Paper Code : 1001CT100116019

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : NURTURE COURSE (PHASE : I)

ANSWER KEY : PAPER-1 TEST DATE : 08-01-2017

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	B	D	B	B	A,B	B,C	A,B,D	A,B,C
	Q.	11	12	13	14	15	16				
	A.	A,D	A,B	A,B,D	A,C	A,C	A,C				
SECTION-IV	Q.	1	2	3	4						
	A.	5	4	5	Bonus						

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	A	C	B	C	B	A,C	A,B,D	B,C,D
	Q.	11	12	13	14	15	16				
	A.	B,C,D	A,D	A,B,D	D	A,B,C	A,B,D				
SECTION-IV	Q.	1	2	3	4						
	A.	6	4	3	4						

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	A	D	D	B	A	A	A,D	A,B,C,D	A,B,D
	Q.	11	12	13	14	15	16				
	A.	A,B,C,D	A,B,C,D	A,B	A,B	D	B				
SECTION-IV	Q.	1	2	3	4						
	A.	5	3	8	4						

Paper Code : 1001CT100116020

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : NURTURE COURSE (PHASE : I)

ANSWER KEY : PAPER-2 TEST DATE : 08-01-2017

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	B,C	C,D or D	A,D	A,C,D	B,C	A,C,D	A,B,C	D	B
	Q.	11	12								
	A.	A	A								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	4	6	5	5	4	9	2		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C,D	A,B,C,D	C	A,C,D	A,B,C	A	A,B,C	B,C,D	C	A
	Q.	11	12								
	A.	B	D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	8	9	9	8	6	3	2	5		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	A,D	A,C	A,C	A,C	A,C	A,B,C,D	A,B,C,D	A	A
	Q.	11	12								
	A.	C	A								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	2	6	1	0	3	1	1	5		

JEE (Main + Advanced) : NURTURE COURSE

PHASE : I

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PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. **Ans. (B)**

Sol. $I = \dfrac{P}{4\pi r^2}$

$dB = 10 \log \dfrac{I}{I_0}$

2. **Ans. (B)**

Sol. $f_1 = nf_0$ $f_2 = (n+1) f_0$

$\Rightarrow f_0 = 100$ Hz

$f_{second} = 2f_0 = 200$ Hz

3. **Ans. (B)**

Sol. At time t

$l' = (l - vt) = \lambda/4$

$\therefore$ Fundamental frequency $f_0 = \dfrac{C}{4\ell}$

$$f_0 = \frac{C}{4(\ell - vt)}$$

$$\therefore \frac{df}{dt} = \frac{-C}{4(\ell - vt)^2}(-v) = \frac{CV}{4\ell^2}$$

4. **Ans. (D)**

Sol. All collisions are elastic hence velocity will be exchanged.

$$\text{for } 0 < t < 4,\ f_1 = f_0\left[\frac{340}{340-40}\right] = f_0\left[\frac{34}{30}\right]$$

$$\text{for } 4 < t < 8,\ f_2 = f_0\left[\frac{340-40}{340}\right] = f_0\left[\frac{30}{34}\right]$$

$$\text{for } 8 < t < 12,\ f_3 = f_0\left[\frac{340+40}{340}\right] = f_0\left[\frac{38}{34}\right]$$

$$\text{for } 12 < t < 16,\ f_4 = f_0\left[\frac{340}{340+40}\right] = f_0\left[\frac{34}{38}\right]$$

5. **Ans. (B)**

Sol. $F = Y\alpha A\Delta T$

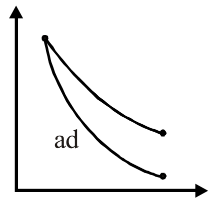
$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2L}\sqrt{\frac{T}{A\rho}}$$

$$f = \frac{1}{2L}\sqrt{\frac{Y\alpha\Delta T}{\rho}}$$

6. **Ans. (B)**

7. **Ans. (A,B)**

Sol.



8. **Ans. (B,C)**

Sol. For adiabatic process, PV^γ = constant.

$\Rightarrow$ $ln P = -\gamma\ ln V$ + constant.

$\Rightarrow$ $|\text{slope}| = \gamma$

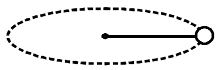
$\Rightarrow \gamma_1 > \gamma_2$

$\Rightarrow f_1 < f_2$ and $C_{V_1} \angle C_{V_2}$

$\because\ f = \dfrac{2}{\gamma = 1}$ and $C_v = \dfrac{R}{\gamma - 1}$

9. **Ans. (A,B,D)**

Sol.



(A) w = 0

10. **Ans. (A,B,C)**

11. **Ans. (A,D)**

Sol. K and m are not known.

12. Ans. (A,B)

13. Ans. (A,B,D)

Sol. $\frac{nR(T_1 - T_2)}{r-1} = \omega$

14. Ans. (A,C)

Sol. $\sqrt{\frac{\frac{4}{3}\times\frac{25}{3}\times 400}{40\times10^{-2}}} = \frac{100\times2\times10}{3\times2} = \frac{1000}{3} = v$

$400 \times (10^{-3})^{1/3} = 300 \times V^{1/3}$

$V^{1/3} = \frac{4}{30}$

$V = \frac{8}{3375}\ m^3$

15. Ans. (A,C)

Sol. $C \to A$	$Q = W + \Delta U$	$g - ve$
	-ve	-ve
$A \to B$	$\Delta U + ve$	$\Rightarrow c + ve$
$B \to C$	$\Delta U + ve$	$\Rightarrow f + ve$
$B \to C$	$W = 0$	

16. Ans. (A,C)

Sol. $\Delta U = nc_v\Delta T = -210$

$= \frac{nf}{2}R\Delta T = \frac{f}{2}(P_2V_2 - P_1V_1)$

$= \frac{f}{2}[203-160] \Rightarrow f \times 70 = 210$

$f = 3$

$\Rightarrow$ monoatomic

$Q = \Delta\omega = \text{Area} = \frac{1}{2}\times -3\times 20 = -30\ J$

SECTION-IV

1. Ans. 5

Sol. $k = 10\ \pi$

$\omega = 100\ \pi$

$S_0 = 20$ min.

