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Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2020 - 2021)

JEE(Main)
AIOT
07-03-2021

JEE(Main+Advanced) : ENTHUSIAST & LEADER COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	B	B	A	D	D	A	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	A	B	B	D	A	C	C	D
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	12.50	1.00	4.00	1.00	0.36	64.00	82.80	1.84 to 1.86	500.00	6.28 to 6.29

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	C	A	B	A	A	B	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	A	A	C	C	B	B	C	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	255.00	8.00	6.00	9.00	5.00	25.00	5.00	6.00	201.66 TO201.67	8.00

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	A	A	D	A	B	A	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	B	A	B	A	C	B	B	C	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	7.00	4.00	6.00	0.50	3.25	6.00	27.00	3.25	19.00	7.00

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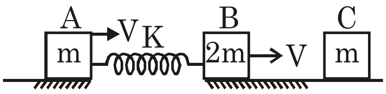
PART-1 : PHYSICS SOLUTION

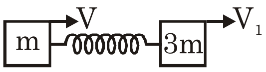
SECTION-I

1. Ans. (A)

Sol.



before collision



Just after collision

$$v_{CM} = \frac{mv + 2mv}{4m} = \frac{3v}{4}$$

for block 'A'

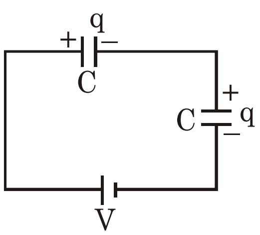
$$-\frac{v}{4} \le v_{A/CM} \le \frac{v}{4}$$

$$\frac{v}{2} \le v_A \le \frac{v}{4} + \frac{3v}{4}$$

$$(v_A)_{min} = (v_A)_{min} = \frac{v}{2} = 5m/s$$

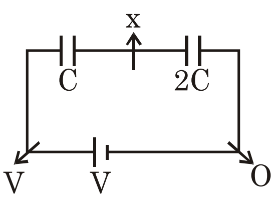
2. Ans. (B)

Sol. For position---(1)



$$q = \frac{C}{2}v$$

For position---(2)



$$(x - V)C + (x - 0)2C = -q = -\frac{CV}{2}$$

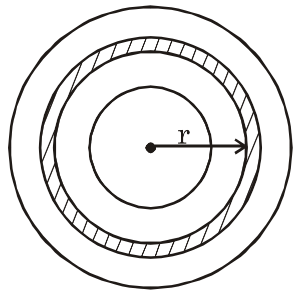
$$3x - V = -\frac{V}{2}$$

$$x = \frac{V}{6}$$

$$q_{2C} = 2C(x - 0) = \frac{CV}{3}$$

3. Ans. (B)

Sol.



Using guass law

$$\int \vec{E}.\vec{ds} = \frac{q_{in}}{\epsilon_0} \text{......(1)}$$

$$q_{in} = q + \int_{R_1}^{r} \frac{b}{r} 4\pi r^2 dr$$

$$= q + 4\pi b \frac{(r^2 - R_1^2)}{2}$$

from equation -1

$$E\ 4\pi r^2 = \frac{1}{\epsilon_0}\left(q + \frac{4\pi b(r^2 - R_1^2)}{2}\right)$$

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{(q - 2\pi b R_1^2)}{r^2} + 2\pi b\right)$$

as $\vec{E}$ is constant, therefore

$$q - 2\pi b R_1^2 = 0$$

$$b = \frac{q}{2\pi R_1^2} = \frac{16 \times 10^{-6}}{2\pi \times \frac{4}{\pi} \times 10^{-6}} = 2$$

4. Ans. (B)

Sol. Time of flight $= \frac{2u}{g} = \frac{50}{10} = 5\,\text{sec}$

$$(\vec{r}_f)_{football} = (2\hat{i} + 5\hat{j}) \times 5 = 10\hat{i} + 25\hat{j}$$

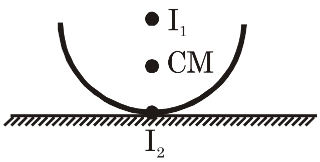
$$(\vec{r}_f)_{player} = (5\hat{i} + 8\hat{j} + 2\hat{i} + 4\hat{j} + 6\hat{j}) + (2\hat{i} + 3\hat{j})$$

$$= 9\hat{i} + 21\hat{j}$$

distance $= \sqrt{1^2 + 4^2} = \sqrt{17}$

5. Ans. (A)

Sol. $r_{CM} = \frac{R}{2}$



$$I_1 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$I_2 = I_{CM} + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$W = \Delta K$

$$mg\frac{R}{2} = \frac{1}{2}\frac{2}{3}mR^2\omega^2 \Rightarrow \omega^2 = \frac{3g}{2R}$$

