ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2020 - 2021)

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Major Test

ANSWER KEY

PART-1 PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	C	C	B	A	C	D	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	A	C	B	B	B	C	D	A	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4.80	9.50	24.00	560.00	23.80	0.00	5.00	576.00	12.00	517.00

PART-2 CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	B	B	C	B	A	D	D	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	D	C	B	D	C	B	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2.00	6.00	1.00	3.00	7.00	4.00	19.00	70.02	16.00	240.00

PART-3 MATHEMATICS

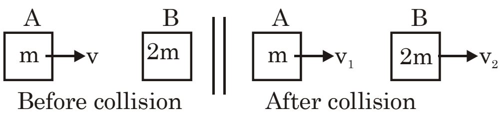
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	A	A	D	A	D	B	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	C	A	D	C	B	C	B	D	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1.00	8.00	0.00	4.00	120.00	0.00	4.00	18.00	9.00	2.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. First collision (between A & B)

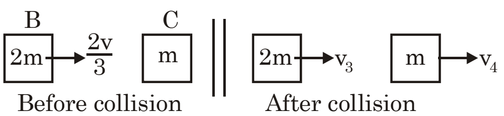


$$mv = mv_1 + 2mv_2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$v_1 = -\frac{v}{3},\ v_2 = \frac{2v}{3}$$

Second collision (between B & C)

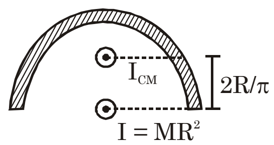


$$2m\cdot\left(\frac{2v}{3}\right) = 2mv_3 + mv_4$$

$$\frac{1}{2}2m\left(\frac{2v}{3}\right)^2 = \frac{1}{2}(2m)v_3^2 + \frac{1}{2}mv_4^2$$

$$v_3 = \frac{2v}{9},\ v_4 = \frac{8v}{9}$$

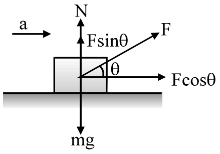
2.



$$I = I_{cm} + Md^2$$

$$MR^2 = I_{cm} + M\left(\frac{2R}{\pi}\right)^2$$

$$\Rightarrow I_{cm} = MR^2 - M\left(\frac{2R}{\pi}\right)^2$$

3. 

$F\cos\theta = ma$

$\Rightarrow \dfrac{mg}{3}\cos\theta = m\dfrac{dv}{dt}$

$\Rightarrow \dfrac{mg}{3}\cos(ks) = m\dfrac{dv}{ds}\dfrac{ds}{dt}$

$\Rightarrow \dfrac{mg}{3}\cos(ks) = mv\dfrac{dv}{ds}$

$\Rightarrow vdv = \dfrac{g}{3}\cos(ks)ds$

$\Rightarrow \dfrac{v^2}{2} = \dfrac{g}{3k}\sin(ks) \Rightarrow v = \sqrt{\dfrac{2g}{3k}\sin\theta}$

4. $F = -\dfrac{dU}{dx}$

$K + U = E$

5. $\lambda_{m_1}T_1 = \lambda_{m_2}T_2$

$\dfrac{T_1}{T_2} = \dfrac{350}{510} = 0.69$

6. Using ideal gas equation for μ gm-mole

$$PV = \mu RT = \frac{m}{M}RT$$

where m is the mass and M the molecular weight of the gas.

For N_2, $P_1V = \dfrac{4}{28}\times R\times 300$ ....(i) ($\because M_{N_2} = 28$)

For CO_2, $P_2V = \dfrac{4}{44}\times R\times 300$ ....(ii)

($\because M_{CO_2} = 12 + 16 \times 2 = 44$)

Adding eqs. (i) and (ii),

$(P_1 + P_2)V = 4 \times R \times 300\left(\dfrac{1}{28}+\dfrac{1}{44}\right)\times 10^3$

Given $P_1 + P_2 = 1$ atm $= 1.013 \times 10^5$ N/m^2

$\therefore \quad V = \dfrac{R\times 300\left(\dfrac{1}{7}+\dfrac{1}{11}\right)\times 10^3}{1.013\times 10^5}$

$= \dfrac{8.3\times 300\times 18\times 10^3}{1.013\times 10^5\times 77}$

$= \dfrac{149.4\times 300\times 10^3}{77\times 1.013\times 10^5} \approx 2\times 300\times 10^{-2}$

$V = 6.0\ m^3$

Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{(4+4)\times 10^{-3}\ kg}{6m^3}$