$\lambda = \frac{2\pi}{10\pi} = \frac{1}{5}\ m$

$f = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50\ Hz$

$v = \frac{\omega}{k} = 10\ m/s$

$v = \sqrt{\frac{B}{\rho}} \Rightarrow v^2\rho = B \Rightarrow 100 \times 1 = 100\ Pa$

$P_0 = BkS_0$

$= 100 \times 10\pi \times 20 \times 10^{-3} = 20\ \pi$ Pa.

$\frac{\lambda}{4} = \frac{1}{20}\ m$

$\frac{p_0^2}{2\rho v} = I_{avg}$

2. Ans. 4

Sol. $14 \times 60 \times 290 = mL = m \times 580 \times 4.2$

$m = \frac{14\times60\times290}{580\times4.2} = 100\ gm$

3. Ans. 5

Sol. $f_A = \frac{340-10}{340-10+10}\times 85 = \frac{330}{4}\ Hz$

$f_B = \frac{340+10}{340+10-10}\times 85 = \frac{330}{4}\ Hz$

$f_{beat} = f_B - f_A = 5Hz$

4. Ans. 5

Sol. Given $\frac{P_{Fe}}{P_{Al}} = \frac{\sqrt{\frac{T}{\rho_{Al}}}}{\sqrt{\frac{T}{\rho_{Fe}}}} = \sqrt{\frac{7.5}{2.7}} = \frac{5}{3}$

Here fifth harmonic of Fe = third harmonic of Al wire.

Using $P_{Fe} = 5$;

$f = \frac{5}{2\times1}\sqrt{\frac{75\pi\times4}{3.14\times10^{-6}\times7.5\times10^{3}}} = 500\ Hz$

PART–2 : CHEMISTRY SOLUTION

SECTION - I

1. Ans. (B)

Sol. According to Boyle's Law

PV= K

(K = nRT)

$P = \frac{K}{V}$

log P = logK – logV

2. Ans.(C)

Total nodes = n – 1

for initial orbit = n –1 = 2 + 1 ⇒ n = 4

for final orbit = n –1 = 1 ⇒ n = 2

$\Delta E = 13.6\ Z^2 \left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)eV = 13.6 \times 1^2$

$\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = 2.55\ eV$

$\therefore \lambda = \frac{1240}{2.55}nm$

3. Ans. (A)

4. Ans. (C)

5. Ans. (B)

6. Ans. (C)

7. Ans.(B)

8. Ans. (A,C)

Sol. $Q_p = \frac{2^2}{4} = 1 < K_p$

Hence, reaction will occur in forward direction.

9. Ans. (A, B, D)

10. Ans. (B, C, D)

11. Ans. (B,C,D)

12. Ans. (A,D)

13. Ans. (A,B,D)

14. Ans. (D)

15. Ans. (A,B,C)

16. Ans. (A,B,D)

SECTION - IV

1. Ans. 6

Sol. $K_C = \frac{\left[\frac{C}{V}\right]^2}{\left[\frac{B}{V}\right]\left[\frac{A}{V}\right]^3} \Rightarrow 9 = \frac{\left[\frac{2}{V}\right]^2}{\left[\frac{2}{V}\right]\left[\frac{2}{V}\right]^3} \Rightarrow V = 6\ L$

2. Ans. 4

3. Ans. 3

4. Ans. Bonus

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (B)

x = y – 15°

tan(y–15°) = tan(y – 5°)tan(y + 5°) tan(y + 15°)

$\frac{\sin(y-15°).\cos(y+15°)}{\cos(y-15°).\sin(y+15°)} = \frac{\sin(y-5°)\sin(y+5°)}{\cos(y-5°)\cos(y+5°)}$

Apply componendo and dividendo

$\frac{\sin 2y}{\sin(-30°)} = \frac{\cos 10°}{-\cos 2y}$

sin4y = cos10°

sin4y = $\cos\frac{\pi}{18} = \sin\frac{8\pi}{18}$ ®

$4y = n\pi + (-1)^n\frac{4\pi}{9}$

$4\left(x+\frac{\pi}{12}\right) = n\pi + (-1)^n\frac{4\pi}{9}$

$4x + \frac{\pi}{3} = n\pi + (-1)^n\frac{4\pi}{9}$

n = 0 $4x + \frac{\pi}{3} = \frac{4\pi}{9} \Rightarrow x = \frac{\pi}{36} = 5°$

n = 1 $4x + \frac{\pi}{3} = \pi - \frac{4\pi}{9} \Rightarrow x = \frac{\pi}{18} = 10°$

Hence : x = 5°,10°

2. Ans. (A)

$\frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\frac{1}{8} = 2\sin\frac{A}{2}\left(\frac{1}{2}-\sin\frac{A}{2}\right)$

