(1001CJA102119028) Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Advanced)
UNIT TEST
05-11-2019

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	D	B	B,C,D	A,D	A,B,D	A,C,D	B,C	A,C,D
	Q.	11	12								
	A.	B,C,D	B,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	112.00	2.98 to 3.02	25.00	–3.00	16.20 to 16.40	6.66 to 6.67				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	C	D	A,C	A,B,D	A	A,B,C	A,D	A,B,C,D
	Q.	11	12								
	A.	A,C	A,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	2.00	1.00	4.00	4.00	9.20	5.00				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	A	D	B	A,B,D	A,C,D	B,C,D	A,B,D	A,C,D	B,C
	Q.	11	12								
	A.	B,D	A,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	10.66 or 10.67	6.25	7.50	0.00	22.50	4.00				

(1001CJA102119029) Test Pattern

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path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Main)
UNIT TEST
05-11-2019

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	C	C	C	A	C	D	B	B	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	D	A	D	A	C	B	C	B
SECTION-II	Q.	1	2	3	4	5					
	A.	184 to 188	0.84 to 0.89	40.00	0.93 to 0.97	78.00 to 79.00					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	C	A	A	D	A	D	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	D	C	A	D	A	C	A	B	C
SECTION-II	Q.	1	2	3	4	5					
	A.	3.12	4.00	4.00	4.00	6.00					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	B	C	B	B	B	B	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	A	C	A	B	A	B	C	D	D
SECTION-II	Q.	1	2	3	4	5					
	A.	8.00	0.75	1.00	3.23 or 3.24	2.00					

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Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Advanced)
UNIT TEST
05-11-2019

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

PAPER-1

PART-1 : PHYSICS — SOLUTION

SECTION-I

1. **Ans. (A)**

Sol. $$\frac{k(ze)e}{r^2}=\frac{mv^2}{r}$$

$$mvr=\frac{nh}{2\pi}$$

$$k(ze^2)=mv^2r=\left(\frac{nh}{2\pi}\right)v$$

$$\therefore v=\frac{k(ze^2)}{nh}(2\pi)$$

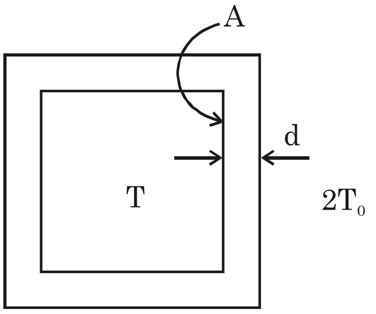
$$r=\left(\frac{nh}{2\pi}\right)\frac{1}{mv}=\left(\frac{nh}{2\pi}\right)\frac{nh}{m(kze^2)(2\pi)}$$

$$=\frac{n^2h^2}{(4\pi^2)ke^2zm}$$

$$\text{for } n=1,\ r=\frac{h^2(4\pi\in_0)}{4\pi^2e^2m}=\frac{h^2\in_0}{e^2m\pi}$$

2. **Ans. (A)**

Sol.



$$\frac{dQ}{dt}=\frac{2T_0-T}{R_{th}}=nCv\frac{dT}{dt}$$

$$\int_{T_0}^{T}\frac{dT}{2T_0-T}=\int_0^t\frac{dt}{nCvR_{th}}$$

$$\left[\frac{\ell n(2T_0-T)}{-1}\right]_{T_0}^{T}=\frac{1}{nCvR_{th}}t$$

$$\ell n(2T_0-T)-\ell n(T_0)=-\frac{t}{nCvR_{th}}$$

$$\left(2-\frac{T}{T_0}\right)=e^{-\frac{t}{R_{th}C_vn}}$$

$$T=T_0\left(2-e^{-\frac{t}{R_{th}C_Vn}}\right)=T_0\left(2-e^{-\frac{t(Ak)(2)}{d(5R)}}\right)$$

$$=T_0\left(2-e^{-\frac{2}{5}\frac{Ak}{Rd}t}\right)$$

3. **Ans. (D)**

Sol. $$\frac{1}{2}\left(\frac{F^2}{4AY}\right)\left(\frac{1}{Y}\right)\left(\frac{2AL}{2}\right)+\frac{1}{2}\left(\frac{F^2}{A^4}\right)\left(\frac{1}{Y}\right)\left(A\times\frac{L}{2}\right)$$

$$\frac{F^2L}{8AY}+\frac{2F^2L}{8AY}=\frac{3F^2L}{8AY}$$

4. **Ans. (B)**

Sol. We divide equation by the time increment Δt and equate it to the (constant speed $v = 100 \times 10^{-9}$ m/s.)

$$v=\alpha L_0\frac{\Delta T}{\Delta t}$$

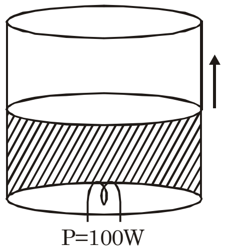
where $L_0 = 0.0200$ m and $\alpha = 23 \times 10^{-6}$/C°. Thus, we obtain

$$\frac{\Delta T}{\Delta t}=0.217\frac{C^\circ}{s}=0.217\frac{K}{s}.$$

5. **Ans. (B,C,D)**

6. **Ans. (A, D)**

Sol.



For water to steam conversion

$PV = nRT$

$$P\frac{dV}{dt}=\left(\frac{dn}{dt}\right)RT$$

$Q = mL_V$

$$\frac{dQ}{dt}=\frac{dm}{dt}L_V$$

$$P_0A\frac{dx}{dt}=\frac{1}{M_W}\left(\frac{dm}{dt}\right)RT$$

$$P_0AV=\frac{1}{M_W}\left(\frac{dQ/dt}{L_V}\right)RT$$

$$v=\frac{PRT}{M_WL_V\left(R_0A\right)}$$

$$=\frac{100\times\frac{25}{3}\times373}{18\times10^{-3}\times2.25\times10^{6}\times10^{5}\times\pi\times\left(\frac{0.1}{\sqrt{\pi}}\right)^2}$$

$$=\frac{100\times25\times373}{18\times10^{-3}\times3\times2.25\times10^{6}\times10^{5}\times0.01}$$

$= 7.67 \times 10^{-3}$ m/s

When temp. of steam starts increasing

$Q = mS_{Steam}\,T$

$$\frac{dQ}{dt}=mS_{Steam}\frac{dT}{dt}$$

PV = nRT

$$PA\frac{dx}{dt}=nR\frac{dT}{dt}$$

$$P_0Av=nR\left(\frac{P}{mS_{Steam}}\right)$$

$$v=\frac{nRP}{mS_{Steam}\times P_0A}$$

$$=\frac{\left(\frac{2}{18\times10^{-3}}\right)\times\frac{25}{3}\times100}{\left(2000\right)\times10^{5}\times\pi\times\left(\frac{0.1}{\sqrt{\pi}}\right)^2}$$

$$=\frac{\frac{2}{18}\times10^{3}\times\frac{25}{3}\times100}{2\times2000\times10^{5}\times0.01}$$