using $F = ma_{CM}$

$$N - mg = m\frac{\omega^2 R}{2}$$

$$N = mg + \frac{3mg}{4} = \frac{7mg}{4}$$

6. Ans. (D)

Sol. $v_{wave} = \frac{30 \times 10}{60} = 5\,m/s$

when boat is moving, suppose n waves are striking per minute

$$n = \frac{(v_{rel} \times time)}{\lambda} = \frac{10 \times 60}{10} = 60$$

7. Ans. (D)

Sol. $\frac{R}{x} = \frac{16}{100 - x}$

$$R = \frac{16x}{100 - x}$$

$\log R = \log 16 + \log x - \log (100 - x)$

$$\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta x}{100 - x}$$

$$\frac{\Delta R}{R} = \frac{0.1}{25} + \frac{0.1}{75}$$

$$\%\text{ error} = 0.4 + \frac{0.4}{3} = \frac{1.6}{3} = 0.53$$

8. Ans. (A)

Sol. Initially no. of active nuclei of $P_{15}^{32} = N_1$

no. of active nuclie of $P_{15}^{33} = N_2$

$$\frac{\lambda_1 N_1}{\lambda_2 N_2} = 9 \quad .....(1)$$

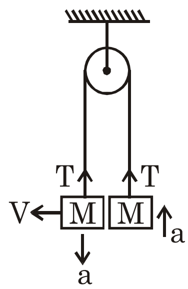
finally $\frac{\lambda_1 N_1 e^{-\lambda_1 t}}{\lambda_2 N_2 e^{-\lambda_2 t}} = \frac{1}{9}$

$$e^{-(\lambda_2 - \lambda_1)t} = 81$$

$$t = \frac{\ell n 81}{-(\lambda_2 - \lambda_1)} = \frac{4\ell n 3}{\left(\frac{\ell n 2}{T_1} - \frac{\ell n 2}{T_2}\right)}$$

9. Ans. (D)

Sol.



before collision with pan

$$V_0 = \sqrt{2g\frac{\ell}{2}} = \sqrt{g\ell}$$

Just after collision

$$\frac{M}{2}V_0 = \left(\frac{M}{2} + M\right)V$$

$$V = \frac{V_0}{3}$$

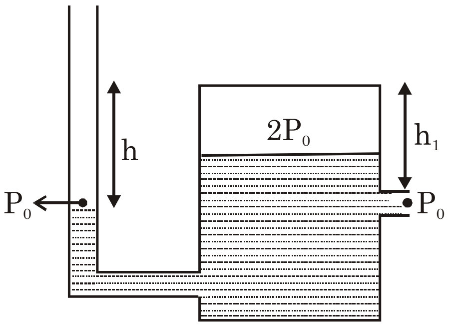
for 'A' $\quad -T + Mg + \frac{MV^2}{\ell} = Ma$ ....(1)

for 'B' $\quad T - Mg = Ma$ ...(2)

$(1) + (2) \Rightarrow a = \frac{V^2}{2\ell} = \frac{g}{18}$

10. Ans. (D)

Sol.



Pressure at same height is same, therefore $h = h_1$

11. Ans. (A)

Sol. $E_2 - E_1 = \frac{hc}{\lambda_\alpha}$ & $E_1 = -23.5$

$E_2 + 23.5 = \frac{12400}{0.71} \Rightarrow E_2 = -6\text{keV}$

energy of atom = $|E_2|$ = 6keV

12. Ans. (C)

Sol. $L = 1 \times 4 \times 0.2 + \frac{1 \times 4 \times 10^{-2}}{2} \times 10$

$= 0.8 + 0.2 = 1$

13. Ans. (A)

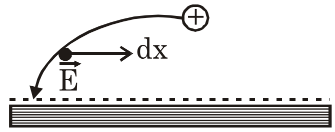
Sol. $\frac{\sqrt{a^2 + h^2}}{V_S} < \frac{a}{V_P} + \frac{h}{V_S}$

$\frac{a^2 + h^2}{V_S^2} < \frac{a^2}{V_P^2} + \frac{h^2}{V_S^2} + \frac{2ah}{V_P V_S}$

$a < \frac{\frac{2h}{V_P V_S}}{\left(\frac{1}{V_S^2} - \frac{1}{V_P^2}\right)} < \frac{2h\left(\frac{V_P}{V_S}\right)}{\left(\frac{V_P}{V_S}\right)^2 - 1}$

14. Ans. (B)

Sol.