$\Rightarrow \sin\frac{A}{2} = \frac{1}{4}, \cos\frac{A}{2} = \frac{\sqrt{15}}{4}$

$\sin A = \frac{\sqrt{15}}{8}$

3. Ans. (D)

Put x = 1

$3^n = 243$ gives n = 5

$\left(x^2+\frac{2}{x^3}\right)^5$, term independent of x is

$$T_{r+1} = {}^5C_r\,(x)^{10-2r}\left(\frac{2}{x^3}\right)^r ; \therefore r = 2$$

$$T_3 = {}^5C_2\,(2)^2 = 40$$

4. Ans. (D)

$({}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8)$

$({}^7C_0 + {}^7C_1x^3 + {}^7C_2x^6 + {}^7C_3x^9 + {}^7C_4x^{12}) \times$

$({}^{12}C_0 + {}^{12}C_1x^4 + {}^{12}C_2x^8 + {}^{12}C_4x^{12})$

$\Rightarrow ({}^4C_0.{}^7C_4x^{12} + {}^4C_3.{}^7C_2x^{12} + {}^4C_1.{}^7C_2x^8$

$+ {}^4C_4.{}^7C_0x^8 + {}^4C_2.{}^7C_0x^{4+} + {}^4C_0.{}^7C_0)$

$\times ({}^{12}C_0 + {}^{12}C_1x^4 + {}^{12}C_2x^8 + {}^{12}C_4x^{12})$

$\Rightarrow {}^4C_0.{}^7C_4.{}^{12}C_0 + {}^4C_3.{}^7C_2.{}^{12}C_0 +$

$+ {}^4C_1.{}^7C_2.{}^{12}C_1 + {}^4C_4.{}^7C_0.{}^{12}C_1 +$

${}^4C_2.{}^7C_0.{}^{12}C_2 + {}^4C_0.{}^7C_0.{}^{12}C_3 = 1755$

5. Ans. (B)

$${}^{300}C_r\,5^{\frac{300-r}{2}}.7^{\frac{r}{5}}$$

where r = 5k (k ≥ 0)

$$\frac{300-r}{2} = \frac{300-5k}{2} = \text{integer and } k \le 60$$

Hence k = 0,2,4,.....60

Thus 31 terms are rational.

Total terms = 301 ∴ Number of irrational terms = 270

6. Ans. (A)

Let C_n denotes the number of codes that have exactly n-digits.

for n ≥ 4 a code with n digits ends with 1 or 10 or 100.

If the code that ends with 1 then the string that remains when the end digit is removed is also a code which is equalto C_{n-1} similarly C_{n-2} and C_{n-3} for code ending with 10 and 100 respectively.

$C_n = C_{n-1} + C_{n-2} + C_{n-3}$

By counting $C_1 = 2$, $C_2 = 4$ and $C_3 = 7$ and so on ∴ $C_{11} = 927$

7. Ans. (A)

For desired combination every button has two options, it can be pressed or not pressed. So for ten buttons total number of ways $= 2^{10}$ ways.

It includes the cases when all buttons are pressed and a case when none of the button is pressed.

so required combinations $= 2^{10} - 2$

8. Ans. (A,D)

$f(x) = [4^x - 2^x + 1]$

$$= \left[\left(2^x - \frac{1}{2}\right)^2 + \frac{3}{4}\right]$$

Hence domain is R

for $x \in (-\infty,2)$, $2^x \in (0,4)$ ∴ $\left(2^x - \frac{1}{2}\right) \in \left[0, \frac{49}{4}\right)$

$$\left(2^x - \frac{1}{2}\right)^2 + \frac{3}{4} \in [0,13)$$

9. Ans. (A,B,C,D)

Put x = –1 to get $\sum_{i=0}^{50} |a_i|$

i.e. $\sum_{i=0}^{50} |a_i| = \left(\frac{3}{2}\right)^5 = \frac{243}{32}$

Put x = 1 to get $\sum_{i=0}^{50} a_i$

i.e. $\sum_{i=0}^{50} a_i = \left(\frac{5}{6}\right)^5 = \frac{3125}{7776}$

10. Ans. (A,B,D)

$f(x)=\frac{1}{\log_{(|x|-1)}[x]}$ domain is $x\in(2,\infty)-\{2\}$

$g(x)=\frac{1}{\log_{[x]}|x-3|}$ domain is $x\in(2,\infty)-\{3,4\}$

11. Ans. (A,B,C,D)

Each of the given options are identical functions.

12. Ans. (A,B,C,D)

(A) y = sin2A + sin2B + sin2C

= 2sinC[cos(A–B) + cosC]

$y\le 2\sin C[1+\cos C]$ {when A = B}

To find maximum value of 2sinC + sin2C differentiate w.r.t. 'C'

2[cosC + cos2C] = 0 gives C = 60°

∴ A = B = C = 60°

maximum value of $y=\frac{3\sqrt{3}}{2}$

similarly : x = sinA + sinB + sinC

$x\le 2\cos\frac{C}{2}\left(1+\sin\frac{C}{2}\right)$

$x\le 2\cos\frac{C}{2}+\sin C$

$\frac{dx}{dc}=-\sin\frac{C}{2}+\cos C$ which gives C = 60°

maximum value of x is $\frac{3\sqrt{3}}{2}$

(B) $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$r\le 4R\left(\frac{1}{8}\right)$ [where A = 60° = B = C]

$R\ge 2r$

(C) since $R\ge 2r$

$\therefore R^2\ge 2Rr$

$R^2\ge 2.\frac{abc}{4\Delta}.\frac{\Delta}{s}$

$R^2\ge\frac{abc}{a+b+c}$

(D) r + 2R = s

$4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}+2R=\frac{a+b+c}{2}$

sinA + sinB – sinC + 2

= sinA + sinB + sinC

2sinC = 2

C = 90°

Solution for Question 13 & 14

Let first term and common difference of A.P are 'a' and 'd' respectively.