$= 2.31 \times 10^{-4} \times 10^{5} \times 10^{-5} \times 100$

= 23.1 mm/sec

7. Ans. (A,B,D)

Sol. $v_e=\frac{E}{B}=\frac{3.7\times10^{+2}}{10^{-3}}=3.7\times10^{5}\,m/s$

$$KE_{max}=\frac{1}{2}mv_e^2$$

$$=\frac{1}{2}\times9.31\times10^{-31}\times\left(3.7\times10^{5}\right)^2$$

= 0.4 eV

$\phi_B = 1.5$ eV

$h\nu = KE_{max} + \phi = 1.5 + 0.4 = 1.9$ eV

${}^nC_2 = 15$

n = 6

$$h\nu=13.6z^2\left(\frac{1}{m^2}-\frac{1}{6^2}\right)$$

$$\frac{1.9}{13.6}=z^2\left(\frac{1}{m^2}-\frac{1}{36}\right)$$

$$=z^2\left(\frac{1}{m^2}-\frac{1}{36}\right)$$

m = 4, z = 2

8. Ans. (A, C, D)

9. Ans. (B,C)

10. Ans. (A,C,D)

Sol. $\phi_e=\frac{hc}{\lambda_1}=\frac{1240.8}{4963}=2.5eV$

$$E_2=\frac{12400}{4133.33}=3eV$$

$E_3 = 2.48$ eV

$$E_4=\frac{12400}{7200}=1.722eV$$

Only λ_2 can emit photoelectrons

No. of photons absorbed by sphere per unit

$$\text{time }=\frac{40\times\pi\left(10^{-2}\right)^2}{4\pi\left(1\right)^2}\times\frac{4960\times10^{-10}}{6.626\times10^{-34}\times3\times10^{8}}$$

$= 10 \times 10^{-4} \times 250 \times 10^{16}$

$= 2500 \times 10^{+12} = 2.5 \times 10^{15}$

No. of electrons per unit time

$= 2.5 \times 10^{15} \times 10^{-6}$

$= 2.5 \times 10^{9}$

Charge per unit time

$= 2.5 \times 10^{+9} \times 1.6 \times 10^{-19}$

$= 4 \times 10^{-10}$

The emission of photoelectron stops when potential of sphere becomes equal to the slopping potential = 0.5 V

$$\frac{kq}{R}=0.5$$

$$\frac{9\times10^{9}\times4\times10^{-10}\times t}{0.01}=0.5$$

$$t=\frac{0.5\times0.01}{9\times10^{9}\times4\times10^{-10}}=1.39\times10^{-3}\sec$$

11. Ans. (B,C,D)

Sol. $\frac{N_0}{5}=N_0e^{-\lambda t_1}$

$$t_1=\frac{2\ell n5}{\ell n2}=2\log_2 5$$

$$N_0-\frac{9N_0}{\ell n2}=N_0e^{-\lambda t_2}$$

$$t_2 = \frac{2\ell n10}{\ell n2} = 2\log_2 10$$

$t_2 - t_1 = 2\ \log_2\ 10 - 2\log_2 5$

$t_2 - t_1 = 2$

$$\frac{N_0}{8} = N_0 e^{-\lambda(6)}$$

$$\frac{1}{8} = e^{-\lambda(6)}$$

$$\ell n8 = \lambda(6) = \frac{\ell n2}{T_{1/2}}(6)$$

$$T_{1/2} = \left(\frac{\ell n2}{\ell n8}\right)6$$

$T_{1/2}$ = 3 sec

12. Ans. (B, C)

Sol. The distribution function gives the fraction of particle with speeds between v and v + dv, so its integral over all speeds is unity : $\int P(v)dv = 1$. The average speed is defined as $v_{avg} = \int_0^{\infty} v^2 P(v) dv$.

(a) We normalize the distribution function as follows:

$$\int_0^{v_0} P(v)dv = 1 \Rightarrow C = \frac{3}{v_0^3}.$$

(b) The average speed as

$$\int_0^{v_0} vP(v)dv = \int_0^{v_0} v\left(\frac{3v^2}{v_0^3}\right)dv = \frac{3}{4}v_0$$

(c) The rms speed is the square root of

$$\int_0^{v_0} v^2P(v)dv = \int_0^{v_0} v^2\left(\frac{3v^2}{v_0^3}\right)dv = \frac{3}{5}v_0^2.$$

Therefore, $v_{rms} = \sqrt{3/5v_0} \approx 0.775v_0$

Note : The maximum speed of the gas is $v_{max} = v_0$, as indicated by the distribution function. Using equation, we find the fraction of molecules with speed between v_1 and v_2 to be

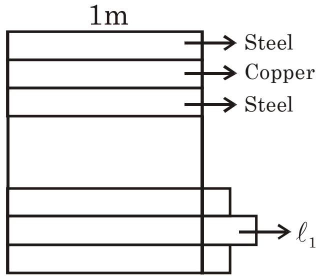
$$\text{frac} = \int_{v_1}^{v_3} P(v)dv = \int_{v_1}^{v_3}\left(\frac{3v^2}{v_0^3}\right)dv = \frac{3}{v_0^3}$$

$$= \int_{v_1}^{v_3} v^2 dv = \frac{v_2^3 - v_1^3}{v_0^3}$$

SECTION-II

1. Ans. 112.00

Sol.



$\Delta\ell_{copper} = \ell\alpha_{cm}\Delta T,\ \Delta\ell_{Steel} = \ell\alpha_{Steel}\Delta T$

Thermal strain in Cu = $\frac{(\ell\alpha_{Cu}\Delta T - \ell_1)}{1}$

Thermal strain in steel = $\frac{(\alpha_{Steel}\Delta T + \ell_1)}{1}$

$F_{Cu} = 2F_{Steel}$

$$\frac{y_{Cu}(\ell\alpha_{Cu}\Delta T - \ell_1)}{1} = 2y_{Steel}\frac{(-\ell\alpha_{Steel}\Delta T + \ell_1)}{1}$$

$2 \times 10^{11} (1 \times 18 \times 10^{-6} \times 20 - \ell_1) = 2 \times 4 \times 10^{11}$

$(\ell_1 - 1 \times 11 \times 10^{-6} \times 20)$

$18 \times 10^{-6} \times 20 - \ell_1 = 4\ell_1 = 44 \times 10^{-6} \times 20$

$5\ell_1 = 62 \times 20 \times 10^{-6}$

$\ell_1 = 248 \times 10^{-6}$

$F_{Cu} = 2 \times 10^{11} \times 50 \times 10^{-4}$

$(1 \times 18 \times 10^{-6} \times 20 - 248 \times 10^{-6})$

$= 112 \times 10^3$ N

2. Ans. 2.98 to 3.02

Sol. Wavelength gap : $|\lambda_{\text{cut off}} - \lambda_{k\alpha}|$

$$= \lambda_{K_a} - \frac{hc}{ev_a}$$

$$\left[\lambda_{K_\alpha} - \frac{hc}{e(20)}\right] = \eta\left[\lambda_{K_\alpha} - \frac{hc}{e(10)}\right]$$

$$\lambda_{K_\alpha}[1-\eta] = \eta\left[\frac{hc}{e(20)} - \frac{hc}{e(10)}\eta\right] \times 10^{-3}$$

$$= \frac{hc}{e}\left[\frac{1}{20} - \frac{\eta}{10}\right] \times 10^{-3}$$

$$\frac{hc}{\lambda_{K_\alpha}} = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(28)^2$$

$$\frac{hc}{\lambda_{K_\alpha}} = 13.6 \times \frac{3}{4} \times 28^2$$

$$\lambda_{K_\alpha} = \frac{hc \times 4}{13.6 \times 3 \times 28^2}$$

$$\frac{4hc}{13.6\times 3\times 28^2 e}[1-\eta]=\frac{hc}{e}\left[\frac{1}{20}-\frac{h}{10}\right]\times 10^{-3}$$

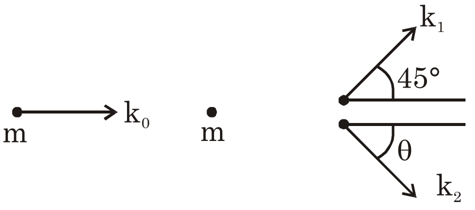
$$\frac{1}{8}(1-\eta)=\left(\frac{1}{20}-\frac{\eta}{10}\right)$$

$$\frac{1-\eta}{2-\eta}=\frac{8}{10}=\frac{4}{5}$$

$\therefore \eta = 3$

3. **Ans. 25.00**

Sol.



$$\sqrt{2mk_1}\sin 45° = \sqrt{2mk_2}\sin\theta$$

$$\sqrt{2mk_0}=\sqrt{2mk_1}\cos 45°+\sqrt{2mk_2}\cos\theta$$

Squaring and adding

$$k_2 = k_1 + k_2 - \sqrt{2k_1k_2}$$

and $k_2 = k_0 - k_1$

$$k_1 = \frac{k_0}{2}$$

After n-collisions

$$k_n = k_0\left(\frac{1}{2}\right)^n$$

$$0.23 = \left(4.6\times 10^6\right)\left(\frac{1}{2}\right)^n$$

$2^n = 2 \times 10^7$

$2^{n-1} = 10^7$

$n = 24.25$

4. **Ans. –3.00**

Sol. For convenience, the "int" subscript fro the internal energy will be omitted in this solution. Recalling equation, we not that

$\sum_{cycle} E = 0$, which gives

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0$$

Now, with $\Delta E_{E\to A}$ = 8.0 J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} = 8.0\ J = 0.$$

In an adiabatic process, $\Delta E = -W$, which leads to

$$-5.0\ J + \Delta E_{C\to D} + 8.0\ J = 0,$$

and we obtain $\Delta E_{C\to D} = -3.0$ J.