$W_{elec} = \int q\vec{E} \cdot dx$

$= \int qE\, dx \cos\theta$

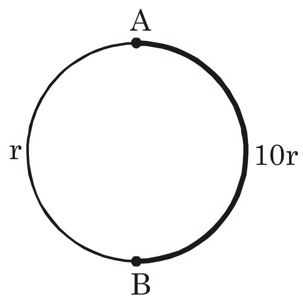
& θ is obtuse angle

$W_{Fext} + W_{elec} = 0$

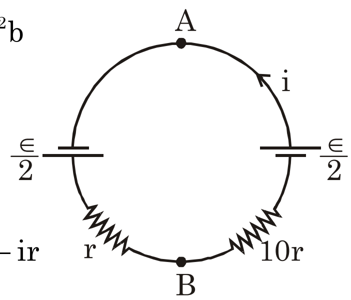
$W_{ext} = +ve$

15. Ans. (B)

Sol.



$\in = \pi R^2 \frac{dB}{dt} = \pi R^2 b$



$i = \frac{\pi R^2 b}{11r}$

$\Delta V_{AB} = E\pi R = \frac{\in}{2} - ir$

$E\pi R = \frac{\pi R^2 b}{2} - \frac{\pi R^2 b}{11}$

$E = \frac{9Rb}{22}$

16. Ans. (D)

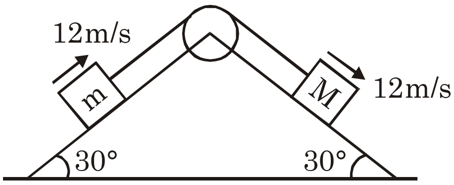
Sol. $F = -\frac{dU}{dx} = -20 \sin 2x$

$a = -40x$ for small 'x'

$T = 2\pi\sqrt{\frac{1}{40}} = \frac{\pi}{\sqrt{10}}$

17. Ans. (A)

Sol.



$a_M = g \sin 30° - \frac{g\cos 30°}{\sqrt{2}} = 5 - 6 = -1 m/s^2$

$a_m = -(g\sin 30° + \frac{g\cos 30°}{\sqrt{2}}) = -11\ m/s^2$

as limiting friction is greater than mgsinθ,

block 'm' will not move after $t = \frac{12}{11}$ sec.

$v_M = 12 - 1 \times 2 = 10$ m/s

18. Ans. (C)

Sol. $a = \omega^2 x$

$V^2 = \omega^2\left(A^2 - x^2\right)$

$V^2 = \omega^2\left(A^2 - \frac{a^2}{\omega^4}\right)$

Slope $= -\frac{1}{\omega^2} = -\frac{\ell}{g}$

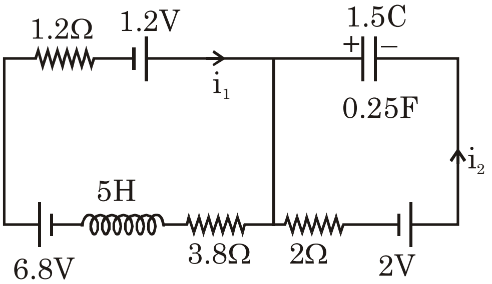
$\tan 30° = \frac{\ell}{g_0}$

$\tan 60° = \frac{\ell\left(1+\frac{h}{R}\right)^2}{g_0}$

$\Rightarrow \left(1+\frac{h}{R}\right)^2 = 3 \Rightarrow h = 0.73R$

19. Ans. (C)

Sol.



Current through key

$i = i_1 + i_2$

$i = \frac{8}{5}\left(1 - e^{-t}\right) + \frac{2}{2}\left(e^{-2t}\right) + \frac{6}{2}e^{-2t}$

for minimum current

$\frac{di}{dt} = 0 \Rightarrow \frac{8}{5}e^{-t} - 8e^{-2t} = 0 \Rightarrow e^{-t} = \frac{1}{5}$

$i_{min} = \frac{32}{25} + \frac{4}{25} = \frac{36}{25}$ amp.

20. Ans. (D)

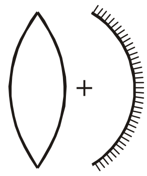
Sol. $f' = \frac{(332-2)}{332} \times \left(\frac{332}{332+2}\right) \times 334$

$f' = 330$ Hz

SECTION-II

1. Ans. 12.50

Sol.



$\frac{1}{f_{eff}} = \frac{2}{f_L} + \left(-\frac{1}{f_m}\right)$

$\frac{1}{f_L} = \left(\frac{\frac{3}{2} - \frac{5}{3}}{\frac{5}{3}}\right)\left(\frac{2}{20}\right) = -\frac{1}{100}$

$\frac{1}{f_{eff}} = -\frac{2}{100} + \frac{1}{10} \Rightarrow f_{eff} = 12.50$ cm

2. Ans. 1.00

Sol. $E = a \cos \omega_0 t + a \cos \omega t \cos \omega_0 t$

$E = a \cos \omega_0 t + \frac{a}{2}\left[\cos(\omega + \omega_0)t + \cos(\omega - \omega_0)t\right]$

