DISTANCE LEARNING PROGRAMME

(Academic Session : 2015 - 2016)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced) 2016

Test Type : UNIT TEST   TEST # 03   Test Pattern : JEE-Advanced

TEST DATE : 09 - 08 - 2015

PART-1 : PHYSICS   **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	C	B	C	D	A	A	A,B,C,D	A,C,D
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,C	B,C	A,B,C,D	B	D	A	D	C		
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	6	6	6	4	5	6	0	4	3	3

SOLUTION

SECTION-I

1. Ans. (B)

Electric field due to single plate is $\frac{\sigma}{2\varepsilon_0}$

2. Ans. (C)

3. Ans. (C)

4. Ans. (B)

5. Ans. (C)

6. Ans. (D)

7. Ans. (A)

$a = \frac{120}{40} = 3$ $f = ma = 30$

$\mu Mg = 30$ $\mu = 0.1$

8. Ans. (A)

$$C_{eq.} = \frac{C_1C_2}{C_1+C_2}$$

$$\text{so } q = C_{eq.}V = \left(\frac{C_1C_2}{C_1+C_2}\right)(E_1+E_2)$$

$$\therefore V_2 = \frac{q}{C_2} = \left(\frac{C_1}{C_1+C_2}\right)(E_1+E_2)$$

9. Ans. (A,B,C,D)

Given combination is equivalent to parallel combination of two capacitance of capacity

$$C_1 = \frac{\epsilon_0 A/2}{d} \text{ and } C_2 = \frac{K\epsilon_0 A/2}{d}$$

Since charging source is removed, thus charge on the plates remains conserved.

$$E = \frac{dV}{d\ell} \text{ [same in both capacitor]}$$

10. Ans. (A, C, D)

There is not force between A & B

$\Rightarrow M_Bg\sin\theta \le \mu_2 m_B g\cos\theta$

$\Rightarrow \tan\theta \le \mu_2 \Rightarrow \theta \le 37°$

Both blocks start moving when

$(m_1+m_2)g\sin\theta = f_{k_1} + f_{k_2}$

$\Rightarrow (m_1+m_2)g\sin\theta = \mu_1 m_1 g\cos\theta + \mu_2 m_2 g\cos\theta$

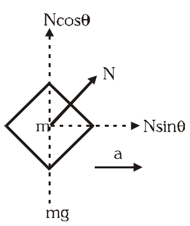
$\because m_1 = m_2$

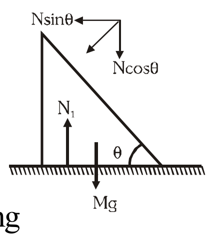
$2\sin\theta = (\mu_1+\mu_2)\cos\theta$

$2\tan\theta = 1 + \frac{3}{4} \Rightarrow \theta = \tan^{-1}\frac{7}{8}$

11. Ans. (A,B,C)

12. Ans. (B,C)





$N\cos\theta = mg$

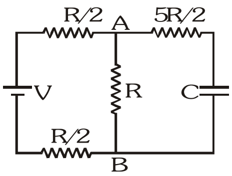
$N\sin\theta = ma$

$a = g\tan\theta$

$N_1 = Mg + N\cos\theta = Mg + mg$

13. Ans. (A,B,C,D)

14. Ans. (B)



In steady state $V_C = V_{AB}$ = capacitor voltage = V/2

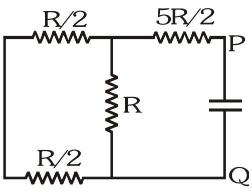
Calculation of time constant (τ_C)

effective resistance across C = 3R

$$q = q_0\left(1 - e^{-\frac{t}{\tau_c}}\right),\ q_0 = C\,\frac{V}{2}$$

$$\Rightarrow q = \frac{CV}{2}\left(1 - e^{\frac{t}{3RC}}\right)$$

15. Ans. (D)



$$V_{AB} = \frac{5}{2}Ri + \frac{q}{C}$$

$$\text{where } i = \frac{dq}{dt} = \frac{dv}{2\times 3RC}e^{-\frac{t}{3RC}},\ i = \frac{V}{6R}e^{-\frac{t}{3RC}}$$

$$V_{AB} = \frac{5V}{12}e^{\frac{t}{3RC}} + \frac{V}{2}\left(1 - e^{-\frac{t}{3RC}}\right) \Rightarrow i_{AB} = \frac{V_{AB}}{R}$$