5. **Ans. 16.2 to 16.4**

Sol. Let p_1, V_1, and T_1 represent the pressure, volume, and temperature of the air at $y_1 = 4267$ m. Similarly, let p, v, and T be the pressure, volume, and temperature of the air at y = 1567 m. Since the process is adiabatic, $p_1V_1^\gamma = pV^\gamma$. Combining with the ideal gas law, pV = NkT, we obtain

$$pV^\gamma = p(T/p)^\gamma = p^{1-\gamma}T^\gamma = \text{constant}$$

$$\Rightarrow p^{1-\gamma}T^\gamma = p_1^{1-\gamma}T$$

With $p = p_0e^{-ay}$ and $\gamma = 4/3$ (which gives $(1-\gamma)/\gamma = -1/4$), the temperature at the end of the descent is

$$T=\left(\frac{p_1}{p}\right)^{\frac{1-\gamma}{\gamma}}T_1=\left(\frac{p_0e^{-ay_1}}{p_0e^{-ay}}\right)^{\frac{1-\gamma}{\gamma}}T_1=e^{-a(y-y_1)/4}$$

$$T = e^{-\frac{\left(1.4\times 10^{-4}(1567-4267)\right)}{4}}\times 263 = 289.3\ K$$

T = 16.3°C.

6. **Ans. 6.66 or 6.67**

Sol. The energy of a photon in terms of the momentum is

$$E = hf = \frac{hc}{gl} = pc$$

The rate at which photons are striking the sail is

$$\frac{N}{\Delta t}=\frac{IA}{E}=\frac{IA}{pc}.$$

Because the photons reflect from the sail, the change in momentum of the photons :

$F\Delta t = N\Delta p$, or

$$F=\left(\frac{N}{\Delta t}\right)\Delta p=\left(\frac{IA}{pc}\right)(2p)=\frac{2IA}{c}$$

$$=\frac{2\left(1000W/m^2\right)\left(1\times 10^3 m\right)^2}{\left(3.00\times 10^8 m/s\right)}=\frac{20}{3}N$$

PART–2 : CHEMISTRY

SOLUTION

SECTION-I

1. Ans.(A)

Sol. $NH_2^{\ominus} \longrightarrow sp^3$

$PCl_6^{\ominus} \longrightarrow sp^3d^2$

$NO_3^- \longrightarrow sp^2$

$OSF_4 \longrightarrow sp^3d$

2. Ans.(B)

Sol. $CO + HCl + AlCl_3 \longrightarrow H - \underset{\uparrow}{C} \equiv \overset{+}{O} + AlCl_4^-$

Attacking electrophile

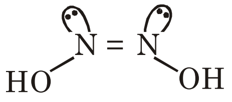
3. Ans.(C)

Sol. B.P. (boiling point) $\propto$ extent of H-bonding

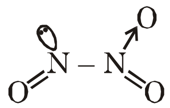
Molecule	Extent of H-bonding
HF	2
H_2O	4
H_2O_2	6
HCl	H-bonding is not present

4. Ans.(D)

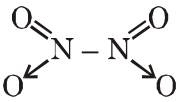
Sol. (A) Hyponitrous acid ($H_2N_2O_2$)



(B) N_2O_3



(C) N_2O_4



(D) $N \equiv N \rightarrow O$ N_2O has max. B.O. thus shortest bond length

5. Ans.(A,C)

Sol. Bromination of phenol in polar solvent give tribromo phenol

6. Ans.(A,B,D)

(I) $NaCl(aq.) \rightleftharpoons Na^+(aq.) + Cl^-(aq.)$

0.1 [1 – 0.9] [0.09] [0.09]

$\Rightarrow C_{total} = 0.19$ M

(II) $CaCl_2(aq.) \rightleftharpoons Ca^{2+}(aq.) + 2Cl^-(aq.)$

0.5 [1 – 0.8] 0.05[0.8] 0.05(0.8)(2)]

$\Rightarrow C_{total} = 0.13$ M

(III) $K_3[Fe(CN)_6] \rightleftharpoons 3K^+ + Fe(CN)_6^{3-}$

0.04 [1 – 0.6] 0.04 × 0.6 × 3 0.04 × 0.6

$\Rightarrow C_{total} = 0.112$ M

(IV) $FeCl \rightleftharpoons 3Fe^{3+} + 3Cl^-$

0.03 [1 –0.75] 0.3 × 0.7 0.03 × 0.7 × 3

$\Rightarrow C_{total} = 0.093$ M

7. Ans.(A)

8. Ans.(A, B, C)

Sol. Aldehyde, α-hydroxyketone, formic acid all gives positive T.R. and F.S. test

9. Ans.(A,D)

Sol. Lactose and maltose both have hemiacetal linkage.

10. Ans.(A,B,C,D)

11. Ans.(A,C)

12. Ans.(A,D)

SECTION-II

1. Ans.(2.00)

2. Ans.(1.00)

3. Ans.(4.00)

4. Ans.(4.00)

5. Ans.(9.20)

6. Ans.(5.00)

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (B)

Sol. $x = \dfrac{-3 \pm \sqrt{3}\,i}{2} = -1 + \omega, -1 + \omega^2$

Let $\alpha = -1 + \omega$ & $\beta = -1 + \omega^2$

$\Rightarrow (\alpha + 1)^{100} + (\beta + 2)^{104} = \omega^{100} + (-\omega)^{104} = -1$

2. Ans. (A)

Sol. $\sum_{r=1}^{100} a_{2r} - \sum_{r=1}^{100} a_{2r-1} = 100d = 50$

& $\sum_{r=1}^{100} a_{2r} + \sum_{r=1}^{100} a_{2r-1} = \dfrac{200}{2}(2a_1 + 199d) = 150$

$\Rightarrow a_1 = -49$

3. Ans. (D)

Sol. $x^3 - 2x + 3 = (x - \alpha)(x - \beta)(x - \gamma)$

$\Rightarrow (2 - \alpha)(2 - \beta)(2 - \gamma) = 7$

$(\because \alpha + \beta + \gamma = 0)$

4. Ans. (B)

Sol. $A^T = A$ & $B^T = -B$

(A) $((A + B)(A - B))^T = (A - B)^T(A + B)^T$

$= (A + B)(A - B)$

(B) $(AB^TA)^T = (-ABA)^T = -A^TB^TA^T = ABA$

(C) $(AB + BA)^T = (AB)^T + (BA)^T = -BA - AB$

(D) $(AB - BA)^T = (AB)^T - (BA)^T = -BA + AB$

5. Ans. (A,B,D)

Sol. $\dfrac{\alpha^3 - \beta^3}{(\alpha - \beta)^3} = \dfrac{73}{3}$

$\dfrac{\alpha^2 + \beta^2 + \alpha\beta}{(\alpha - \beta)^2} = \dfrac{73}{3}$

$\Rightarrow \dfrac{3\alpha\beta}{(\alpha - \beta)^2} = \dfrac{70}{3}$

$\Rightarrow \dfrac{4\alpha\beta}{(\alpha - \beta)^2} = \dfrac{280}{9} \Rightarrow \dfrac{(\alpha + \beta)^2}{(\alpha - \beta)^2} = \dfrac{289}{9}$