$\nu_{max} = \frac{\omega + \omega_0}{2\pi} = \frac{4.8}{2\pi} \times 10^{15}$

$KE_{max} = h\nu - \phi$

$= \frac{6.6 \times 10^{-34} \times \frac{4.8}{2\pi} \times 10^{15}}{1.6 \times 10^{-19}} - 2.15$

$= \frac{6.3}{2} - 2.15 = 1eV$

3. Ans. 4.00

Sol. $\frac{hc}{\lambda}-\frac{hc}{\lambda_{max}}=eV$

$$\frac{hc}{2\lambda}-\frac{hc}{\lambda_{max}}=\frac{eV}{3}=\frac{1}{3}\left(\frac{hc}{\lambda}-\frac{hc}{\lambda_{max}}\right)$$

$$\frac{1}{2\lambda}-\frac{1}{\lambda_{max}}=\frac{1}{3\lambda}-\frac{1}{3\lambda_{max}}$$

$$\frac{1}{6\lambda}=\frac{2}{3\lambda_{max}}\Rightarrow\frac{\lambda_{max}}{\lambda}=4$$

4. Ans. 1.00

Sol. $v_{PJ}=(i\lambda 30)=\left(\frac{10}{R+1}\right)R$ for 1st case

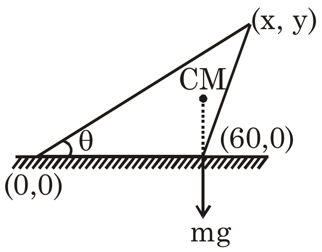
$v_{PJ}=i\lambda 10=\left(\frac{5R}{R+2}\right)$ for 2nd case

taking ratio

$$3=\frac{2(R+2)}{(R+1)}\Rightarrow R=1$$

5. Ans. 0.36

Sol.



Just to prevent from toppling, mg must pass through toppling point.

$$x_{CM}=60=\frac{0+60+x}{3}\Rightarrow x=120\text{ cm}$$

$$\text{Area}=\frac{1}{2}\times\frac{60}{100}\times y=1\Rightarrow y=\frac{10}{3}\text{m}$$

$$\cot\theta=\frac{x}{y}=\frac{120}{100\times10}\times3=0.36$$

6. Ans. 64.00

Sol. ratio = $n=\frac{8^2}{9^2}$

$$n=\frac{64}{81}$$

7. Ans. 82.80

Sol. $I_C=\frac{5}{2}mA$

$$I_B=\frac{2.3}{R_B}$$

$$\beta=90=\frac{2.5\times10^{-3}}{2.3}R_B$$

$$R_B=\frac{90\times23}{25}k\Omega$$

$= 82.80\ k\Omega$

8. Ans. 1.84 to 1.86

Sol. $B=\frac{\mu_0\varepsilon_0 A}{2\pi r}\frac{dE}{dt}$

$B = 1.85\times10^{-18}$ T

9. Ans. 500.00

Sol. $\Delta I_E=\Delta I_B+\Delta I_C$

$2.1=\Delta I_B+2\Rightarrow\Delta I_B=0.1$ mA

$v_{in}=\Delta I_B R_{in}$

$0.05=10^{-4}R_{in}$

$R_{in}=500\ \Omega$

10. Ans. 6.28 to 6.29

Sol. $I=\frac{\vec{M}}{V}=\frac{5}{10^{-6}}$ $\qquad H=0.5\times10^4$

$B=\mu_0(H+I)$

$=4\pi\times10^{-7}(0.5\times10^4+5\times10^6)$

$= 6.28$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (C)

in air, $P_{N_2} = 0.8\,atm, P_{O_2} = 0.2$ atm

$P = K_H X$ (Henry's law)

$0.8 = 2 \times 10^4\ X_{N_2}$ ....(1)

$0.2 = 10^4\ X_{O_2}$ ....(2)

Ratio of solubility of N_2 & O_2 is 2 : 1

2. Ans. (A)

Specific conductance ,

$K = \frac{1}{R}.\frac{\ell}{A} = 10^{-2}\,S\,cm^{-1}$

$\therefore$ Equivalent conductance , $\wedge eq = \frac{K \times 1000}{N}$

$= \frac{10^{-2}}{10^{-1}} \times 10^3$

$= 10^2\ S\ cm^2\ eq^{-1}$

3. Ans. (C)

4. Ans. (A)

$A \rightarrow B\ +$		C
P_0	0	0
$P_0 - x$	x	x
0	p_0	p_0

Given :

$p_0 - x = 200$

$p_0 + x = 300$

$\Rightarrow$ $p_0 = 250$

$x = 50$

$K = \frac{1}{t}\ell n\frac{p_0}{p_0 - x}$

$= \frac{1}{600}\ell n\frac{250}{250-50}\,sec^{-1}$

$= \frac{1}{600}\ \ell n\ 1.25\ sec^{-1}$

5. Ans. (B)

A = Aromatic

B = Non-aromatic (exist as nonplanar structure)