16. Ans. (A)

$$\text{At } t \to \infty\ V_{AB} = \frac{V}{2},\ i_{AB} = \frac{V}{2R}$$

17. Ans. (D)

18. Ans. (C)

SECTION-IV

1. Ans. 6

$Q_0 = Cv_0$

$$V = \frac{Q_0}{kC} = \frac{v_0}{k} = \frac{12}{2} = 6$$

$\Delta V = 12 - 6 = 6$ volts

2. Ans. 6

$$i_2 = \frac{\varepsilon}{r + R_2} = \frac{5}{1+4} = 1A$$

$Pd = i_2R_2 = 4V$

Pd. A = 2V

$Q = 3 \times 2 = 6\ \mu C$

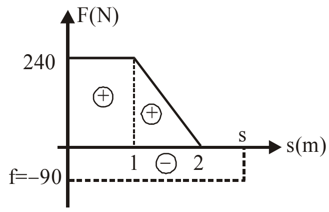
3. Ans.6

$$q = q_1\,e^{-\frac{t}{R_1C_1}} = q_2\,e^{-\frac{t}{R_2C_2}}$$

$$\frac{q_1}{q_2} = e^{-t\left(\frac{1}{R_1C_1} - \frac{1}{R_2C_2}\right)}$$

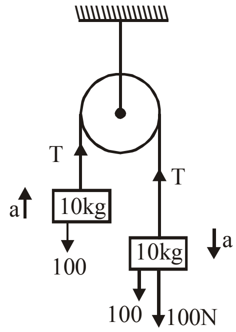
$\Rightarrow$ $R = 6\ \Omega$

4. Ans. 4



The block will come to rest when total area will become zero $\frac{1}{2}(20)(1+2) - 90(s) = 0 \Rightarrow s = 4m$

5. Ans. 5



200 – T = 10 a ⇒T – 100

= 10 a ⇒ a = 5

6. Ans. 6

7. Ans. 0

8. Ans. 4

9. Ans. 3

10. Ans. 3

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \left(5\cos 37^\circ\hat{i} + 5\sin 37^\circ\hat{j}\right)$$

$$+\left(4\hat{j}\right) + \left(-5\cos 53^\circ\hat{i} - 5\sin 53^\circ\hat{j}\right)$$

$$= 4\hat{i} + 3\hat{j} + 4\hat{j} - 3\hat{i} - 4\hat{j} = \hat{i} + 3\hat{j}$$

Therefore minimum additional force needed = 3N

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	D	B	A	A	C	D	A,B	A,B,C
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,C	A,C,D	A,B,C,D	D	B	B	D	D		
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	2	2	4	2	4	6	5	7	5

SOLUTION

SECTION-I

1. **Ans. (C)**
2. **Ans. (D)**
3. **Ans. (D)**
4. **Ans. (B)**
5. **Ans. (A)**
6. **Ans. (A)**
7. **Ans. (C)**
8. **Ans. (D)**
9. **Ans. (A,B)**
10. **Ans. (A,B,C)**

    $AlCl_3$ and BH_3 are electrophile due to incomplete octet.

    $$O = C = O \longrightarrow O = \overset{\oplus}{C} - \overset{\ominus}{O}$$

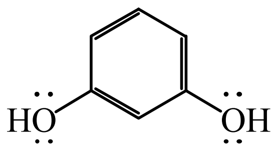
11. **Ans. (A,B,C)**
12. **Ans. (A,C,D)**
13. **Ans. (A,B,C,D)**