$\Rightarrow \dfrac{\alpha + \beta}{\alpha - \beta} = \dfrac{17}{3} \Rightarrow \alpha = 10, \beta = 7$

p, q, r can be rational & irrational

6. Ans. (A,C,D)

Sol. $\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = a$

$\alpha\beta\gamma = -a$ &

$\alpha^3\gamma + \beta^3\alpha + \gamma^3\beta = -8\alpha\beta\gamma$

$\because \alpha^3 + a\alpha + a = 0$

$\therefore \sum(-a\alpha - a)\gamma = -8\alpha\beta\gamma$

$\Rightarrow -a(a) - a(0) = 8a$

$\Rightarrow a = -8$

$\Rightarrow$ roots are $-2,\ 1 \pm \sqrt{5}$

7. Ans. (B,C,D)

Sol. $A - G = 3,\ G - H = \dfrac{12}{5}$ & $AH = G^2$

$\Rightarrow G = 12, A = 15$ & $H = \dfrac{48}{5}$ &

$a_1 = 24, a_2 = 6$ & $r = \dfrac{1}{4}$

$\Rightarrow a_3 = a_1r^2 = \dfrac{24}{16} = \dfrac{3}{2}$

& $\sum_{i=1}^{\infty} a_i = \dfrac{24}{3/4} = 32$

8. Ans. (A,B,D)

Sol. $S_{99} = (-1^2 + 4^2) + (-2^2 + 5^2) + (-3^2 + 6^2) + \ldots.$

$= 3(1 + 2 + 3 + 4 + 5 + 6 + \ldots.\ 99 \text{ times})$

$= 3(4950)$

$S_{100} = S_{99} + T_{100} = S_{99} + (100)^2$

(A) $S_{100} > S_{99}$ &

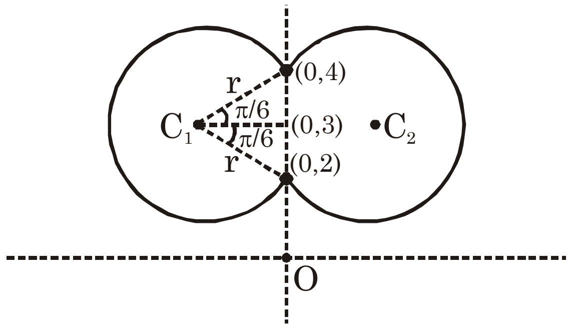
(B) $S_{98} = S_{99} - T_{99} = S_{99} + (99^2)$

$S_{98} > S_{99}$

(D) $T_{100} - S_{99} = 100^2 - 3(4950) = 4850$

9. Ans. (A,C,D)

Sol.



$r = 2$ & $C_1(-\sqrt{3}, 3)$ and $C_2(\sqrt{3}, 3)$

(A) $|z|_{\max} = OC_1 + r = \sqrt{12} + 2$

$|z|_{\min} = OC_1 - r = \sqrt{12} - 2$

(C) $|z-\sqrt{3}-3i|_{\min}=r=2$

(D) Area $= 2\left(\frac{1}{2}.4.\frac{5\pi}{3}+\frac{\sqrt{3}}{4}.4\right)$

10. Ans. (B,C)

Sol. $(z^2+1)(z^2+z+1)=0$

roots are i, –i, ω & ω^2

11. Ans. (B,D)

Sol. $A^2 \text{ adj } (A) = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ k & 0 & 2 \end{bmatrix}$

$\Rightarrow A \,.\, A \text{ adj } (A) = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ k & 0 & 2 \end{bmatrix}$

$\Rightarrow |A|\ A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ k & 0 & 2 \end{bmatrix}$

$\Rightarrow |A|^4 = \begin{vmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ k & 0 & 2 \end{vmatrix} \Rightarrow k = 2$

$\Rightarrow A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$

13. Ans. (A,D)

Sol. $D = pq - 3$

$D_1 = q + 3$

$D_2 = -pq - 6p - 3$

$D_3 = 3p + 3$

(A) Unique solution when $D \neq 0 \Rightarrow pq \neq 3$

(B) Infinite solution when $pq = 3$, $q = -3$ & $p = -1$

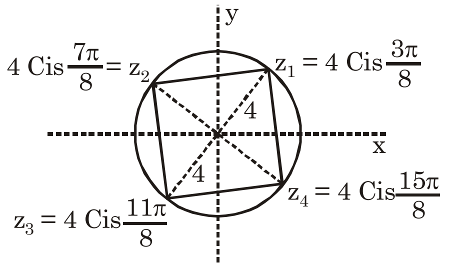
(D) No solution when $pq = 3$ but $p \neq -1$ or $q \neq -3$

SECTION–II

1. Ans. 10.66 or 10.67

Sol. $\arg(z^4) = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \arg(z) = (4n-1)\frac{\pi}{8}$



Area (A) $= \frac{d^2}{2} = \frac{64}{2} = 32$

2. Ans. 6.25

Sol. $|2z_1+\bar{z}_2|^2 - |1+2z_1z_2|^2 = 8-9$

$\Rightarrow 4\ |z_1|^2 + |z_2|^2 - 1 - 4|z_1|^2|z_2|^2 = -1$

$\Rightarrow \frac{4}{|z_2|^2} + \frac{1}{|z_1|^2} = 4$

A.M. $\geq$ H.M.

$$\frac{|z_1|^2 + 4|z_2|^2}{5} \geq \frac{5}{\frac{1}{|z_1|^2} + \frac{4}{|z_2|^2}}$$

$\Rightarrow |z_1|^2 + 4|z_2|^2 \geq \frac{25}{4}$

3. Ans. 7.50

Sol. $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$\begin{vmatrix} x^3 & 8+6x(x+2) & -8-6x(x-2) \\ y^3 & 8+6y(y+2) & -8-6y(y-2) \\ z^3 & 8+6z(z+2) & -8-6z(z-2) \end{vmatrix} = 0$$

$C_3 \to C_3 + C_2$

$$\begin{vmatrix} x^3 & 8+6x(x+2) & x \\ y^3 & 8+6y(y+2) & y \\ z^3 & 8+6z(z+2) & z \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x^3 & 8+6x^2 & x \\ y^3 & 8+6y^2 & y \\ z^3 & 8+6z^2 & z \end{vmatrix} = 0$$

$$\Rightarrow 8\begin{vmatrix} x^3 & 1 & x \\ y^3 & 1 & y \\ z^3 & 1 & z \end{vmatrix} + 6xyz\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-y)(y-z)(z-x)(8x+8y+8z-6xyz) = 0$$

$$\Rightarrow \frac{x+y+z}{xyz} = \frac{3}{4}$$

4. Ans. 0.00

Sol. $A^2 = I$

$\Rightarrow B = 1010I + 1010A$

$$B = \begin{bmatrix} 1010 & 1010 \\ 1010 & 1010 \end{bmatrix} \Rightarrow |B| = 0$$