C,D = Antiaromatic

6. Ans. (A)

7. Ans (A)

8. Ans. (B)

9. Ans. (A)

10. Ans. (C)

11. Ans. (B)

12. Ans. (B)

13. Ans. (A)

$= \frac{\frac{4}{3}\pi r_+^3 + \frac{4}{3}\pi r_-^3}{(2r_-)^3} \times 100 \quad = \frac{\pi}{6}\left[\left(\frac{r_+}{r_-}\right)^3 + 1\right] \times 100$

$= 67.15$

14. Ans.(A)

(i) α – D – glucose (aq) $\longrightarrow$ β – D – glucose (aq) $\Delta H° = -1.16\ kJ\ mol^{-1}$

(ii) α – D – glucose (s) $\longrightarrow$ α – D – glucose (aq)

$\Delta H° = 10.72\ kJ\ mol^{-1}$

(iii) β – D – glucose (s) $\longrightarrow$ β – D – glucose (aq) $\Delta H° = 4.68\ kJ.mol^{-1}$

To obtain reaction

(iv) α – D – glucose (s) $\longrightarrow$ β – D – glucose (s)

(iv) = (ii) + (i) – (iii)

$\therefore \Delta H° = 10.72 - 1.16 - 4.68 = 4.88\ kJ\ mol^{-1}$

15. Ans. (C)

16. Ans. (C)

17. Ans. (B)

18. Ans. (B)

19. Ans. (C)

20. Ans. (C)

SECTION-II

1. Ans. (255.00)

$1 \times \frac{3}{2}R\ (T_2 - 300) = -P_{ext}\left(\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1}\right)$

$1.5\ (T_2 - 300) = -1\left(\frac{T_2}{2} - \frac{300}{5}\right)$

$1.5\ T_2 - 450 = -0.5\ T_2 + 60$

$2T_2 = 510$

$T_2 = 255$ K

2. Ans. (8.00)

sol. $K_{sp} = 4 \times (5 \times 10^{-6})^3 = (0.5 \times 10^{-3})\,[OH^-]^2$

$[OH^-]^2 = 100 \times 10^{-18+4} = 10^{-12}$

$[OH^-] = 10^{-6}$

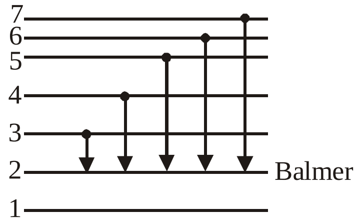
pOH = 6

pH = 8

3. Ans. (6.00)

4. Ans.(9.00)

5. Ans.(5.00)



6. Ans. (25.00)

7. Ans (5.00)

8. Ans. (6.00)

9. Ans.(201.66 to 201.67)

10. Ans.(8.00)

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (A)

Sol. $\sqrt{(a-1)^2+(b-3)^2} = AB$

$\sqrt{(4-a)^2+(7-b)^2} = BC$

$\sqrt{(4-1)^2+(7-3)^2} = AC = 5$

$\therefore$ k cannot be 4

2. Ans. (B)

Sol. Let circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

orthogonality condition

c = 8

circle passes through (1,1)

$2 + 2g + 2f + 8 = 0$

$g + f + 5 = 0$

$-h - k + 5 = 0$

$x + y = 5$

3. Ans. (A)

Sol. Normal

$y = mx - 2m - m^3$ ...(i)

and $y = mx - km - \frac{m}{2} - \frac{m^3}{4}$ ...(ii)

(i) & (ii) same

$\frac{1}{1} = \frac{m}{m} = \frac{-2m-m^3}{-km-\frac{m}{2}-\frac{m^3}{4}}$

$\Rightarrow 1 = \frac{-8-4m^2}{-m^2-4k-2}$

$\Rightarrow 3m^2 = 4k - 6$

$m^2 \geq 0 \Rightarrow 4k - 6 \geq 0$

$\Rightarrow k \geq \frac{3}{2}$

4. Ans. (A)

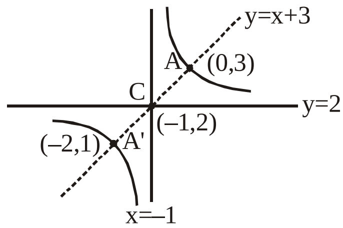
Sol. $\frac{b^2}{a^2} = 1 - e^2$ and $a^2e^2 = b^2(1 - e_1^2)$

$\Rightarrow 1 - e_1^2 = \frac{e^2}{1-e^2} \Rightarrow e_1^2 = 1 - \frac{e^2}{1-e^2}$