$= \dfrac{4}{3}\times 10^{-3}\ kg/m^3 = \dfrac{4}{3}\ gm/cm^3$

7. $MS\Delta T = M'L_f$

(2) (400) (500) = $M'(3.5 \times 10^5)$

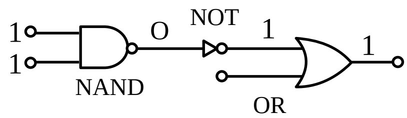
$M' = \dfrac{4\times 10^5}{3.5\times 10^5} = \dfrac{40}{35} = \dfrac{8}{7}kg$

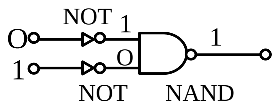
8. Electronic current

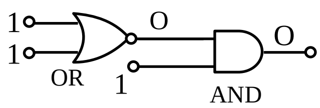
$I_e = neA\mu_e\left(\dfrac{V}{l}\right)$

$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times 10^{-4} \times 0.14 \times \dfrac{2}{10^{-1}}$

$= 6.72 \times 10^{-7}A$

9. (A) 

(B) 

(C) 

The outputs of A,B,C are respectively 1,1,0.

Hence option (C) is correct

10. $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-6}}$

$= 3.31 \times 10^{-28}$ kg –m/sec.

11. Here $V_{max} = \frac{24mV}{2} = 12$ mV

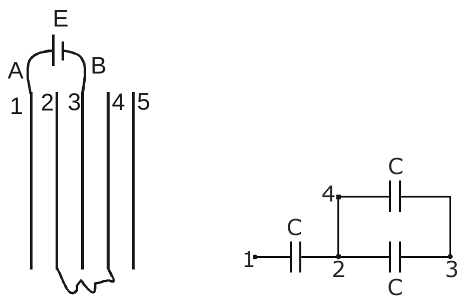
and $V_{min} = \frac{8mV}{2} = 4$ mV

$\therefore \quad m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-4}{12+4} = \frac{1}{2}$

$= 0.5 = 50\%$

12. Both cuts equal number of electric lines.

14.



$C_{eq} = \frac{2C \times C}{3C} = \frac{2 \epsilon_0 A}{3d}$

$Q = \frac{2}{3} \times \frac{\epsilon_0 A}{d} \times E$

15. Particle will move in x-y plane

$r = \frac{mv}{qB}$

if r > (b – a) then particle will enter in region x > b

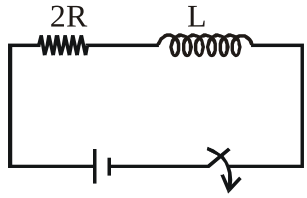
$(b-a) < \frac{mv}{qB}$

$v < \frac{(b-a)qB}{m}$

minimum velocity

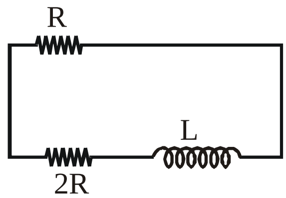
$v = \frac{(b-a)qB}{m}$

16. Build up circuit



$\tau_B = L/2R$

Decay



$\overline{C}_D = L/3R$

Ratio = 3 : 2

SECTION-II

1. $S = ut_R + \frac{u^2}{2a}$

$= 4 \times 0.2 + \frac{(4)^2}{2 \times 2} = 4.8$ m

2. At bottom point

vel. $u = \sqrt{5g\ell}$

by energy conservation

TE at bottom = TE at point P

$\frac{1}{2}mu^2 = \frac{1}{2}mv_p^2 + mgh$

$5g\ell = v_p^2 + 2g \times \frac{3\ell}{2}$

so $v_p = \sqrt{2g\ell}$

now $a_{cp} = \frac{v_p^2}{\ell} = 2g$

$a_t = g\sin 60 = \frac{\sqrt{3}}{2}g$

Hence $a_{net} = \sqrt{(2g)^2 + \left(\frac{\sqrt{3}g}{2}\right)^2}$

$a_{net} = \frac{g\sqrt{19}}{2} = \frac{g\sqrt{\alpha}}{\beta}$

$\alpha = 19$

B = 2 and $\frac{\alpha}{\beta} = \frac{19}{2} = 9.5$

3. $F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = 24N$

4. By adjoining graph $W_{AB} = 0$ and

$W_{BC} = 8\times10^4[5-2]\times10^{-3} = 240$ J

$\therefore$ $W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240$ J

Now,
$\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800$ J

From FLOT $\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$

$\Rightarrow 800 = \Delta U_{AC} + 240 \Rightarrow \Delta U_{AC} = 560$ J.