5. Ans. 22.50

Sol. $a - 2d + a - d + a + a + d + a + 2d = 10$

$\Rightarrow a = 2$ &

$$\frac{1}{a-2d} + \frac{1}{a+2d} + \frac{1}{a-d} + \frac{1}{a+d} + \frac{1}{a} = \frac{29}{10}$$

$$\Rightarrow \frac{4}{4-4d^2} + \frac{4}{4-d^2} + \frac{1}{2} = \frac{29}{10}$$

$$\Rightarrow d^2 = \frac{1}{4} \text{ or } \frac{8}{3} \text{ (reject)}$$

$\Rightarrow$ numbers are $1, \frac{3}{2}, 2, \frac{5}{2}, 3$

6. Ans. 4.00

Sol. $F(x) \,.\, F(-x) = I$

$\Rightarrow p^2\cos^2x + \sin^2x = 1, \;\; (p-q)\sin x\cos x = 0$

$\sin^2x + q^2\cos^2x = 1$ & $r^2 = 1$

$\Rightarrow p = q \,,\, p^2 = q^2 = r^2 = 1$

$\Rightarrow$ 4 triplets

(1001CJA102119029) **Test Pattern**

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Main)
UNIT TEST
05-11-2019

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (D)

Sol. $r_N = \dfrac{n^2 r_0}{z}$

$0.847 \times 10^{-9} = n^2 (0.529 \times 10^{-10}$

$n^2 = \dfrac{8.47}{0.529}$

$n^2 = 16$

$n = 4$

2. Ans. (C)

Sol. $KE_{loss} = \dfrac{1}{2}\left(\dfrac{m}{2}\right)v_0^2 = 10.2\text{eV}$

$\dfrac{1}{2}\left(mv_0^2\right) = 20.4\text{eV}$

3. Ans. (C)

Sol. $n_{N_2} = \dfrac{m}{28}$

$n_{O_2} = \dfrac{m}{32}$

$KE_{N_2} = \dfrac{5}{2}k(300)$

$KE_{O_2} = \dfrac{5}{2}k(600)$

$P_1V_1 = n_1RT$

$P_1 = \dfrac{m}{28}\dfrac{R(300)}{V}$

$P_2 = \left(\dfrac{m}{32}\right)\dfrac{R(600)}{V}$

$(P_1 < P_2)$

4. Ans. (C)

Sol. For A → B : $v \propto T$

P is constant

For C → D : $v = mT + C$

$\dfrac{nRT}{P} = mT + C$

$P = \dfrac{nRT}{mT + C} = \dfrac{nR}{\left(m + \dfrac{C}{T}\right)}$

P increases, as T increases.

For E → F : $P = \dfrac{nRT}{mT - C} = \dfrac{nRT}{m - \dfrac{C}{T}}$

P decreases, as T increases.

5. Ans. (A)

Sol. Total number of photons in one second

$= 10^{12} \times 2 \times 10^{-4} = 2 \times 10^8$

Total number of photoelectrons in one second

$= 2 \times 10^8 \times 10^{-5} = 2 \times 10^3$

Total number of photoelectrons in 25 seconds

$= 2 \times 10^3 \times 25$

$= 5 \times 10^4$

Charge on plate in 25 seconds

$= 5 \times 10^4 \times 1.6 \times 10^{-19} = 8 \times 10^{-15}$ C

6. Ans.(C)

Sol. $N = N_0e^{-\lambda t}$

$\dfrac{dN}{dt} = \lambda N_0 e^{-\lambda t}$

$A = \lambda N_0 e^{-\lambda t_0}$

$3Ae = \lambda(3N_0)e^{-\lambda t}$

$\left(\lambda N_0 e^{-\lambda t_0}\right)e = \lambda N_0 e^{-\lambda t}$

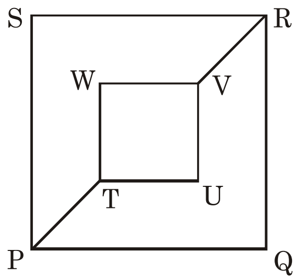
$e^{-\lambda t_0 + 1} = e^{-\lambda t}$

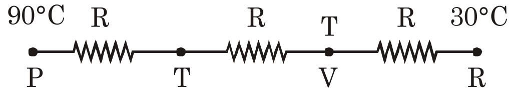
$\lambda t = \lambda t_0 - 1$

$t = t_0 - \dfrac{1}{\lambda}$

7. Ans. (D)

Sol.





$\dfrac{90 - T}{2R} = \dfrac{T - 30}{R}$

$90 - T = 2T - 60$
$3T = 150$
$T = 50°C$

8. Ans. (B)

Sol. Pressure = $\frac{2I}{C}$

Force = $\frac{2IA}{C}$

$\frac{2IA}{C} = Rx_{eq}$

$x_{eq} = \frac{2IA}{KC}$

Amplitude = $\frac{2IA}{KC}$

9. Ans. (B)

10. Ans. (B)

Sol. $N_A = N_0 e^{-\lambda_A t}$

$N_B = N_0 e^{-\lambda_B t}$

at t = 12 hrs,

$N_A = \frac{N_0}{2^6} = \frac{N_0}{64}, \frac{dN}{dt} = \lambda_A \frac{N_0}{64}$

$N_B = \frac{N_0}{2^3} = \frac{N_0}{8}, \frac{dN}{dt} = \lambda_B \frac{N_0}{8}$

at t = 0, $\left.\frac{dN}{dt}\right|_{total} = \lambda_A N_0 + \lambda_B N_0$

$= \left(\frac{\ell n2}{2} + \frac{\ell n2}{4}\right) N_0 = \frac{3}{4}(\ell n2) N_0$

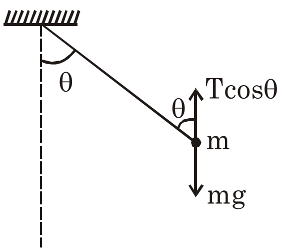
at t = 12, $\left.\frac{dN}{dt}\right|_{total} = \frac{\ell n2}{2} \cdot \frac{N_0}{64} + \frac{\ell n2}{4} \cdot \frac{N_0}{8}$

$= \frac{5}{128}(\ell n2) N_0$

Ratio = $= \frac{5/128}{3/4} = \frac{5}{96}$

11. Ans. (A)

Sol.



$T\cos\theta = mg$
$T = mg\sec\theta$

$\frac{T}{A} = \frac{Y\Delta\ell}{\ell}$

$\Delta\ell = \frac{(mg\sec\theta)4\ell}{\pi d^2 Y} = \frac{4mg\ell\sec\theta}{Y\pi d^2}$

12. Ans. (C)

Sol. $\sqrt{v} = k(z-b)$

$\frac{1}{\sqrt{\lambda}} = k(z-b)$

$\frac{1}{\sqrt{\lambda}} = k(11-1)$

$\frac{1}{\sqrt{4\lambda}} = k(z-1) = \frac{1}{2\sqrt{\lambda}} = \frac{1}{2}(k(10))$

$k(z-1) = \frac{1}{2}k(10)$

(z = 6)

13. Ans. (D)

Sol. $T = \alpha V^2$

$PV = nR(\alpha V^2)$

$P = nR\alpha V$

$P = 80V$

$W = \int PdV$

$\int_{1.5}^{9} 80VdV = \frac{80}{2}[81 - 2.25] = 3150 \text{ J}]$

$\Delta U = \frac{f}{2}nR\Delta T = \frac{4}{2}(1)(R)\alpha\left[9^2 - 1.5^2\right]$

$= 2(80)\ [78.75] = 12600 \text{ J}$

Heat loss = 12600 + 3150 = 15750 J

14. Ans. (A)

15. Ans. (D)

Sol. $\vec{E} = \frac{100}{2}\left[\sin\left(3\times10^{15}t\right) - \sin\left(9\times10^{15}t\right)\right]$

Maximum frequency is $\frac{9\times10^{15}}{2\pi}$

$h\nu - f = KE_{max}$

$KE_{max} = \left(\frac{6.626\times10^{-34}\times9\times10^{15}}{1.6\times10^{-19}\times2\pi} - 2\right) = 3.93 \text{ eV}$

16. Ans. (A)

Sol. For isotropic materials, the coefficient of linear expansion α is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$. The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$\Delta R_E = E_E\alpha\Delta T = \left(6.4\times10^3 \text{km}\right)$

$\left(\frac{1}{3}\right)\left(3.0\times10^{-5}/K\right)$

(3000 K – 300 K) = 1.7 × 10^2 km.