$e_1 = \sqrt{\frac{1-2e^2}{1-e^2}}$

5. Ans. (D)

Sol. $y = 2 + \frac{1}{x+1}$



$\Rightarrow (y-2)(x+1) = 1$

$a = \sqrt{2}$

Length of L.R. = $2\sqrt{2}$

6. Ans. (A)

Sol. $\frac{\cos 20^\circ}{\sin 50^\circ.\sin 110^\circ} + \frac{\cos 50^\circ}{\sin 110^\circ} + \frac{\cos 110^\circ}{\sin 20^\circ \sin 50^\circ}$

$= \frac{1}{2}\left[\frac{\sin 40^\circ + \sin 100^\circ + \sin 220^\circ}{\sin 20^\circ \sin 50^\circ \sin 110^\circ}\right]$

$= \frac{1}{2}\left[\frac{4\sin 20^\circ \sin 50^\circ \sin 110^\circ}{\sin 20^\circ \sin 50^\circ \sin 110^\circ}\right] = 2$

7. Ans. (B)

Sol. $\sqrt{\sin(1-x)} = \sqrt{\cos x}$

$\sin(1 - x) = \cos x$

$\cos x - \cos\left(\frac{\pi}{2} - (1-x)\right) = 0$

$$-2\sin\left(x+\frac{\pi}{4}-\frac{1}{2}\right)\sin\left(\frac{1}{2}-\frac{\pi}{4}\right)=0$$

$$\therefore \quad \sin\left(x+\frac{\pi}{4}-\frac{1}{2}\right)=0$$

$$\therefore \quad x+\frac{\pi}{4}-\frac{1}{2}=n\pi$$

$x=\frac{1}{2}-\frac{\pi}{4}$ (negative), $x=\frac{1}{2}+\frac{3\pi}{4}$ (rejected)

$x=\frac{1}{2}+\frac{7\pi}{4}$ (least positive)

8. Ans. (A)

Sol. Area $=5\sin\frac{\pi}{5}$

$$\therefore \quad 10\left(\frac{1}{2}.R^2\sin\frac{\pi}{5}\right)=5\sin\frac{\pi}{5}$$

$\therefore \quad R=1$

R is circumradius of polygon.

Use $A_1A_r=\sqrt{2-2\cos\left(\frac{(r-1)\pi}{5}\right)}$

$$=2\sin\left(\frac{(r-1)\pi}{10}\right)$$

$\therefore \quad (A_1A_2)(A_1A_3)$ ...... (A_1A_{10})

$$=2^9\sin\frac{\pi}{10}.\sin\frac{2\pi}{10}.......\sin\frac{9\pi}{10}=10$$

9. Ans. (A)

Sol. $(A\cup B)\cap B'=A$

$\Rightarrow \quad A'\cup((A\cup B)\cap B')=A'\cup A=U$

10. Ans. (B)

Sol. a R b $\Leftrightarrow a=2^k.a$ it is true for k = 0

$\therefore$ reflexive

$(2,1)\in R$ but $(1,2)\notin R\Rightarrow$ it is not symmetric

if $a=2^{k_1}b$ and $b=2^{k_2}c$, then $a=2^{k_1+k_2}c$

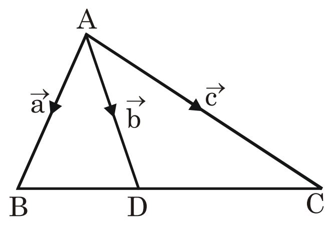
$\Rightarrow$ it is transistive.

11. Ans. (A)

Sol. $\vec{a}=\hat{i}-\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}+3\hat{k}$

$\vec{b}=\frac{\vec{a}+\vec{c}}{2}$



$\therefore$ Area $=\frac{1}{2}|\vec{a}\times\vec{c}|$

$$=\frac{1}{2}\left|\vec{a}\times\left(2\vec{b}-\vec{a}\right)\right|$$

$$=\left|\vec{a}\times\vec{b}\right|=\sqrt{86}$$

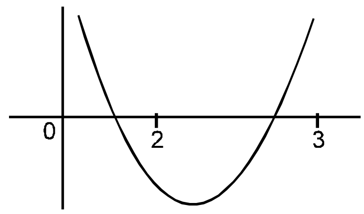
12. Ans. (B)

Sol. $\sqrt{3}=\left|(\vec{b}-\vec{a})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$\Rightarrow \sqrt{3}=|\vec{b}-\vec{a}|\cos 30^\circ\Rightarrow|\vec{b}-\vec{a}|=AB=2$

13. Ans. (A)

Sol. $x^2-\frac{2p}{p-5}x+\frac{p-4}{p-5}=0$



f(0) > 0, f(2) < 0, f(3) > 0

$f(0)>0\Rightarrow\frac{p-4}{p-5}>0$...... (1)

$f(2)<0\Rightarrow\frac{p-24}{p-5}<0$..........(2)