5. Mass of $_2He^4$ = 4.00388 a.m.u

Mass of two deuterium = 2 × 2.01478 = 4.02956

Energy equivalent to $2\,_1H^2$ = 4.02956 × 1.112 MeV = 4.48 MeV

Energy equivalent to $_2H^4$ = 4.00388 × 7.047 MeV = 28.21 MeV

Energy released = 28.21 – 4.48 = 23.73 MeV

= 24 MeV

6. Minimum kinetic energy is always zero.

7. $\frac{Q^2}{2C}$ = 640 μJ ... (1) $\frac{Q}{C}$ = 16 V ... (2)

from (1) & (2) : $\frac{Q}{2} = \frac{640\times10^{-6}}{16}$

$\Rightarrow Q = 80 \times 10^{-6}$ C

$\therefore C = \frac{Q}{16} = \frac{80}{16}$ μF = 5 μF

8. $\phi = (E \sin 53° \times \pi R^2)$

10. After filling frequency increases, so n_A increases(↑). Also it is given that beat frequency increases (i.e., x↑)

Hence $n_A\uparrow - n_B = x\uparrow$ ...(i) $\longrightarrow$ Correct

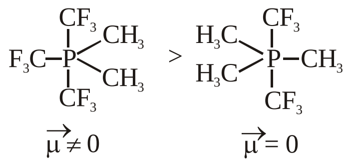
$n_B - n_A\uparrow = x\uparrow$ ...(ii) $\longrightarrow$ Wrong

$\Rightarrow n_A = n_B + x = 512 + 5 = 517$ Hz.

PART-2 : CHEMISTRY

SECTION-I

2.



5. Ag, Al, Pb, Cu are isolated from sulphide ores.

6. $Co^{3+} \Rightarrow d^6 \rightarrow$ 4 unpaired electron

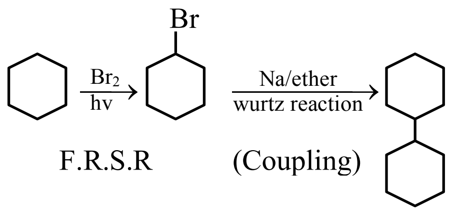
Magnetic moment = $\sqrt{n(n+2)}$

$= \sqrt{4(4+2)}$

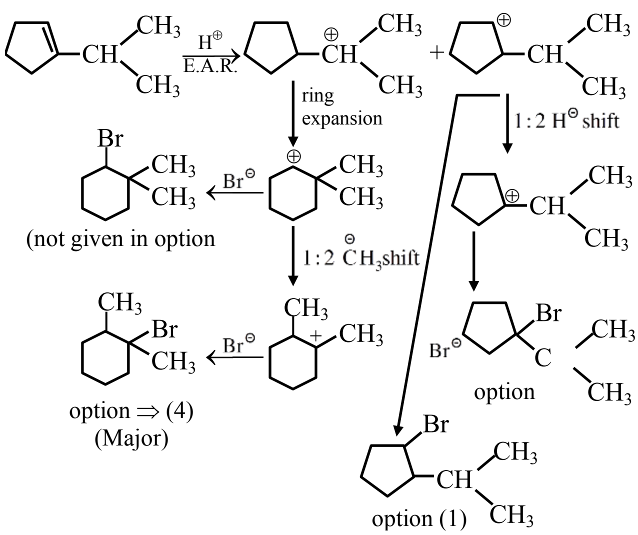
= 4.90 MB

7. Minimum oxidation of Cu = +1

13.



14.



SECTION-II

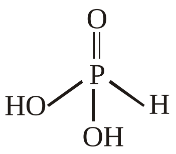
1. $K_2[HgI_4]$

$2(1) + x + 4(-1) = 0$

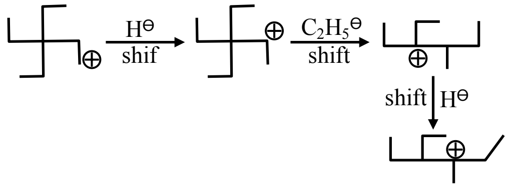
$x = +2$

2. $O = \overset{+5}{N} - O - O - H$, with $N \rightarrow O$

3. H_3PO_3



4.