17. Ans. (C)

Sol. (a) The average speed is

$$v_{avg} = \frac{\Sigma n_i v_i}{\Sigma n_i}$$

$$= \frac{\left[2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)\right]\text{cm/s}}{2+4+6+8+2}$$

= 3.2 cm/s.

(b) From $v_{rms} = \sqrt{En_i v_i^2 / En_i}$ we get

$$v_{rms} = \sqrt{\frac{2(1.0)^2+4(2.0)^2+6(3.0)^2+8(4.0)^2+2(5.0)^2}{2+4+6+8+2}}\,\text{cm/s}$$

= 3.4cm/s.

(c) There are eight particles at v = 4.0 cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

18. Ans. (B)

19. Ans. (C)

Sol. We assume that the elapsed time is much smaller than the half-life, so we can use a constant decay rate.

Because ^{87}Sr is stable, and there was none present when the rocks were formed, every atom of ^{87}Rb that decayed is now an atom of ^{87} Sr. Thus we have

$N_{Sr} = -\Delta N_{Rb} = \lambda N_{Rb}\Delta t$, or

$$\frac{N_{Sr}}{N_{Rb}} = \left(\frac{0.693}{T_{1/2}}\right)\Delta t\ ;$$

$0.0160 = \left(\dfrac{0.693}{4.75\times10^{10}\,\text{yr}}\right)\Delta t$, which gives Δt = 1.1×10^9 yr

This is = 2% of the half-life, so our original assumption is valid.

20. Ans. (B)

Sol. The heat that must be removed from the water (Q_L) is found in three parts – cooling the liquid water to the freezing point, freezing the liquid water, and then cooling the ice to the final temperatures.

$$Q_L = m(c_{liquid}\Delta T_{liquid} + L_{fusion} + c_{ice}\Delta T_{ice})$$

$$= (0.50\text{kg})\left[\begin{array}{l}\left(\dfrac{4200\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)(27\text{C}^\circ)+\left(3.33\times10^5\,\text{J/kg}\right)\\ +\left(2100\text{J/kg}\cdot\text{C}^\circ\right)(23\text{C}^\circ)\end{array}\right]$$

= 247350J

The Carnot efficiency can be used to find the work done by the refrigerator.

$$e = 1-\frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W+Q_L} \rightarrow$$

$$W = Q_L\left(\frac{T_H}{T_L}-1\right) = 247350$$

$$\left(\frac{(27+273)\text{K}}{(-23+273)\text{K}}-1\right) = 49470\text{J}$$

SECTION-II

1. Ans. 1.84 to 1.88

Sol. Maximum tension in string

$= 2 \times 10^9 \times 6.5 \times 10^{-9} = 13$ N

$$\frac{2(1)(M)}{M+1}\times10 = 13$$

$$\frac{2M}{M+1} = \frac{13}{10} = 13$$

2M = 1.3 M + 1.3

0.7 M = 1.3

M = 1.857

2. Ans. 0.84 to 0.89

Sol. $\dfrac{F_t}{A} = \eta\phi$

$$\phi = \frac{F_t}{\eta A} = \frac{25}{1.9\times10^4\times15\times10^{-4}} = 0.87$$

3. Ans. 40.00

4. Ans. 0.93 to 0.97

5. Ans. 78.00 to 79.00

Sol. From ice at –12°C to ice at 0°C

$$\Delta S = \int\frac{dQ}{T} = \int mS_{ice}\frac{dT}{T}$$

$$= (0.03)\ (2100)\ \ell n\left(\frac{273}{261}\right) = (0.03)(2100)\,\ell n\left(\frac{273}{261}\right)$$

= 0.03 × 2100 × 0.04 = 2.52 J/K

From ice at 0°C to water at 0°C

$$\Delta S = \int\frac{dQ}{T} = \frac{mL}{T} = \frac{0.03\times3.33\times10^5}{273} = 36.59\ \text{J/K}$$

From water at 0°C to water at 100°C

$$\Delta S = \int\frac{dQ}{T} = mS_{water}\int\frac{dT}{T}$$

$$= (0.03)(4200)\,\ell n\left(\frac{373}{273}\right) = 39.312\ \text{J/K}$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

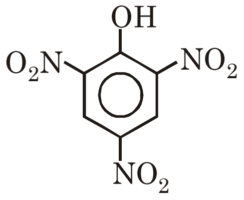
1. **Ans.(D)**

2. **Ans.(D)**

3. **Ans.(C)**

4. **Ans.(A)**

Sol. X =



Picric acid-(Ka of picric acid is more than H_2CO_3)

5. **Ans.(A)**

Sol. If R is $-CH_3$ (lowest alkyl) then above method gives amino acid of more than 2-carbon atoms while glycine is

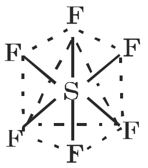
$HOOC - CH_2 - NH_2$.

6. **Ans.(D)**

Sol. In Lassaigne test if both 'N' and 'S' are present then blood red colour is obtained due to formation of $[Fe(SCN)]^{+2}$.

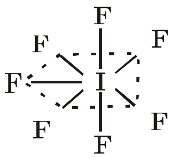
7. **Ans.(A)**

Sol. (A) SF_6 (sp^3d^2)



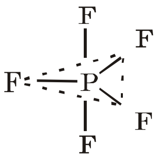
no. of 90° angle ⇒ 12

(B) IF_7 (sp^3d^3)



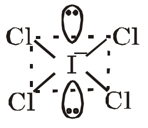
no. of 90° angle ⇒ 10

(C) PF_5 (sp^3d)



no. of 90° angle ⇒ 6

(D) IF_7 (sp^3d^3)



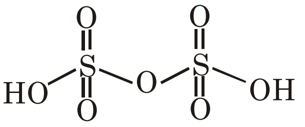
no. of 90° angle ⇒ 4

8. **Ans.(D)**

Sol. H_2O, acetic acid and fumaric acid all having inter molecular H-bonding.

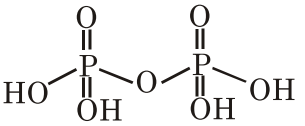
9. **Ans.(B)**

Sol. (A) Pyrosulphuric acid ($H_2S_2O_7$)



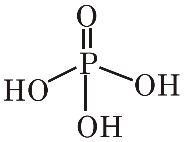
2 ionizable H-atom

(B) Pyrophosphoric acid ($H_4P_2O_7$)



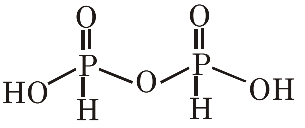
4 ionizable H-atom

(C) Orthophosphoric acid (H_3PO_4)



3 ionizable H-atom

(D) Pyrophosphorous acid ($H_4P_2O_5$)



2 ionizable H-atom

10. **Ans.(A)**

11. **Ans.(B)**

12. **Ans.(D)**

13. **Ans.(C)**

14. Ans.(A)

Sol. Hanny - Smith equation

% ionic character = [(ΔEN)16 + 3.5 $(\Delta EN)^2$]

ΔEN ∝ % ionic character

ΔEN is max for HF thus % ionic character is max in case of HF

15. Ans.(D)

Sol. $XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$

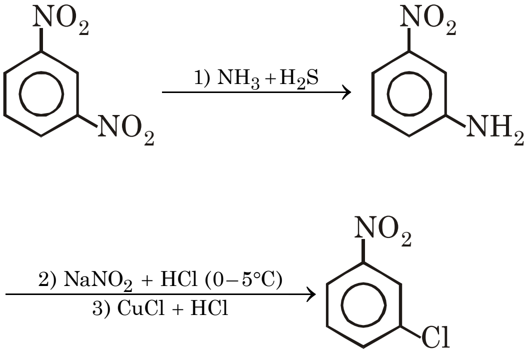
$XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

16. Ans.(A)

17. Ans.(C)

Sol.