$f(3)>0\Rightarrow\frac{4p-49}{p-5}>0$..........(3)

Intersection of (1) (2) & (3) gives $\left(\frac{49}{4},24\right)$

14. Ans. (B)

Sol. $a^{\left(\frac{1}{a}+\frac{1}{2a}+\frac{1}{4a}+......\infty\right)}.2^{\frac{1}{2a}+\frac{1}{4a}+\frac{1}{8a}+......\infty}=\frac{8}{27}$

now $\frac{1}{a}\left(1+\frac{1}{2}+\frac{1}{2^2}+........\infty\right)=\frac{2}{a}$ and

$\frac{1}{2a}+\frac{2}{4a}+\frac{3}{8a}+........\infty=\frac{2}{a}$ (use AGP)

$\therefore \quad a^{\frac{1}{a}}.2^{\frac{1}{a}}=\frac{8}{27}=\left(\frac{1}{3}\right)^3.2^3\Rightarrow a=\frac{1}{3}$

15. Ans. (A)

Sol. negation is $p\wedge\sim\left((\sim p)\vee(\sim q)\right)$

$p\wedge(p\wedge q)$

$p\wedge q$

16. Ans. (C)

Sol. Let us define some events

A : 1R & 1G ball is drawn

B : 1R & 1W ball is drawn

C : 1W & 1G ball is drawn

D : 2 balls drawn are of different colour

Let number of white balls = x

$$P\left(\frac{A}{D}\right)=\frac{2}{9}$$

From Baye's theorem

$$P\left(\frac{A}{D}\right)=\frac{(A\cap D)}{P(D)}$$

$$=\frac{P(A).P\left(\frac{D}{A}\right)}{P(A).P\left(\frac{D}{A}\right)+P(B)P\left(\frac{D}{B}\right)+P(C)P\left(\frac{D}{C}\right)}$$

$$=\frac{\frac{{}^{3}C_1\times{}^{4}C_1}{{}^{x+7}C_2}}{\frac{{}^{3}C_1\times{}^{4}C_1}{{}^{x+7}C_2}+\frac{{}^{x}C_1\times{}^{3}C_1}{{}^{x+7}C_2}+\frac{{}^{x}C_1\times{}^{4}C_1}{{}^{x+7}C_2}}$$

$$\Rightarrow \quad \frac{12}{12+7x}=\frac{2}{9} \quad \Rightarrow \quad 108=24+14x$$

$$\Rightarrow \quad 14x=84 \ \Rightarrow \ x=6$$

17. Ans. (B)

Sol. $T_{r+1}={}^{n}c_{r}\,(2x)^{n-r}\,(5y)^{r}$

$T_{r}={}^{n}c_{r-1}\,(2x)^{n-r+1}\,(5y)^{r-1}$

$$\frac{T_{r+1}}{T_r}=\frac{(n-r+1)}{r}\ \frac{(5y)}{(2x)}$$

$$=\left(\frac{34-5+1}{r}\right)\left(\frac{10}{6}\right)>1$$

$$\Rightarrow \quad \frac{35-r}{r}>\frac{3}{5} \qquad 3r<175-5r$$

$8r<175$

$r<\left(\frac{175}{8}\right) \qquad r=0, 1, \ldots\ldots, 21$

$\Rightarrow \ T_{22}$ is largest.

18. Ans. (B)

Sol. $f(x)=\frac{4x}{3}-\left\{\frac{x}{3}\right\}=\frac{4}{3}x-\frac{x}{3}+\left[\frac{x}{3}\right]=x+\left[\frac{x}{3}\right]$

$\because \quad x\in(3, 6) \ \Rightarrow \left[\frac{x}{3}\right]=1$

$\Rightarrow \ f(x)=x+1$, whose inverse is $x-1$.

19. Ans. (C)

Sol. $$\lim_{x\to\infty} x^{-x^2}\left\{(x+1)\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2^2}\right)\ldots\ldots\left(x+\frac{1}{2^{x-1}}\right)\right\}^x$$

$$=\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)\left(1+\frac{1}{2x}\right)\left(1+\frac{1}{2^2x}\right)\ldots\ldots\left(1+\frac{1}{2^{x-1}x}\right)\right\}^x$$

$$=e^1.e^{1/2}.e^{1/2^2}\ldots\ldots\infty=e^{1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\infty}=e^2$$

20. Ans. (A)

Sol. $\left[\frac{x^2-4x+3}{5\sin x+e^x+7}\right]$ is always an integer

$\Rightarrow \ f(x)=\tan n\pi=0$, always continous.

