JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Unit Test # 04, 05 & 06

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	C	A	D	C	B	C	B	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	C	A	B	A	A	A	A	B
SECTION-II	Q.	1	2	3	4	5					
	A.	5	3.39	0.40	0.25	1.40					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	B	C	B	C	C	D	C	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	B	A	D	C	B	A	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	3	4	7	8	4					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	C	C	D	A	B	A	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	C	A	A	C	A	B	A	C
SECTION-II	Q.	1	2	3	4	5					
	A.	6	0.50	30.60	0	2.41 or 2.42					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

2. If current in all wire are same, then magnetic field at centre will be zero.

$$B = \frac{\mu_0 2I_0}{2\pi a}$$

4. M = iA

$$i = \frac{dq}{T} = \frac{\lambda dx}{2\pi}\omega$$

$A = \pi x^2$

$$dM = \frac{\lambda dx\omega}{2\pi}\pi x^2 \Rightarrow \quad M = \int_0^{\ell}\frac{\lambda\omega x^2}{2}dx$$

$$M = \frac{\lambda\omega\ell^3}{6}$$

5. $X = \frac{C}{T}$

7. At t = 0, no current across inductor

$$\therefore \quad i_{(t=0)} = \frac{100}{R_2} \Rightarrow 20 = \frac{100}{R_2} \Rightarrow R_2 = 5\Omega$$

At t = ∞, current across battery

$$= \quad i_{(t=\infty)} = \frac{100}{\left(\frac{R_1R_2}{R_1+R_2}\right)} = 40A$$

$$\therefore \quad \frac{5R_1}{5+R_1} = \frac{5}{2} \Rightarrow 10R_1 = 25 + 5R_1$$

$\therefore \quad R_1 = 5\Omega$

8. $\phi = \frac{5\times10^{-4}}{10}\times500 = 2.5\times10^{-2}\,\text{wb}$

$L = \frac{\phi}{i} = 5\times10^{-3}\,\text{H}$

9. ϕ = constant

$\mu_0 n^2 A\ell = L$

$\frac{\mu_0 N^2 A}{\ell} = L$

Li = constant

$\frac{\mu_0 N^2 A}{L_0} i_0 = \frac{\mu_0 N^2 A i_1}{L_1}$

$i_0 = \frac{L_1 i_0}{L_0}$

10. Let $q = q_0 \sin \omega t$

$I = \frac{dq}{dt} = q_0\omega\cos\omega t$

$\frac{dI}{dt} = \frac{d^2q}{dt^2} = -q_0\omega^2 \sin\omega t$

$\frac{dI}{dt}$ maximum at $\sin \omega t = -1$

$\left(\frac{dI}{dt}\right)_{max} = q_0\omega^2 \qquad \omega = \frac{1}{\sqrt{LC}}$

11. Instantaneous sum of potential difference

$V_R = V_L + V_C$

14. 

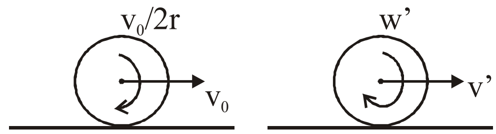
α is maximum when rod is horizontal.

$$\alpha = \frac{\tau}{I} = \frac{mg\times\frac{L}{2} + mg\times L}{\frac{m\ell^2}{3} + m\ell^2}$$

$$\alpha = \frac{9g}{8L}$$

16. Since the rod will rotate

$\therefore$ velocity will change direction atleast

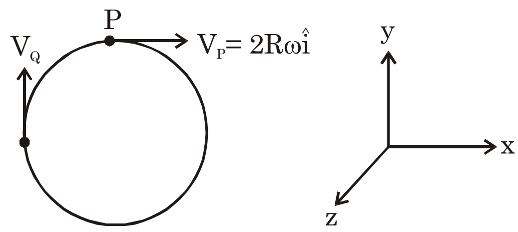
17. 

Angular momentum can be conserved about lowest point

$$mv_0r + \frac{2}{5}mr^2\frac{v_0}{2r} = mv'r + \frac{2}{5}mr^2\frac{v'}{r}$$

$$\frac{10mv_0r + 2Mv_0r}{10} = \frac{7mv'r}{5}$$

$$v' = \frac{6v_0}{7}$$

18. 