∴ First term of GP and common ratio is 'd' and 'a'.

$\frac{10}{2}[2a+9d]=155$, 2a + 9d = 31 ...(1)

and d + da = 9 ....(1)

solving (1) and (2)

a = 2, $\frac{25}{2}$ and $d=3,\frac{2}{3}$

13. Ans. (A,B)

14. Ans. (A,B)

Paragraph for Question 15 & 16

15. Ans. (D)

For x + 2y – a = 0 to intersect the circle

$x^2+y^2=4$ $\therefore \left|\frac{-a}{\sqrt{5}}\right|\le 2$

$\therefore -2\sqrt{5}<a<2\sqrt{5}$

Also : Radical axis : $S_1-S_2=0$

i.e 4x + 2y – 5 = 0

for A,B,C,D to be concyclic radical axis x + 2y – a = 0 and 12x – 6y – 41 = 0 must be concurrent.

$$\therefore \begin{vmatrix}1 & 2 & -a\\ 12 & -6 & -41\\ 4 & 2 & -5\end{vmatrix}=0$$

$\therefore a=-2$

16. Ans. (B)

Let the equation of circle passing through A,B,C,D be $S_3 : x^2 + y^2 + 2gx + 2fy + c$

$S_1 : x^2 + y^2 - 4 = 0$ and S_2 :

$$x^2 + y^2 - 4x - 2y + 1 = 0$$

$S_3 - S_1 = 0$ and $S_3 - S_2 = 0$

$2gx + 2fy + c + 4 = 0$ and $(2g + 4)x + (2f + 2)y + c - 1 = 0$

comparing with

$x + 2y + 2$ and $12x - 6y - 41 = 0$

$$\frac{2g}{1} = \frac{2f}{2} = \frac{c+4}{2} \text{ and } \frac{2g+4}{12} = \frac{2f+2}{-6} = \frac{c-1}{-41}$$

$$c = -\frac{36}{5} \quad g = -\frac{8}{10} \quad f = -\frac{16}{10}$$

SECTION-IV

1. Ans. 5

$x^2 - 4 \geq 0$

$\therefore\ x \in (-\infty, -2) \cup (2, \infty)$

Also $36 - x^2 \geq 0$

$x^2 - 36 \leq 0$

$x \in (-6,6)$

But $x^2 \neq 11$

interval right to origin is

$$\left[2, \sqrt{11}\right) \cup \left(\sqrt{11}, 6\right]$$

2. Ans. 3

(sinx + 1) (cosx + 1)

$E : \frac{1}{2}[\sin x + \cos x + 1]^2$ ...(1)

$$-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$$

$$\therefore\ E \in \left[0, \frac{1}{2}\left(\sqrt{2}+1\right)^2\right]$$

$\therefore$ Number of integers : 3

3. Ans. 8

Let $a = 5^j$ and $b = 5^k$

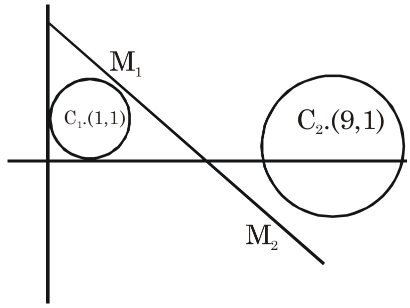
$$\log_a b = \frac{k}{j}$$

for each j number of integral multiples are at most 25 also $j \neq k$, the number of possible values of 'k' for each j is $\left[\frac{25}{j}\right] - 1$

Hence total possible values of (a,b) is

$$\sum_{j=1}^{25}\left(\left[\frac{25}{j}\right] - 1\right)$$ Hence '62'.

4. Ans. 4



$S_1 : x^2 + y^2 - 2x - 2y + 1 = 0 \quad C_1(1,1)\ r_1 = 1$

$S_2 : x^2 + y^2 - 18x - 2y + 78 = 0 \quad C_2(9,1)\ r_2 = 2$

$C_1M_1 \geq r_1$

$$\frac{|3 + 4 - \lambda|}{5} \geq 1$$

$|7 - \lambda| \geq 5$

$7 - \lambda \geq 5$ or $\lambda - 7 \geq 5$

$\lambda \leq 2$ (reject) or $\lambda \geq 12$

similarly $C_2M_2 \geq r_2$

$$\frac{|27 + 4 - \lambda|}{\sqrt{3^2 + 4^2}} \geq 2$$

$31 - \lambda \geq 10,\ \lambda \leq 21$

Hence $12 \leq \lambda \leq 21$

JEE (Main + Advanced) : NURTURE COURSE

PHASE : I

Test Type : MINOR Test Pattern : JEE-Advanced

TEST DATE : 08 - 01 - 2017

PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (B,C)

Sol. Due to wind the wavelength and speed of sound w.r.t. ground changes. The frequency & time period also remain unchanged.

2. Ans. (B, C)

Sol. $\frac{dQ}{dt} = eS \times A = ms\frac{dT}{dt}$

$\Rightarrow \frac{dT}{dt}$ is more in sprit

In steady state

$eSA = e \times \sigma(T_4 - T_0^4)$

$\Rightarrow$ T is same in both

3. Ans. (C,D or D)

4. Ans. (A,D)

Sol. $C_V = \frac{fR}{2}$

f is = 3 for monoatomic but f is 5 for diatomic at normal temperature. But f > 5 for diatomic gases.