PART-3 : MATHEMATICS

SECTION-I

1. Equation can be written as

$$3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$ and $\tan\theta = -\sqrt{3}$

$\Rightarrow \quad \theta = n\pi + \frac{\pi}{6}$ or $\theta = n\pi - \frac{\pi}{3}$

$\Rightarrow \quad |r - s| = |6 + 3| = 9$

2. Let p : New Delhi is in India

q : London is in England

$\therefore$ We have p q

It's negation is ~ (p q) ≡ ~ p ~ q

$\therefore$ New Delhi is not in India or London is not in India.

3. $20x^2 + 210x + 400 = 4500$

$2x^2 + 21x - 410 = 0$

$2x^2 + 41x - 20x - 410 = 0$

$x(2x + 41) - 10(2x + 41) = 0$

$(2x + 41)(x - 10) = 0$

$x = -\frac{41}{2}, 10$

4. {Linear = Excluding I & last}

$20 - 5 = (15)$

X 0 X 0 X 0 X ….. X0X

out of 15, these are is places

$\therefore\ ^{16}C_5 - ^{14}C_3$ {Excluding I & last}

$^{16}C_{11} - ^{14}C_3$

5. $a = 1, d = 10$

$T_{100} = 1 + 99\times 10 = 991$

$a = 31, d = 5$

$T_{100} = 31 + 99 \times 5 = 526$

31, 41, 51, 61...526

$T_n \le 526$

$31 + (n - 1) 10 \le 526$

$10n \le 505$

$n \le 50.5$

largest common terms

$31 + 49 \times 10$

$= 521$

6. $R_2 \Rightarrow R_2 - R_1$

$$\begin{vmatrix} d & e & f \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

Does not depend on x.

7. $A^TA = 3I_3$

$$\begin{pmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x = \pm\sqrt{\frac{3}{8}}; y = \pm\sqrt{\frac{1}{2}}$

8. $\alpha^3 + \alpha^2 - \alpha + 22 = (\alpha^2 - 2\alpha + 5)(\alpha + 3) + 7 = 7$

$\beta^3 + 4\beta^2 - 7\beta + 35 = (\beta^2 - 2\beta + 5)(\beta + 6) + 5 = 5$

$x^2 - 12x + 35 = 0$

10. $g'(x) = 3(f(3f(x) + 6))^2 \cdot f'(3f(x) + 6) \cdot 3f'(x)$

$\therefore g'(0) = 3(f(3f(0) + 6))^2 \cdot f'(3f(0) + 6) \cdot 3f'(0)$

$= 3(f(0))^2 \cdot f'(0) + 3f'(0)$

$= 3.4(-1) \cdot 3 \cdot (-1) = 36$

11. By newton's Leibniz formula.

$$= \lim_{x\to\pi/4} \frac{f\left(\sqrt{2}\sec x\right).\sqrt{2}\sec x.\tan x - 0}{2x}$$

$$= \frac{f(2).\sqrt{2}.\sqrt{2}.1}{2.\pi/4} = \frac{4}{\pi}f(2)$$

12. L.H.L. $= \lim_{x\to 0^-} 3(1+|\tan x|)^{\frac{6}{|\tan x|}} = 3e^{\alpha} ....(E_1)$

R.H.L $= \lim_{x\to 0^+} 3\left(1+\left|\frac{\sin x}{3}\right|\right)^{\frac{6}{|\sin x|}} = 3e^2 ....(E_2)$

at $x = 0 \Rightarrow f(0) = \beta$ …(E_3)

Function is cont.

$\therefore 3e^{\alpha} = 3e^2 = \beta$

$\therefore \alpha = 2, \beta = 3e^2$

14. $$I = \int \frac{\sec^2 x\ dx}{(4\tan x - 5)^2} = \frac{1}{4}\int \frac{d(4\tan x - 5)}{(4\tan x - 5)^2}$$

$$= -\frac{1}{4(4\tan x - 5)} + C$$

16. Given lines are –

$L_1 \equiv (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$L_2 \equiv (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

$$S.D. = \left|\frac{(a_2 - a_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Where $a_1 = 3\hat{i} - 15\hat{j} + 9\hat{k}$ & $a_2 = -\hat{i} + \hat{j} + 9\hat{k}$

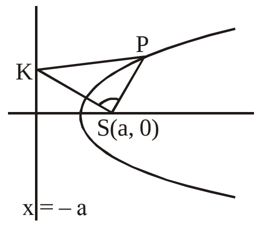
and $b_1 = 2\hat{i} - 7\hat{j} + 5\hat{k}$ & $\vec{b}_2 = (2\hat{i} + \hat{j} - 3\hat{k})$