$V_Q = R\omega\hat{j} + R\omega\hat{i}$

$\therefore \frac{|V_P|}{|V_Q|} = \frac{\sqrt{2}}{1}$

19. $A = \frac{F_0/m}{\sqrt{\omega^2 - r^2}}$

20. $\omega' = 0.8\omega = \sqrt{\omega^2 - r^2}$

$r = 0.6\,\omega$

$A = A_0\,e^{-rT}$

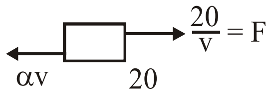
$= A_0 e^{-2\pi\frac{r}{\omega'}}$

$= A_0 e^{-1.5\pi}$

SECTION-II

1. Ans. 5

$\frac{20}{v} = \alpha v$

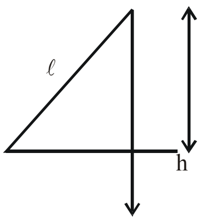


2. Ans. 3.39

$mg\ell\ (1 - \cos\theta) = \frac{1}{2}mv^2$

$mgl\ (1 - \cos\theta) = \frac{1}{2}mv^2_{max}$

$v_{max} = \sqrt{2g\ell(1-\cos\theta)}$



$= \sqrt{2\times10\times2\times\left(1-\frac{\sqrt{2}}{2}\right)}$

$= \sqrt{40\times.3} = \sqrt{12} = 3.4$

3. Ans. 0.40

$\vec{F} = q(\bar{v}\times\bar{B}) \Rightarrow \vec{F} = -e(\bar{v}\times\bar{B})$

$\bar{v} = 5\times10^5\hat{j}(m/s)$

$\bar{a} = \frac{\vec{F}}{m}$

4. Ans. 0.25

Direction of impulse $= \left(\hat{j}/2\right) - \hat{i}$

$\Rightarrow$ Unit vector along impulse $= \frac{\hat{j}-2\hat{i}}{\sqrt{5}}$

$\Rightarrow$ v_a = velocity of approach

$= -\hat{i}.\left(\frac{\hat{j}-2\hat{i}}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}}$

$\Rightarrow$ v_S = velocity of separation

$= \frac{\hat{j}}{2}.\left(\frac{\hat{j}-2\hat{i}}{\sqrt{5}}\right) = \frac{1}{2\sqrt{5}}$ $\Rightarrow e = v_S/v_a = \frac{1}{4}$

5. Ans. 1.40

$\frac{f_{max}}{\Delta f_{\text{half of max power}}}$ = Quality factor

PART-2 : CHEMISTRY

SECTION-I

1. M_{avg} always lies between mol. mass of species having highest and lowest mol. mass.

$SO_3 \rightarrow SO_2 + \frac{1}{2}O_2$	$32 < M_{avg} < 80$
$N_2O_4 \rightleftharpoons 2NO_2$	$46 < M_{avg} < 92$
$Cl_2 \rightleftharpoons 2Cl$	$35.5 < M_{avg} < 71$
$2NH_3 \rightleftharpoons N_2 + 3H_2$	$2 < M_{avg} < 28$]

2. $b = 4\ V_m \Rightarrow 4 \times VN_0$

4. ${}_ZX \longrightarrow {}^{12}_{6}C + {}^{0}_{1}e$

A = 12, Z = 7

7. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

t = 0	1 mole		
t=eq	(1–α)	α	α

$d = \frac{Mass}{Vol} = \frac{1\times M}{\frac{nRT}{P}}$

$d = \frac{M\times P}{(1+\alpha)\times RT}$

$\Rightarrow \alpha = \frac{PM}{dRT} - 1$

SECTION-II

1. Ans. 3

Eq. of MnO_4^- = Eq of FeC_2O_4

$5 \times nKMnO_4 = 3 \times 1$

$nKMnO_4 = \dfrac{3}{5}$

2. Ans. 4

3. Ans. 7

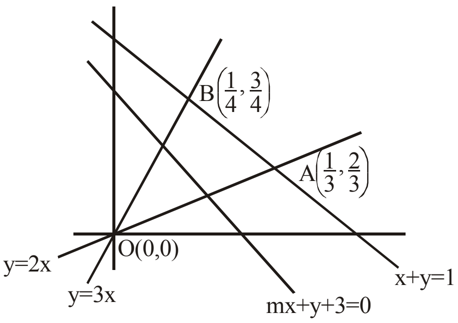
4. Ans. 8

5. Ans. 4

PART-3 : MATHEMATICS

SECTION-I

1.



Sides of triangle are y = 2x; y = 3x & x + y = 1

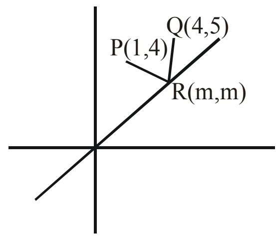
let line mx + y + 3 = 0 divide the sides OA & OB in ratio k : 1 & λ : 1 respectively, then

$$k = -\left(\frac{3}{\frac{m}{3}+\frac{2}{3}+3}\right) > 0 \quad \Rightarrow \quad m < -11 \quad ......(1)$$

$$\lambda = -\left(\frac{3}{\frac{m}{4}+\frac{3}{4}+3}\right) > 0 \quad \Rightarrow \quad m < -15 \quad ....(2)$$

by (1) & (2), $m < -15$

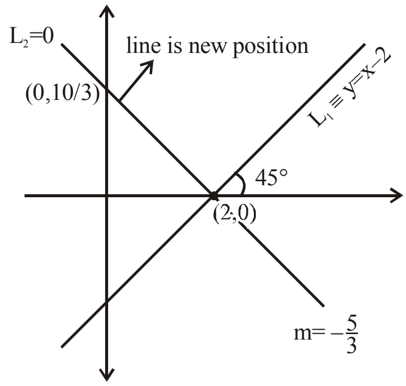
2.