SECTION–II

1. Ans. 7.00

Sol. $a, b\in\{2, 3, 5, 7\}$

$f'(x)=\ 3x^2+2ax+b\geq 0 \quad \forall\ x\in R$

$D\leq 0$

$4a^2-12b\leq 0$

$4(a^2-3b)\leq 0$

$a=2 \qquad b=2,3,5,7$

$a=3 \qquad b=3,5,7$

$a=5 \qquad$ No solution

$a=7 \qquad$ No solution

Ordered pairs (a,b) are (2,2), (2,3), (2,5), (2,7), (3, 3), (3,5) & (3,7)

2. Ans. 4.00

Sol. 1 2 4

Applying LMVT in [1, 2]

$\frac{f(2)-f(1)}{2-1}=f'(c_1)\ \forall\ c_1\in(1, 2)$

$f(2)-2\leq 2\ \{\because f'(x)\leq 2\}\Rightarrow f(2)\leq 4 \quad \ldots(1)$

Similarly applying LMVT in [2, 4]

$\frac{f(4)-f(2)}{4-2}=f'(c_2)\ \forall\ c_2\in(2, 4)$

$\frac{8-f(2)}{2}\leq 2\Rightarrow f(2)\geq 4 \quad \ldots(2)$

from (1) & (2) $\quad f(2)=4$

3. Ans. 6.00

Sol. $\int_{\alpha}^{\beta} f(x)dx + \int_{a}^{b} f^{-1}(x)dx = b\beta - a\alpha$

(if $f(\alpha) = a$, $f(\beta) = b$)

$\therefore \quad I_1 + I_2 = 2 \times 3 - 0 \times 1 = 6$

4. Ans. 0.50

Sol. Using king in 'm'

$$m = \int_{-1}^{2} (1-x)\sin((1-x)x)dx$$

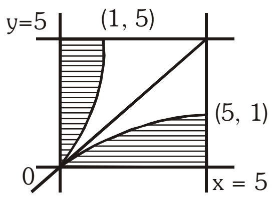
$$= \int_{-1}^{2} \sin((1-x)x)dx - \int_{-1}^{2} x\sin((1-x)x)dx$$

$m = n - m \quad \Rightarrow \quad 2m = n$ or $\frac{m}{n} = \frac{1}{2}$

5. Ans. 3.25

Sol. $A = \int_{0}^{1} (5 - 3x^3 - 2x)\ dx$

$= \frac{13}{4}$



6. Ans. 6.00

Sol. $y^2(x^2dy + 2xydx) - dy = 0$

$\Rightarrow d(x^2y) - \frac{dy}{y^2} = 0$

$\Rightarrow x^2y + \frac{1}{y} = c = 2$

$\Rightarrow 2k^2 + \frac{1}{2} = 2 \Rightarrow 4k^2 = 3$

7. Ans. 27.00

Sol. If A is skew symmetric matrix whose entries are 0, 1, –1, then possible matrices are

$a_{11} = 0 \quad a_{22} = 0 \quad a_{33} = 0$

we can fill a_{12} in 3 ways (0,1,–1)

we can fill a_{13} in 3 ways (0,1,–1)

we can fill a_{23} in 3 ways (0,1,–1)

remaining a_{21}, a_{31}, a_{32} filled by only one way. Hence possible matrix A are $3 \times 3 \times 3 = 27$

and all gives infinite solutions due to $D = 0$, $D_x = 0$, $D_y = 0$, $D_z = 0$

hence $\quad pq = 27$

$\therefore \quad p + q = 9.$

8. Ans. 3.25

Sol.
$$\begin{vmatrix} 2 & -3 & 4 \\ 7 & -2 & 2 \\ [3\pi] & -[2e] & [4a] \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 7 & 2 & 2 \\ 9 & 5 & [4a] \end{vmatrix} = 0$$

$4[4a] - 20 - 3(7[4a] - 18) + 4(17) = 0$

$-17[4a] + 102 = 0$

$[4a] = 6$

$6 \le 4a < 7$

$\frac{3}{2} \le a < \frac{7}{4}$

$\alpha = 3,\ \beta = 2,\ \gamma = 7,\ \delta = 4$

9. Ans. 19.00

Sol. Let x be random variable of blue marbles choosen

x	0	1	2	3
Px(x = x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$Var[x] = E[x^2] - [E[x]]^2$

$E[x]^2 = 0^2 \times \frac{1}{6} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{3}{10} + 3^2 \times \frac{1}{30}$

$E(x) = \left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{30}\right)$

$= 1.2$

$[E(x)]^2 = 1.44$

Variance $= 2 - 1.44 = 0.56$

S.D. $= V = \sqrt{0.56} = \frac{\sqrt{14}}{5} = 14 + 5 = 19$

10. Ans. 7.00

Sol. Required number of ways

= Total when all A's seperated

– Total when A's separated and H's are together

$= \frac{7!}{2!}({}^8C_4) - 6!({}^7C_4)$

$= \frac{7!.6!}{4!.3!}(6) = 4^1.5^2.6^3.7^1$