3 - (4 + 4 + 2) + 1$

$= 48 - 10 + 1 = 39$

(Principle of inclusion & execlusion)

10. Ans. (A)

sinx cosx + sinx + cosx

$= 1 + \sin^2 x + \cos^2 x$

$\Rightarrow$ sinx = cosx = 1 (Not possible)

Paragraph for Question 11 & 12

$$E = \frac{(x^2+y)+(y^2+x)+|(x^2+y)-(y^2+x)|}{2}$$

$$E = \begin{cases} x^2+y\ ; & x^2+y \ge y^2+x \\ y^2+x\ ; & y^2+x \ge x^2+y \end{cases}$$

$E = \max(x^2+y,\ y^2+x)$

$E \ge x^2 + y$

$E \ge y^2 + x$

$$2E \ge \left(x+\frac{1}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 - \frac{1}{2}$$

$$E \ge -\frac{1}{4}$$

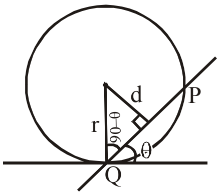
Equality holds for $x = y = -\frac{1}{2}$

11. Ans. (B)

12. Ans. (B)

Paragraph for Question 13 & 14

13. Ans. (C)



$$\cos\theta = \frac{d}{r}$$

14. Ans. (A)

Required locus is perpendicular bisector of PQ

Paragraph for Question 15 & 16

$P(x) = C(x-1)^2 + 1$

$181 = C(11-1)^2 + 1$

$180 = C100$

$$C = \frac{9}{5}$$

$$P(x) = \frac{9}{5}(x-1)^2 + 1$$

$$P(16) = \frac{9}{5} \times 15 \times 15 + 1 = 406$$

$P(\sin^{-1}x) = P(\cos^{-1}x)$

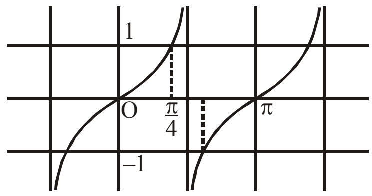
$$\Rightarrow \sin^{-1}x = \cos^{-1}x \quad \Rightarrow \quad x = \frac{1}{\sqrt{2}}$$

15. Ans. (D)

16. Ans. (B)

17. Ans. (A)

For (P,Q) $-1 \le \tan(\cos^{-1}x) \le 1$



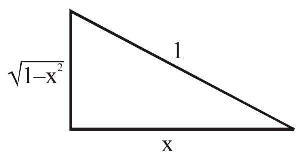
$$0 \le \cos^{-1}x \le \frac{\pi}{4} \text{ or } \frac{3\pi}{4} \le \cos^{-1}x \le \pi$$

$$\frac{1}{\sqrt{2}} \le x \le 1 \text{ or } -1 \le x \le -\frac{1}{\sqrt{2}}$$

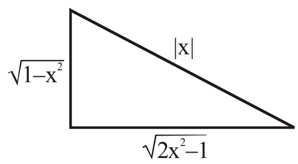
$$D_f : \left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$$

$R_f : [0, 1]$

For (R,S) $f(x) = \cos\left(\sin^{-1}\left(\tan\left(\cos^{-1}x\right)\right)\right)$



$$f(x) = \cos\left(\sin^{-1}\frac{\sqrt{1-x^2}}{x}\right)$$



$$f(x) = \frac{\sqrt{2x^2-1}}{|x|}$$

$$f(f(x)) = \sqrt{2 - \frac{1}{f^2(x)}}$$

$$=\sqrt{2-\frac{x^2}{2x^2-1}}=\sqrt{\frac{3x^2-2}{2x^2-1}}$$

Now, $\frac{(2x^2-1)}{x^2}=\frac{3x^2-2}{2x^2-1}$

$\Rightarrow$ $4x^4 - 4x^2 + 1 = 3x^4 - 2x^2$

$x^4 - 2x^2 + 1 = 0$

$x = \pm 1$

Domain $f(f(x))$

$x^2<\frac{1}{2}$ or $x^2\geq\frac{2}{3}$

$$\left(-\infty,-\frac{\sqrt{2}}{\sqrt{3}}\right]\cup\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\cup\left[\frac{\sqrt{2}}{\sqrt{3}},\infty\right)\cap D_f$$

18. Ans. (C)

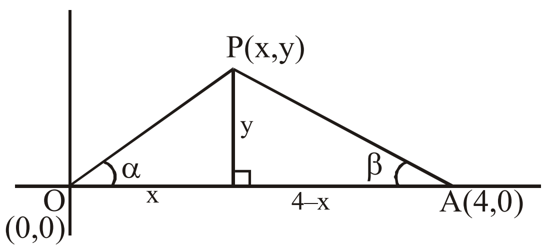
$$P(n+1)=\sum_{k=0}^{n}(-1)^{n-k}\left\{1-\frac{1}{k+1}\right\}\frac{\lfloor n+1}{\lfloor k\lfloor n+1-k}$$

$$=\sum_{k=0}^{n}(-1)^{n-k}\,{}^{n+1}C_k-\sum_{k=0}^{n}(-1)^{n-k}\frac{\lfloor n+1(n+2)}{\lfloor k+1\lfloor n+1-k}\cdot\frac{1}{n+2}$$

$$=1-\frac{1}{n+2}\sum_{k=0}^{n}(-1)^{n-k}\,{}^{n+2}C_{n+1-k}$$

$$=1-\frac{1+(-1)^n}{n+2}$$

19. Ans. (B)



$\tan\alpha\ \tan\beta = 1$

$\Rightarrow$ $\frac{y}{x}\cdot\frac{y}{4-x}=1$

$x^2 + y^2 - 4x = 0$

$x = 2 + 2\cos\theta$

$y = 2\sin\theta$

$x + y = 2 + 2(\cos\theta + \sin\theta)$

20. Ans. (A)

$a_n(n + 1) - n\ a_{n+1} = n^2(n + 1)$

$\frac{a_n}{n}-\frac{a_{n+1}}{n+1}=n$

$\Rightarrow$ $a_1-\frac{a_{n+1}}{n+1}=\left(\frac{n(n+1)}{2}\right)$

$\Rightarrow$ $50-n\left(\frac{n+1}{2}\right)=\frac{a_{n+1}}{n+1}$

Now check