At high temperature energy of vibration also increases taken that into account C_V increases since molecules of monoatomic gas do not vibrate, its C_V remains same.

5. Ans. (A,C,D)

6. Ans. (B,C)

Sol. $\frac{F}{A} = y\frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \left(\frac{\ell F}{r}\right)\frac{1}{A}$

$\Rightarrow U = \frac{1}{2}\left(\frac{\Delta r}{\ell}\right)\Delta\ell^2 = \frac{1}{2}\frac{\Delta r}{\ell}\left(\frac{\ell F}{r}\right)^2\frac{1}{A^2}$

$\Rightarrow U \propto \frac{1}{A}$

7. Ans. (A,C,D)

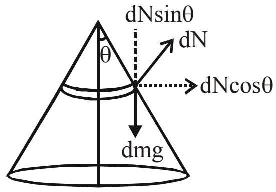
Sol. Rightward shift of junction

$= \frac{(\alpha_1 Y_1 - \alpha_2 Y_2)\ell_1 \Delta T}{Y_1 + Y_2}$

8. Ans. (A,B,C)

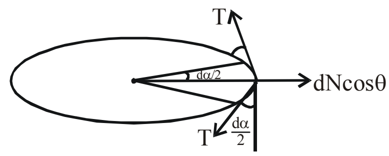
Sol. $dN\sin\theta = (dm)g$

$N\sin\theta = mg$



$\frac{N}{\sqrt{2}} = mg$

$N = \sqrt{2}mg$ ....(i)



$2T\sin\left(\frac{d\alpha}{2}\right) = dN\cos\theta$

$2T\left(\frac{d\alpha}{2}\right) = dN\cos\theta$

(1) $T(2\pi B) = \sqrt{2}mg \times \frac{1}{\sqrt{2}}$

$T = \frac{mg}{(2\pi)}$

(2) $\frac{T}{A} = Y \times \frac{\Delta\ell}{\ell}$ $\frac{mg}{2\pi A} = \frac{Y\Delta\ell}{2\pi R}$

$\Delta\ell = \frac{mgR}{Ay}$

9. Ans. (D)

Sol. (A) $C = C_v - \dfrac{R}{m-1} = \dfrac{3}{2}R - \dfrac{R}{\frac{5}{3}-1}$

$= \dfrac{3}{2}R - \dfrac{R}{2/3} = 0$

(B) PV^3 = const. $\Rightarrow C = \dfrac{3}{2}R - \dfrac{R}{3-1} = R$

10. Ans. (B)

11. Ans. (A)

Sol. $P = \dfrac{\rho}{M_W}RT$

For (A) : For AB, $P \propto V \Rightarrow T \propto V^2 \Rightarrow T \propto \rho^{-2}$

For BC, V = constant $\Rightarrow \rho$ = constant

For CA, P = constant $\Rightarrow \rho T$ = constant

For (B) : For AB, $P \propto T \Rightarrow \rho$ = constant

For BC, T = constant $\Rightarrow P \propto \rho$

For CA, P = constant $\Rightarrow \rho T$ = constant

For (C) : For AB, P = constant

$\Rightarrow \rho T$ = constant

For BC, T = constant $\Rightarrow P \propto \rho$

For CA, V = constant $\Rightarrow \rho$ = constant

For (D) : For AB, $\rho \propto T \Rightarrow P \propto T^2$

For BC, T = constant $\Rightarrow P \propto \rho$

For A, ρ = constant $\Rightarrow P \propto T$

12. Ans. (A)

Sol. (P) No medium $\Rightarrow$ Radiation

(Q) No medium $\Rightarrow$ Radiation

(R) Conduction

SECTION-IV

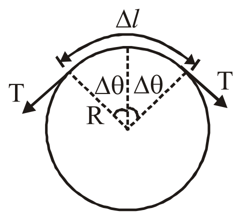
1. Ans. 1

Sol. $64 = \sigma T^4 (2\pi r l)$

$r = 10^{-5}$ m = 10 mm)

2. Ans. 4

Sol. $2T\sin\Delta\theta = \Delta m R\omega^2$



$2T\left(\dfrac{\Delta l}{2R}\right) = (\rho A \Delta l)R\omega^2$

$\dfrac{T}{A} = \rho R^2\omega^2$

$Y\dfrac{\Delta \ell}{\ell} = \rho R^2\omega^2$

$\dfrac{\Delta \ell}{\ell} = \dfrac{\rho R^2\omega^2}{Y} = \dfrac{\Delta R}{R}$ $\quad [l = 2\pi R, \dfrac{\Delta \ell}{\ell} = \dfrac{\Delta R}{R}]$

$\Delta R = \dfrac{\rho\omega^2 R^3}{Y} = \dfrac{(10^4)(10^2)(8)}{2\times 10^{11}}$

On solving

$\Delta R = 4 \times 10^{-5}$ m

3. Ans. 6

Sol. Hint : $\dfrac{1}{2}\rho A^2\omega^2 \times \dfrac{\lambda}{4}$

4. Ans. 5

Sol. $\dfrac{k_1A_1(T_1 - T)}{L_1} = \dfrac{k_2A_2(T - T_2)}{L_2}$

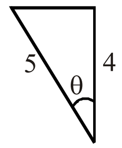
$300 - T = \left(\dfrac{L_1}{L_2}\right)\left(\dfrac{k_2}{k_1}\right)\left(\dfrac{A_2}{A_1}\right)(T - 0)$

$300 - T = 2T$

$T = 100°C$

5. Ans. 5

Sol.