    Acid stronger than H_2O are soluble in aqueous NaOH so all options are soluble

14. **Ans. (D)**
15. **Ans. (B)**
16. **Ans. (B)**
17. **Ans. (D)**
18. **Ans. (D)**

SECTION-IV

1. **Ans. 4**
2. **Ans. 2**
3. **Ans. 2**
4. **Ans 4**



resorcinol

5. **Ans. 2**

    Acid base equilibrium shift into the direction of weak acid weak base.

    Acidic strength $PhSO_3H > MeCOOH$

    $EtOH > CH_4$

    $MeCOOH > Ph\text{–}OH$

    $NH_3 > CH_4$

6. **Ans. 4**
7. **Ans. 6**

    Priority order : –I > –Br > –Cl > $-SO_3H$ > –SH > –F > –OH > NH_2 > –CN > –D > –H

8. **Ans. 5**
9. **Ans. 7**
10. **Ans. 5**

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	B	C	D	D	D	D	A,D	A,B,C,D
	Q.	11	12	13	14	15	16	17	18		
	A.	A,C	A,B,C	A,B	B	A	A	B	A		
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	2	1	6	3	5	4	6	1	1

SOLUTION

SECTION-I

1. Ans. (C)

Let z = 5k

2x + y = 10k

7x + 6y = 45k

$\Rightarrow$ x = 3k, y = 4k

$x^3 + y^3 + z^3 = 216$

$\Rightarrow$ x = 3, y = 4 & z = 5

$\Rightarrow$ $a\cos\theta + b\sin\theta = c$ is $3\cos\theta + 4\sin\theta = 5$

or $\sin(\theta+\alpha) = 1$, where $\tan\alpha = \frac{3}{4}$

2 solution in $[0, 3\pi]$

2. Ans. (A)

$\cos^2 x = 1 + \sin^{2014} x$

$\Rightarrow$ $\cos^2 x = 1$ & $\sin x = 0$

$\Rightarrow$ $x = n\pi$ & $0, \pi, 2\pi, 3\pi......99\pi$

$\Rightarrow$ $\pi\left(\frac{99.100}{2}\right) = 4950\pi$

3. Ans. (B)

$$\sin A = \frac{\sin C}{\sin B} = \frac{c}{b}$$

3b – 3c = 0

$$\frac{b}{c} = \frac{5}{3} \quad \Rightarrow \quad \frac{c}{b} = \frac{3}{5}$$

$$\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}} = \frac{3}{5} \qquad \text{Let } \tan\frac{A}{2} = x$$

(3x – 1)(x – 3) = 0

$$\tan\frac{A}{2} = \frac{1}{3} \text{ or } \tan\frac{A}{2} = 3$$

4. Ans. (C)

$$\because \left(1+\sqrt{2}\right)^n = A_n + B_n\sqrt{2}$$

$$\Rightarrow \left(1-\sqrt{2}\right)^n = A_n - B_n\sqrt{2}$$

on solving, $A_n = \frac{\left(1+\sqrt{2}\right)^n + \left(1-\sqrt{2}\right)^n}{2}$,

$$B_n = \frac{\left(1+\sqrt{2}\right)^n - \left(1-\sqrt{2}\right)^n}{2\sqrt{2}}$$

$$\therefore \lim_{n\to\infty}\frac{A_n}{B_n} = \lim_{n\to\infty}\sqrt{2}.\left(\frac{1+\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)^n}{1-\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)^n}\right) = \sqrt{2}$$

$$\because \left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)^n \to 0 \text{ as } n\to\infty$$