18. Ans.(A)

Sol. (A) $N_2^{\oplus}$

$$B.O. = \frac{1}{2}(\text{bonding } e^{\ominus} - \text{anti bonding } e^{\ominus})$$

$$= \frac{1}{2}(9-4) = 2.5$$

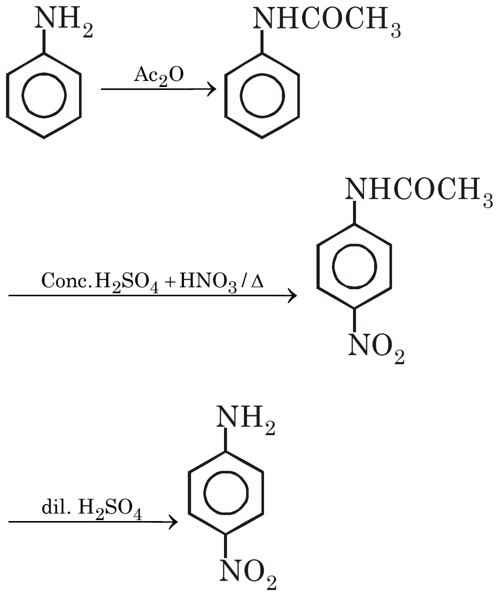
(B) O_2 B.O. ⇒ 2

(C) O_2^{2-} BO = 1

(D) $C_2^{\oplus}$ BO = 1.5

19. Ans.(B)

Sol.



20. Ans.(C)

Sol. Grphite & diamond contains dangling bond while fullerene does not contain dangling bond.

SECTION-II

1. Ans.(3.12)

$$\frac{0.5\alpha^2}{1-\alpha^2} = 2.5\times10^{-2} \Rightarrow \alpha = 0.2$$

⇒ i = 1.2

ΔT_b = 1.2 × 5.2 × 0.5

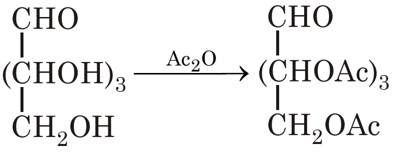
3.12°C = 3.12 K

2. Ans.(4.00)

3. Ans.(4.00)

4. Ans.(4.00)

Sol. (A) ,(E) , (F), (G)



$$\begin{array}{c}CHO\\|\\(CHOH)_3\\|\\CH_2OH\end{array} \xrightarrow{Ac_2O} \begin{array}{c}CHO\\|\\(CHOAc)_3\\|\\CH_2OAc\end{array}$$

Ac = CH_3CO–

Molecular mass increased by 168 unit

5. Ans.(6.00)

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (B)

Sol. Let d_1, d_2, d_3 be common difference of the A.Ps then

$a_1 + b_1 + c_1 = 10$

$a_1 + b_1 + c_1 + d_1 + d_2 + d_3 = 20$

$\therefore d_1 + d_2 + d_3 = 10$

Value of $a_1 + b_1 + c_1 + 2018(d_1 + d_2 + d_3)$

$= 10 + 20180$

$= 20190$

2. Ans. (D)

Sol. $[2(k - a)]^2 - 4(k^2 + 4^2 - 16k - b + 12) = 0$

$4[k^2 + a^2 - 2ak - k^2 - a^2 + 16k + b - 12] = 0$

$16k - 2ak + b - 12 = 0$

$2k(8 - a) + b - 12 = 0$

$a = 8, \; b = 12$

3. Ans. (B)

Sol. $4\log_2\lambda = 8$

$2\log_2\lambda = 3$

$\lambda = 2\sqrt{2}$

4. Ans. (C)

Sol. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2 + 9n + 13)$

$$S_{n-1} = \frac{1}{6}(n-1)\left[2(n-1)^2 + 9(n-1) + 13\right]$$

$$= \frac{1}{6}(n-1)(2n^2 + 5n + 6)$$

$$= \frac{1}{6}(2n^3 + 5n^2 + 6n - 2n^2 - 5n - 6)$$

$t_n = S_n - S_{n-1}$

$$= \frac{1}{6}[2n^3 + 9n^2 + 13n - 2n^3 - 3n^2 - n + 6]$$

$$t_n = \frac{1}{6}[6n^2 + 12n + 6] = (n+1)^2$$

$$\sum_{r=1}^{n}\sqrt{t_r} = (n+1) = \frac{n(n+1)}{2} + n \Rightarrow \frac{n(n+3)}{2}$$

5. Ans. (B)

Sol. $\frac{a_1 r^2}{a_1} = 9 \quad \therefore r = 3$

$a_1 + a_1 r = \frac{4}{3} \;\therefore\; 4a_1 = \frac{4}{3} \;\therefore\; a_1 = \frac{1}{3}$

$a_4 = \frac{1}{3}(3)^3 = 9$

6. Ans. (B)

Sol. $p + q = \frac{5}{3}$, $pq = \frac{-2}{3}$

Sum of roots = $3[p + q] - 2(p + q)$

$= p + q = \frac{5}{3}$

Product $(3p - 2q)(3q - 2p)$

$\Rightarrow 13pq - 6(p^2 + q^2)$

$\Rightarrow 13pq - 6((p + q)^2 - 2pq)$

$\Rightarrow 25 \times \frac{-2}{3} - 6 \times \frac{25}{9}$

$\Rightarrow \frac{-50}{3} - \frac{50}{3} = \frac{-100}{3}$

$x^2 - \frac{5}{3}x - \frac{100}{3} = 0$

$3x^2 - 5x - 100 = 0$

7. Ans. (B)

Sol. $\alpha^2 = 2\alpha + 1$

$\alpha^3 = 5\alpha + 2$

$\alpha^4 = 12\alpha + 5$

$5\alpha^4 = 5(12\alpha + 5) + 12(5\beta + 2)$

$\Rightarrow 60(\alpha + \beta) + 49 = 169$

8. Ans. (B)

Sol. $\frac{2ac}{a+c} = b$

Now $a - \frac{b}{c} = a - \frac{ac}{a+c} = \frac{a^2}{a+c}$

$c - \frac{b}{2} = c - \frac{ac}{a+c} = \frac{a^2}{a+c}$

$\frac{b}{2} = \frac{ac}{a+c}$

$\frac{a^2}{a+c}, \frac{ac}{a+c}, \frac{c^2}{a+c}$ are in G.P.

9. Ans. (B)

Sol. $(N^T M^{-1} N^{-1})^T = (-N M^{-1} N^{-1})^T$

$= (-N(NM)^{-1})^T = (-N(MN)^{-1})^T$

$= (-N N^{-1} M^{-1})^T$

$= (-M^{-1})^T = (-MT)^{-1} = M^{-1}$

10. Ans. (A)

Sol. $x^2 - 2x + 4 = 0 \Rightarrow \alpha = 1 - i\sqrt{3}$ and $\beta = 1 + i\sqrt{3}$

$\Rightarrow \alpha = -2\omega$, $\beta = -2\omega^2$ and $z_0 = \omega$ (given)

$\Rightarrow \alpha^{15} + \beta^{10}, z_0 = (-2\omega)^{15} + (-2\omega^2)^{10}, \omega = -2^{15} + 2^{10}$

11. Ans. (A)

Sol. $\because L L^T = I \Rightarrow L^{-1} = L^T$ and $|L| = \pm 1$

Now det. $(2L^{-1}) = \dfrac{8}{\det(L)} = \pm 8$

$\Rightarrow |\det(2L^{-1})| = 8$

12. Ans. (A)

Sol. $\text{adj } A = B - A \Rightarrow A.\text{adj}A = AB - A^2$

$\Rightarrow |A|I = I - A^2 \Rightarrow A^2 = I(1 - |A|)$

$\Rightarrow |A|^2 = (1 - |A|)^3$

$\Rightarrow |A|^2 = 1 - |A|^3 + 3|A|^2 - 3|A|$

Let $f(x) = x^3 - 2x^2 + 3x - 1 = 0$ (where $|A| = x$)