$\cos\theta = \dfrac{4}{5}$

$mg = 2T\cos\theta$

$T = \dfrac{mg}{2\cos\theta}$, $v = \sqrt{\dfrac{T}{\mu}}$ = 100 m/s,

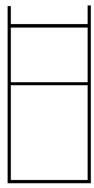
$\dfrac{3\lambda}{2} = 6m$

$\lambda = 4m$

$\upsilon = \dfrac{3}{2L}\sqrt{\dfrac{T}{\mu}}$ = 25 Hz

6. Ans. 4

Sol. $(p_0 + h_{\rho g})\, v_0 = (p_0 - h_{\rho g})v$



$(H + 8) \times 4 = (H - 8) \times 5$

$4H + 32 = 5H - 40$

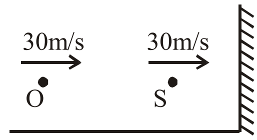
$72 = H$

7. Ans. 9

8. Ans. 2

Sol. $f_1 = f$

$f_w = \dfrac{C}{C-30}f = \dfrac{330}{300}\times 1000\text{Hz}$



$f_2 = \dfrac{C+v_0}{C}f_w = \dfrac{360}{330}\times\dfrac{330}{300}\times 1000 = 1200\text{Hz}$

Frequency Band width = $f_2 - f_1$ = 200 Hz

PART–2 : CHEMISTRY — SOLUTION

SECTION-I

1. Ans.(B,C,D)

Sol. * no of radial node = $n - l - 1 = 2 - 0 - 1 = 1$

* Probability density ψ^2

* Since ψ contain no θ or ϕ terms – probability of finding electron do not depend upon direction

* Also since at r = 0,

$$\psi = \left(\frac{1}{4\sqrt{2}}\right)\left(\frac{1}{a_0}\right)^{3/2} .(2) ;$$

ψ^2 is non zero at r = 0

* $\psi = 0$ when the term $\left(2 - \frac{r}{a_0}\right)$ vanishes

putting $2 - \frac{r}{a_0} = 0$

$\Rightarrow \quad r = 2a_0$

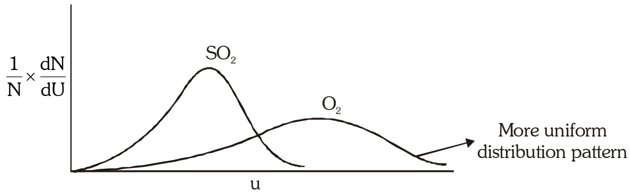
at $r = 2a_0$, $\psi = 0$: Corresponds to radial node.

2. Ans. (A,B,C,D)

3. Ans.(C)

4. Ans. (A,C,D)

Sol. (A)



(B) In every gas whether light or heavy some molecules move with very less speed & some move with high speed. Only average speed of lighter & heavier molecules can be compared.

(C) $\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 N^*}$,

$\therefore$ on changing temperature in a rigid vessel neither σ nor N changes.

(D) AKE = $\frac{3}{2}RT$

5. Ans. (A, B, C)

6. Ans. (A)

7. Ans. (A,B,C)

8. Ans. (B,C,D)

9. Ans.(C)

10. Ans. (A)

11. Ans. (B)

12. Ans. (D)

SECTION-IV

1. Ans. 8

$$\frac{1}{\lambda} = R_H z^2 \left(\frac{1}{4} - \frac{1}{9}\right) = R_H Z^2 \left(\frac{5}{4\times 9}\right)$$

$$\lambda = \frac{4\times 9}{5\times R_H Z^2} = \frac{9\times 4}{5\times 10^7 \times 9}$$

$$= \frac{4}{5}\times 10^{-7} = 80 \times 10^{-9} \text{ m}$$

= 80 nm

2. Ans. 135 [OMR Ans. 9]

let mmoles of each is = x

n-factor of FeO = 1

n-factor of $Fe_{0.80}O$ = 0.4

m_{eq} of FeO + $m_{eq\ of}$ $Fe_{0.80}O$ = eq of $KMnO_4$

x × 1 + x × 0.4 = 70 × 0.3 × 5

x = 75 mmoles

mmoles of Fe^{3+} produced = 75 + 75 × 0.8

= 135 mmoles

3. Ans. 9

Sol. $\underset{5-x}{A(g)} \rightleftharpoons \underset{x+y}{B(g)} + \underset{2x-y}{2C(g)}$

$\underset{2x-y}{C(g)} \rightleftharpoons \underset{x+y}{B(g)} + \underset{y}{D(g)}$

2x – y = 3 ....(i)

5 – x + x + y + 2x + y = 12 .....(ii)

5 + y + 2x = 10

–y 2x = 3

3 – 2y = 7

4. Ans. 8

5. Ans. 6

6. Ans. 3

7. Ans. 2

8. Ans. 5

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (B,C)

Domain of g(x) and h(x) are $(0, \infty)$

and g(x) = h(x).