Take reflection of Q in the line y = x. Let it be Q' ≡ (5,4) so, the equation of line PQ' is y = 4 Now, R must be the point of intersection of line PQ' & y = x.

$\therefore$ m = 4

3.



Equation of line in new position

$$y - 0 = -\frac{5}{3}(x-2) \Rightarrow \quad 3y = -5x + 10$$

$\Rightarrow \quad 5x + 3y = 10$

Area of triangle made by line with co-ordinate

axes $= \dfrac{1}{2}(2)\left(\dfrac{10}{3}\right) = \dfrac{10}{3}$ square units

4. $$2p = \left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{9p^2} = \frac{2}{8p^2}$$

$\dfrac{1}{a^2}, \dfrac{1}{8p^2}, \dfrac{1}{b^2}$ are in A.P.

5. Let line be $y - 8 = m(x - 6)$

$mx - y + 8 - 6m = 0$

for tangency $\frac{|3m-2+8-6m|}{\sqrt{1+m^2}} = 3$

$\Rightarrow \quad 9(m-2)^2 = 9(m^2+1)$

$\Rightarrow \quad m = \frac{3}{4}$, not defined ($\infty$)

Equation of tangent will be $x = 6$

& $\quad 3x - 4y + 14 = 0$

6. Let P(h,k) be mid point of one such chord. Equation of chord AB in terms of middle point is $T = S_1$.

$xh + ky - 2(x + h) - 4(y + k) + 11 = h^2 + k^2 - 4h - 8k + 11$.

it passes through (0,0)

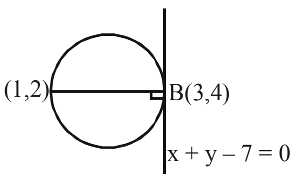
$\Rightarrow x = 0, y = 0$

$-2h - 4k = h^2 + k^2 - 4h - 8k$

$x^2 + y^2 - 2x - 4y = 0$

7. $r = \sqrt{S_1} \Rightarrow m^2 = n^2$

8.



For smallest circle AB is diameter

$(x - 1)(x - 3) + (y - 2)(y - 4) = 0$

$x^2 + y^2 - 4x - 6y + 11 = 0$

$\therefore\ g = -2,\ f = -3,\ c = 11$

9. The natural numbers in r^{th} group is r^3

$\Rightarrow$ Total number of natural numbers in the first (n – 1) groups $= \sum_{1}^{n-1} r^3 = \frac{(n-1)^2 n^2}{4}$

The n^{th} group, there for consists of the numbers

$\frac{1}{4}(n-1)^2 n^2, \frac{1}{4}(n-1)^2 n^2 + 2, \ldots, \frac{1}{4}(n-1)^2 n^2 + n^3$

Required sum $= n^3 \frac{1}{4}(n-1)^2 n^2 + \frac{1}{2} n^3 (n^3 + 1)$

$= \frac{n^3}{4}(n^4 + n^2 + 2)$

10. x, xr, xr^2 & $4y = x + 3z$

$\Rightarrow \quad 4xr = x + 3xr^2$

$\Rightarrow \quad 3r^2 - 4r + 1 = 0$

$\therefore \quad r = \frac{1}{3}$

11. $\int \frac{(x+1)(x^2+1+x)}{x(1+x^2)} dx$

$= \int \frac{(x+1)}{x} dx + \int \frac{x+1}{x^2+1} dx$

$= \int dx + \int \frac{1}{x} dx + \frac{1}{2}\int \frac{2x}{x^2+1} dx + \int \frac{dx}{x^2+1}$

$= x + \ell n x + \frac{1}{2}\ell n(x^2+1) + \tan^{-1} x + C$

12. $\int \frac{\tan^{-1} x . \cot^{-1} x}{1+x^2} dx$

put $\tan^{-1} x = t$

$\frac{1}{1+x^2} dt = dt$

$\int t.\left(\frac{\pi}{2} - t\right) dt = \frac{\pi}{4} t^2 - \frac{t^3}{3} + C$

$= \frac{\pi}{4}(\tan^{-1} x)^2 - \frac{(\tan^{-1} x)^3}{3} + C$

13. $I_1 + I_2 = \int\left(\frac{\tan x}{1-\cot x} + \frac{\cot x}{1-\tan x}\right)dx$