$f(0) < 0$ and $f(1) > 0$, $f(x)$ is increasing function

$\Rightarrow x^3 - 2x^2 + 3x - 1 = 0$ has exactly one root and it lies in $(0, 1) \Rightarrow 0 < |A| < 1 \Rightarrow |B| > 1$

13. Ans. (C)

Sol. $\because A^2 = I$ and $\text{tr}(A) = -2$

$\therefore A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ only(A is invalutory)

$\Rightarrow A = -I$

$\Rightarrow |A^{-1} + A^2| = |-I + I| = 0$

14. Ans. (A)

Sol. $\dfrac{1}{1-\omega} + \dfrac{1}{\omega(1-\omega)} + \dfrac{1}{(\omega-1)(\omega+1)}$

$= \dfrac{\omega + \omega^2 + 1 + \cancel{\omega} - \cancel{\omega}}{\omega(1-\omega)(1+\omega)}$

$= 0$

15. Ans. (B)

Sol. $T_r = \dfrac{1}{\omega - r} - \dfrac{1}{\omega - r + 1}$

$T_r = v_r - v_{r-1}$

$$\Rightarrow \sum_{r=1}^{n} T_r = v_n - v_0$$

$$\Rightarrow \sum_{r=1}^{n} T_r = \frac{1}{\omega - n} - \frac{1}{\omega} = \frac{10\omega}{1 - 10\omega^2}$$

$$\Rightarrow \frac{1}{\omega - n} = \frac{10\omega}{1 - 10\omega^2} + \frac{1}{\omega}$$

$$\frac{1}{\omega - n} = \frac{\cancel{10\omega^2} + 1 - \cancel{10\omega^2}}{\omega - 10}$$

$n = 10$

16. Ans. (A)

Sol. $D = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \alpha(\alpha^2 - 1) - (\alpha - 1) + (1 - \alpha) = 0$

$\Rightarrow (\alpha - 1)(\alpha^2 + \alpha - 2) = 0$

$\Rightarrow \alpha = 1, -2$

at $\alpha = 1$, all planes are coincident planes

$\therefore$ infinite solutions

at $\alpha = -2$

$D_1 = \begin{vmatrix} -2 & 1 & 1 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{vmatrix}$

$= -2(4 - 1) - 1(4 + 2) + 1(-2 - 4)$

$= -6 - 6 - 6 = -18 \neq 0$

17. Ans. (B)

Sol. $c_{ij} = \dfrac{2}{3} a_{ij}$

$\Rightarrow |A|^2 = \dfrac{8}{27}|A|$

$\Rightarrow |A| = \dfrac{8}{27}$

Now $A^T A = \lambda I \Rightarrow |A|^2 = \lambda^3 \Rightarrow \lambda = \dfrac{4}{9}$

18. Ans. (C)

Sol. $\text{LHS} = \begin{vmatrix} \alpha^2 & 2\alpha & 1 \\ \alpha^4 & 2\alpha^2 & 1 \\ \alpha^6 & 2\alpha^3 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & \alpha^2 & \alpha^4 \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \end{vmatrix}$

$= 2\alpha^5 (\alpha - 1)^6 (\alpha + 1)^2$

19. Ans. (D)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & b & a \end{vmatrix} = 0$

$\Rightarrow 1(a^2 - b) - 1(a - 1) + 1(b - a) = 0$

$\Rightarrow a^2 - 2a + 1 = 0 \Rightarrow a = 1$

$\therefore$ Equation (1) and (2) are coincident for $b = 1$, 3rd equation will be parallel

$\Rightarrow$ no solution

$\therefore b \neq 1$

20. Ans. (D)

Sol. $x^3 + ax^2 + b = 0$

$\alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = 0$

$\alpha\beta\gamma = -b$

$$\Delta = \frac{1}{\alpha^6\beta^6\alpha^6}\begin{vmatrix} \beta\gamma & \gamma\alpha & \alpha\beta \\ \alpha\gamma & \alpha\beta & \beta\gamma \\ \alpha\beta & \beta\gamma & \gamma\alpha \end{vmatrix}^2$$

$$\Delta = \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2}{\alpha^6\beta^6\alpha^6}\begin{vmatrix} 1 & \gamma\alpha & \alpha\beta \\ 1 & \alpha\beta & \beta\gamma \\ 1 & \beta\gamma & \gamma\alpha \end{vmatrix}^2$$

$\Delta = 0$

SECTION–II

1. Ans. 8.00

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \Rightarrow A^n = \begin{bmatrix} 1 & 0 & 0 \\ 2n & 1 & 0 \\ 2n^2 & 2n & 1 \end{bmatrix}$

and $A^{-n} = (A^n)^{-1} = (\text{adj } A^n)$ ($\because$ $|A| = 1$)

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2n & 1 & 0 \\ 2n^2 & -2n & 1 \end{bmatrix}$$

$$\Rightarrow |A^{10} + A^{-10}| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 400 & 0 & 2 \end{vmatrix} = 8.00$$

2. Ans. 0.75

Sol. $z = x + iy$

$$\text{Re}\left(\frac{1}{x + i(y+1)}\right) = 1$$

$$\text{Re}\left(\frac{x - i(y+1)}{x^2 + (y+1)^2}\right) = 1$$

$$\Rightarrow \frac{x}{x^2 + (y+1)^2} = 1$$

$\Rightarrow x^2 + y^2 + 2y - x + 1 = 0$

$\Rightarrow r = \frac{1}{2}$

3. Ans. 1.00

Sol. Let $z_2 = r_2\ e^{i\theta_2}$

$\because$ arg z_1 = arg z_2 + $\frac{2\pi}{3}$

$$z_1 = \frac{1}{r_2}\ .\ e^{i\left(\theta_2 + 2\frac{\pi}{3}\right)}$$

$$= \frac{1}{r_2} . e^{i\theta_2} . \omega$$

Now $\left|z_1\bar{z}_2 + e^{i4\frac{\pi}{3}}\right| = \left|\frac{1}{r_2} . e^{i\theta_2} . \omega . r_2 e^{-i\theta_2} + \omega^2\right|$

$= |\omega + \omega^2| = 1$

4. Ans. 3.23 or 3.24

Sol. Let $z = \frac{t + 2i}{t - 2i}$, $t \in R$

$\Rightarrow |z| = 1 \Rightarrow$ curve f(z) is circle $|z| = 1$

Now $|z + i - 2| \le |z| + |i - 2|$

$\le 1 + \sqrt{5}$

≤ 3.23 or 3.24

5. Ans. 2.00

Sol. $z^2 + z = i\bar{z}$

$z(z+1) = i\bar{z}$

$|z|\ |z + 1| = |z|$

$\Rightarrow |z + 1| = 1$ ($\because z \ne 0$)

$\Rightarrow (z + 1)\ (\bar{z} + 1) = 1$

$$\Rightarrow \bar{z} = \frac{-z}{z+1}$$

$$\Rightarrow z(z+1) = i\left(\frac{-z}{z+1}\right)$$

$\Rightarrow (z + 1)^2 = -i$

$\Rightarrow z + 1 = \pm\left(e^{-i\,\pi/2}\right)^{1/2}$

$$z + 1 = \pm\frac{1-i}{\sqrt{2}}$$

$$z_1 + 1 = \frac{1-i}{\sqrt{2}}$$

$$z_2 + 1 = -\frac{1-i}{\sqrt{2}}$$

$z_1 + z_2 = -2$

$|z_1 + z_2| = 2$