2. Ans. (A,D)

$f(x) = [x]^2 + [x] - 22 = ([x] - 1)([x] + 2)$

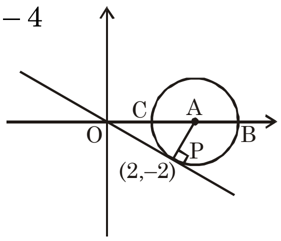
$f(x) = 0 \Rightarrow [x] = 1, -2 \Rightarrow x \in [-2,-1) \cup [1,2]$

$f(x) \le 0 \Rightarrow [x] \in [-2,1] \Rightarrow x \in [-2,2)$

3. Ans. (A,C)

Equation of AP is $y = x - 4$

$\Rightarrow$ A(4, 0)



radius $= \sqrt{8} = 2\sqrt{2}$

$B(4 + 2\sqrt{2}, 0)$

greatest value of $\alpha = 4 + 2\sqrt{2}$

least value of $\alpha = 4 - 2\sqrt{2}$

4. Ans. (A,C)

$S_1 : x^2 + y^2 - 9 = 0$

$S_2 = x^2 + y^2 - 2x + 4y + 1 = 0$

$S_1 - S_2 = 2x - 4y - 10 = 0$

$x - 2y - 5 = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & -5 \\ 2 & 1 & 1 \\ 3 & k & -4 \end{vmatrix} = 0$$

$k = -1$

radical centre is $\left(\frac{3}{5}, -\frac{11}{5}\right)$

5. Ans. (A,C)

Girls and boys sit alternatly in 8!7! (option A)

for C option :

first arrange boys in 7! boys,

it creates 6 gaps between boys

Let these gaps are 2ℓ, 2m, 2n, 2p, 2q, 2r.

$2\ell + 2m + 2n + 2p + 2q + 2r = 8$

$\ell + m + n + p + q + r = 4$

non negetive integral solutions

${}^{4+5}C_5 = {}^{9}C_4$

Total arrangements $= 7!8!.{}^{9}C_4$.

6. Ans. (A,C)

$$f(x) = 1 - \left(\cos^2\left(x + \frac{\pi}{3}\right) - \sin^2 x\right) + \left(\cos(x)\cos\left(x + \frac{\pi}{3}\right)\right)$$

$$f(x) = \frac{5}{4}$$

$$f\left(\frac{\pi}{8}\right) = \frac{5}{4}$$

7. Ans. (A,B,C,D)

$a\cos x - \cos 2x = 2a - 7$

$a\cos x - (2\cos^2 x - 1) = 2a - 7$

$a\cos x - 2\cos^2 x = 2a - 8$

$2\cos^2 x - a\cos x + 2a - 8 = 0$

$$\cos x = \frac{a \pm |a-8|}{4} = \frac{a-4}{2} \text{ or } 2 \text{ if } a \ge 8$$

$$-1 \le \cos x \le 1 \Rightarrow -1 \le \frac{a-4}{2} \le 1$$

$\Rightarrow \quad 2 \le a \le 6$

8. Ans. (A,B,C,D)

$N = 3^{400}$

$= (9)^{200}$

$= (10 - 1)^{200}$

$= {}^{200}C_0 10^{200} - {}^{200}C_1 10^{199} + ... {}^{200}C_{197} 10^3$

$+ {}^{200}C_{198} 10^2 - {}^{200}C_{199} 10 + {}^{200}C_{200}$

$= 10^3(\lambda) + {}^{200}C_{198} 100 - (200)...10 + 1$

last digit of N is 1

last two digits are 01

last three digits are 001.

$N = (81)^{100}$

$= (80 + 1)^{100}$

$= {}^{100}C_0(80)^{100} + {}^{100}C_1(80)^{99} + ... + {}^{100}C_{99} 80 + 1$

$= 80\lambda + 1$

if N divided by 80 then remainder is 1.

9. Ans. (A)

$(1 - x + x^2)^{30}$

$= a_0 + a_1x + a_2x^2 + ... + a_{60}x^{60}$ ......(1)

$x \to -x$

$(1 + x + x^2)^{30} = a_0 - a_1x + a_2x^2 + ... + a_{60}x^{60}$

$$x \to \frac{1}{x}$$

$(1 + x + x^2)^{30}$

$= a_0x^{60} - a_1x^{59} + ... + a_{60}$ ......(2)

Multiply (1) and (2)

$(1 + x^2 + x^4)^{30}$

$= a_0 + a_1x^2 + a_2x^4 + ... + a_{60}x^{120}$ .....(3)

(P) $\left(a_0^2 - a_1^2 + a_2^2 - a_3^2 + ...\right)$

= coff. of x^{60} in $(1 + x^2 + x^4)^{30} = a_{30}$

$\left(\frac{a_0^2 - a_1^2 + a_2^2 - a_3^2 + ...}{a_{30}}\right) = 1$

(Q) Put x = 1 & –1 (1) add

$2(a_0 + a_2 + ...) = 1 + 3^{30}$

$\frac{4(a_0 + a_2 + ...)}{3^{30} + 1} = 2$

(R) Put x = 1 & –1 in (1) and subtract

$2(a_1 + a_3 + ...) = 1 - 3^{30}$

(S) $a_0a_1 - a_1a_2 +$ .... is coeff. of x^{61} in $(1 + x^2 + x^4)^{30} = 0$

10. Ans. (A)

(P) $f(x) = \frac{x^2 - 3x - 4}{x^2 - 3x + 4} \in \left[-\frac{25}{7}, 1\right)$

(Q) $g(x) = x^2 - (b + 1)x + b - 1$

(1) $D \geq 0$

(2) $-\frac{b}{2a} > -1$

(3) $f(-1) > 0$

(R) $g(x) > -2$

$x^2 - (b + 1)x + b + 1 > 0$

$D < 0$

(S) $|f(x)| \leq 1$

$|x^2 - 3x - 4| \leq x^2 - 3x + 4$

$\Rightarrow 2x^2 - 6x \geq 0$

$x \in (-\infty, 0] \cup [3, \infty)$

11. Ans. (C)

$f(x) = \left[\sqrt{x-2} + \sqrt{4-x}\right]$

$\sqrt{x-2} + \sqrt{4-x} \in [\sqrt{2}, 2]$

$f(x) = \{1, 2\}$ ......(1)