$= \int\left(\frac{\tan^2 x}{\tan x - 1} - \frac{1}{\tan x(\tan x - 1)}\right)dx$

$= \int \frac{\tan^2 x + \tan x + 1}{\tan x}dx$

$= \int\left(\frac{1}{\sin x \cos x} + 1\right)dx = 2\int \text{cosec}2x\,dx + \int dx$

$= x + \ell n\left|\text{cosec}2x - \cot 2x\right| + C$

14. $\because$ {x} is periodic with period 1

$\therefore\ I = 2015\int_0^1 \cos(\pi t)dt = 0$

15. Apply king property & add

$2I = \int_0^{\pi}\left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2}\right)(\ell n \sin x)dx$

$2I = 2\int_0^{\pi}(\ell n \sin x)dx$

$I = -\frac{\pi}{2}\ell n 2$

16. $f(x)\,f(1-x) = 1 \Rightarrow f(1) = 1\ \{\because f(0) = 1\}$

$f'(x) = f'(1-x) \Rightarrow f(x) + f(1-x) = 2$

$\Rightarrow f(x) + \frac{1}{f(x)} = 2$

$\Rightarrow f(x) = 1$

17. $f(1) = 3 \Rightarrow \beta - \alpha = 3$

$f'(1) = -1 \Rightarrow 2\alpha = 3$

$\Rightarrow \alpha = \frac{3}{2}\ \&\ \beta = \frac{9}{2}$

18. Apply L.M.V.T to $f(x)$ in [1,3]

$f'(x) = \frac{f(3) - f(1)}{3-1}$

$\Rightarrow \frac{a^2 - a}{2} \le 6$

$\Rightarrow -3 \le a \le 4$

19. $f(x^2 - x) > f(2) \quad \Rightarrow \quad x^2 - x < 2$

$\Rightarrow (x-2)(x+1) < 0$

or $x \in (-1,2)$

20. $S = 4\pi r^2 \Rightarrow \frac{ds}{dt} = 8\pi r\frac{dr}{dt} = 16$

$\Rightarrow \frac{dr}{dt} = \frac{2}{\pi r}$

Now, $v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi r^2 \times \frac{2}{\pi r} = 8r$

SECTION-II

1. **Ans. 6**

Let $C_1 \equiv (5, 6)$: $C_2 \equiv (2, 3)$ and $r_1 = r_2 = r$ is radius of circles then for orthogonal circle.

$r_1^2 + r_2^2 = C_1C_2^2$

$\Rightarrow 2r^2 = 18$

$\Rightarrow r = 3$

2. **Ans. 0.50**

$$\sum_{n=2}^{\infty}\frac{1}{t_n}=\sum\frac{2}{n(n-1)(n+1)}$$

$$=\sum\left(\frac{(n+1)-(n-1)}{n(n-1)(n+1)}\right)$$

$$=\sum\left(\frac{1}{n(n-1)}-\frac{1}{(n)(n+1)}\right)$$

$$=\left(\frac{1}{2.1}-\frac{1}{2.3}+\frac{1}{3.2}-\frac{1}{3.4}+....\right)=\frac{1}{2}$$

3. **Ans. 30.60**

$S_n = \quad 1 + 5 + 11 + 19 + 29 + ... + T_n$

$S_n = \qquad 1 + 5 + 11 + 19 + ... + T_n$

$- \quad - \quad - \quad - \quad - \quad -$

____

$0 = 1 + \{4 + 6 + 8 + 10 + ... (n-1) \text{ terms}\} - T_n$

$$\Rightarrow T_n = 1+\frac{n-1}{2}\{8+(n-2)2\} = n^2+n-1$$

$$\Rightarrow S_n = \Sigma T_n = \frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}-1$$

$\Rightarrow \lambda = 3060$

$$\Rightarrow \frac{\lambda}{100} = 30.60$$

4. **Ans. 0**

$$\int\frac{x^4-(x^4-1)}{x^4(x^2+1)}dx$$

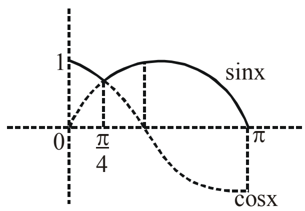
$$=\int\left(\frac{1}{1+x^2}-\frac{1}{x^2}+\frac{1}{x^4}\right)dx$$

$$=\tan^{-1}x+\frac{1}{x}-\frac{1}{3x^3}+C$$

$$\Rightarrow f(x)=\tan^{-1}x+\frac{1}{x}-\frac{1}{x^3}$$

which is an odd function.

5. **Ans. 2.41 or 2.42**



$$\Rightarrow I = \int_0^{\pi/4}\cos x\,dx + \int_{\pi/4}^{\pi}\sin x\,dx$$