17. Derector circle

$x^2 + y^2 = 9 + 4$

19. Here, $P(at^2, 2at)$ and $S(a, 0)$.

If the tangent at P, $ty = x + at^2$, meets the directrix $x = -a$ at k, then $k = \left(-a, \frac{at^2 - a}{t}\right)$



m_1 = slope of SP $= \frac{2at}{a(t^2 - 1)}$

m_2 = slope of SK $= \frac{a(t^2 - 1)}{-2at}$

Clearly $m_1m_2 = -1$, $\therefore \angle PSK = 90°$

20. Point (0, 0) satisfies $(x - 7)^2 + (y + 1)^2 = 50$ which is equation of director circle hence point (0, 0) lies on it, and from any point lie on director circle angle between tangents is $\pi/2$.

SECTION-II

1. $\alpha + \beta = 90°$ and $\alpha - \beta = 30°$

$\Rightarrow \alpha = 60°, \beta = 30°$

$\Rightarrow \tan 120° \times \tan 150° = (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right) = 1$

2. $\Sigma f_i x_i = 4 + 10 + 3K + 4 + 10 = 3K + 28$

$\Sigma f_i = 12 + K$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 2.6 = \frac{3K+28}{12+K}$$

$$\Rightarrow K = 8$$

3. $${}^{40}C_{15}\sum_{r=0}^{15} {}^{15}C_r\,{}^{20}C_r - {}^{35}C_{15}\sum_{r=0}^{15} {}^{15}C_r\,{}^{25}C_r$$

$$= {}^{40}C_{15}[{}^{15}C_0\,{}^{20}C_{20} + {}^{15}C_1\,{}^{20}C_{19} + {}^{15}C_2\,{}^{20}C_{18} + ... + {}^{15}C_{15}\,{}^{20}C_5]$$
$$- {}^{35}C_{15}[{}^{15}C_0\,{}^{25}C_{25} + {}^{15}C_1\,{}^{25}C_{24} + ... + {}^{15}C_{15}\,{}^{25}C_{10}]$$

$$= {}^{40}C_{15}\,{}^{35}C_{20} - {}^{35}C_{15}\,{}^{40}C_{25}$$

$$= {}^{40}C_{15}\,{}^{35}C_{20} - {}^{35}C_{20}\,{}^{40}C_{15} = 0$$

4. P(hitting the target at least once) > 0.99

$$\Rightarrow \Rightarrow 1 - \underbrace{\frac{1}{4}.\frac{1}{4}.\frac{1}{4}........\frac{1}{4}}_{\text{n times}} > 0.99$$

$$\Rightarrow 1 - \left(\frac{1}{4}\right)^n > 0.99 \Rightarrow 4^n > \frac{1}{0.01} \Rightarrow 4^n > 100$$

So minimum value of n to satisfy the inequality is 4.

5. $$\lim_{x\to 0}\frac{3f(x) - 4f(3x) + f(9x)}{x^2}\left(\frac{0}{0}\text{form}\right)$$

$$\lim_{x\to 0}\frac{3f'(x) - 12f'(3x) + 9f'(9x)}{2x}\left(\frac{0}{0}\text{form}\right)$$

$$\lim_{x\to 0}\frac{3f''(x) - 36f''(3x) + 81f''(9x)}{2}$$

$$\frac{3f''(0) - 36f''(0) + 81f''(0)}{2}$$

$$= 24f''(0) = 24(5) = 120$$

9. $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors. Now

$$x = |\hat{a} - \hat{b}|^2 + |\hat{b} - \hat{c}|^2 + |\hat{c} - \hat{a}|^2$$

$$= 2\left(\hat{a}\cdot\hat{a} + \hat{b}\cdot\hat{b} + \hat{c}\cdot\hat{c}\right) - 2\hat{a}\cdot\hat{b} - 2\hat{b}\cdot\hat{c} - 2\hat{c}\cdot\hat{a}$$

$$= 6 - 2\left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right) \qquad ..(i)$$

Also, $|\hat{a} + \hat{b} + \hat{c}| \ge 0$

or $\hat{a}\cdot\hat{a} + \hat{b}\cdot\hat{b} + \hat{c}\cdot\hat{c} + \left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right)2 \ge 0$

or $3 + 2\left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right) \ge 0$

or $2\left(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}\right) \ge -3$

from (1) $\quad x \ge 9$

10. Use $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 1 \times 2 + 2 \times 1 + \lambda \times (-2) = 0 \Rightarrow \lambda = 2$