$g(x) - 1 = \text{sgn}(x)$ ......(2)

$Dg(x) = 2, -2, 1, -1$

Solution of equation (2) is x = 1 only

Number of solution $f(x) = g(x)$ is 0

$g(x) = \left\{2, \frac{1}{4}\right\}$

12. Ans. (A)

$(1 + x)^{10} = {}^{10}C_0 + {}^{10}C_1x + ... + {}^{10}C_{10}x^{10}$

(P) ${}^{10}C_1 + 2{}^{10}C_2 + ... + 10{}^{10}C_{10} = 10.2^{10}$

(Differentiate w.r.t. x and put x = 1)

(Q) ${}^{10}C_0 + \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{3} + ... + \frac{{}^{10}C_{10}}{11} = \frac{2^{11} - 1}{11}$

(Integrate w.r.t. x from x = 0 to x = 1)

(R) ${}^{10}C_0{}^{10}C_{10} + {}^{10}C_1{}^{10}C_9 + ... + {}^{10}C_{10}{}^{10}C_0$

= coeff. of x^{10} in $(1 + x)^{20} = {}^{20}C_{10}$

(S) $S = 1({}^{10}C_0)^2 + 3({}^{10}C_1)^2 + ... + 21({}^{10}C_{10})^2$

$S = 21({}^{10}C_{10})^2 + ... + 1.({}^{10}C_0)^2$

$S = \frac{22}{2}\left(\left({}^{10}C_0\right)^2 + \left({}^{10}C_1\right)^2 + ... + \left({}^{10}C_{10}\right)^2\right)$

$= 11.{}^{20}C_{10}$

SECTION-IV

1. Ans. 2

If N_1 and N_2 are two integers in order that the product N_1N_2 will have 0, 1, 5, or 6 as its last digit the following possiblitities

N_1	N_2
0	0 to 9
1	0,1,5,6
2	0,3,5,8
3	0,2,5,7
4	0,4,5,9
5	0 to 9
6	0,1,5,6
7	0,3,5,8
8	0,2,5,7
9	0,4,5,9

N is 52

2. **Ans. 6**

$^{n}C_{r} + {}^{n}C_{r-1} + \ldots\ldots + {}^{2n}C_{r-1} = {}^{2n+1}C_{r^2-132}$

$^{2n+1}C_{r} = {}^{2n+1}C_{r^2-132}$

r = 12

$\Rightarrow$ n $\geq$ 12

3. **Ans. 1**

$(1 + x + x^2 + \ldots\ldots + x^9)^{-1}$

$= \frac{1-x}{1-x^{10}} = (1-x)(1-x^{10})^{-1}$

cofficient of $x^{401} = -1$

4. **Ans. 0**

$(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots.. + a_{2n}x^{2n}$

$(1 + x)^n(1 - x + x^2)^n$

$= (1 + x)^n(a_0 + a_1x + \ldots\ldots. + a_{2n}x^{2n})$

$(1 + x^3)^n = (C_0 + C_1x + C_2x^2 + \ldots\ldots + C_2x^n)$

$(a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{2n}x^{2n})$

$\sum_{r=0}^{n} {}^{n}C_{r}.a_{2n-r}$ = coff. of x^{2n} in $(1 + x^3)^n$

Since $n \neq 3k$

Therefore there is no term containing x^{2n} in $(1 + x^3)^n$.

5. **Ans. 3**

Let $2^{2x} = t$

$2^{2x} + 64^{\frac{x-2}{3}} - \frac{72+2^{2x}}{2} \geq 0$

$2^{2x} + 2^{2x-4} - \frac{72+2^{2x}}{2} \geq 0$

$t + \frac{t}{16} - \frac{72+t}{2} \geq 0$

$16t + t - 576 - 8t \geq 0$

$9t \geq 576$

$t \geq 64$

$2^{2x} \geq 2^6$

$x \geq 3$

$x \in [3,\infty)$

6. **Ans. 1**

$f(x) = \ell n\left(\frac{x}{\{x\}}\right)$

$sgn(f(x)) = 1$

$\Rightarrow$ $f(x) > 0$

$\ell n\left(\frac{x}{\{x\}}\right) > 0$

$\frac{x}{\{x\}} > 1$

$x > \{x\}$

$x - \{x\} > 0 \Rightarrow [x] > 0$

$\Rightarrow$ $[x] \geq 1$

$\Rightarrow$ $x \in (1, \infty)$, $x \neq$ Integer

$\Rightarrow$ p = 1

7. **Ans. 1**

$\log_{3/4}(x^2 + x + 1) = \log_{\frac{3}{4}}\left[\frac{3}{4},\infty\right) = (-\infty,1]$

8. **Ans. 5**

$\sin x \neq \frac{2(p-1)}{p^2-3p+2} = \frac{2}{p-2}, p \neq 1$

$\left|\frac{2}{p-2}\right| > 1 \Rightarrow p \in (0,4) - \{1\}$

$p \in (0,1) \cup (1,4)$