& (A) $\to$ limit = 1

(B) $\to$ limit = 0

(C) $\to$ limit = $\sqrt{2}$

5. Ans. (D)

$$\lim_{x\to\infty}\frac{a^2x^2 - x^2 - bx}{ax - \sqrt{x^2+bx}} = 3 \Rightarrow$$

$$\lim_{x\to\infty}\frac{(a^2-1)x^2 - bx}{ax - x\sqrt{1+\frac{b}{x}}} = 3$$

For existance of limit

$\Rightarrow$ $a^2 - 1 = 0$ $\Rightarrow$ a = 1 or –1

$$\Rightarrow -\frac{b}{a-1} = 3$$

a = –1; b = 6

6. Ans. (D)

$$f(x)=\begin{cases}2x+\tan^{-1}x+b & x<0\\ x^3+x^2+ax+c & x\ge 0\end{cases}$$

$\because$ Continuous at $x=0 \Rightarrow b=c \Rightarrow \dfrac{b}{c}=1$

$\because$ differentiable at $x=0$

$\Rightarrow 2+1=a \Rightarrow a=3$

$$\frac{b^2}{c^2}+a=4$$

7. Ans. (D)

(A) $\dfrac{f(a)+f(b)}{2}=f(c)\Rightarrow f(c)\in(f(a),\ f(b))$
according to IMVT. $c\in(a, b)$

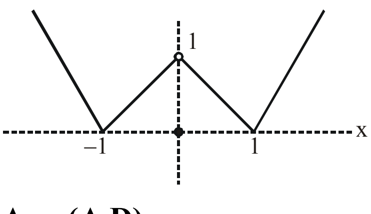
(B) $f(c)=\sqrt{f(a)f(b)}\Rightarrow f(c)\in(f(a), f(b))$ then $c\in(a,b)$

(C) $f(c)=\dfrac{3f(a)+2f(b)}{5}$, $f(c)\in(f(a),\ f(b))$
then $c\in(a,b)$

(D) External divisors & $f(c)\notin(f(a), f(b))$ then not always true.

8. Ans. (D)

$$f(x)=\begin{cases}|x-1| & x>0\\ 0 & x=0\\ |x+1| & x<0\end{cases}$$



9. Ans. (A,D)

$f(x)=\sin^2x\cos x-\sin^2x-\cos^3x+\cos x+\sin x\cos x-\sin x$

$=(\cos x-1)(\sin x-\cos x)(\sin x+\cos x+1)=0$

$\Rightarrow f(x)=0\Rightarrow \cos x=1$ or $\sin x=\cos x$
or $\sin x+\cos x=-1$

$\Rightarrow x=0,2\pi$ or $x=\dfrac{\pi}{4},\dfrac{5\pi}{4}$ or $x=\pi,\dfrac{3\pi}{2}$

$\Rightarrow$ Total 6 solutions in $[0,2\pi]$

10. Ans. (A,B,C,D)

Let, $a=\cos\alpha$, $b=\tan\alpha$, $c=\operatorname{cosec}\alpha$

$\Rightarrow\ a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)$

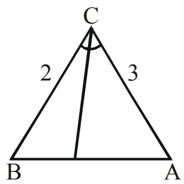
now using AM > GM

(AM $\ne$ GM because $\cos\alpha\ne\tan\alpha\ne\operatorname{cosec}\alpha$)

$$\Rightarrow\frac{a\left(b^2+c^2\right)+b\left(c^2+a^2\right)+c\left(a^2+b^2\right)}{6}>\sqrt[6]{a^6b^6c^6}$$

$\Rightarrow a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)>6$
$(\therefore abc=1)$

11. Ans. (A,C)



$c^2=a^2+b^2-2ab\cos C$

$=13-6=7$

Now, length of internal angle bisector through vertex

$$C=\frac{2ab}{a+b}\cos\frac{C}{2}=\frac{12}{5}\times\frac{\sqrt3}{2}=\frac{6\sqrt3}{5}\ \therefore\ (A)$$

& length of median through vertex C is

$$\frac12\sqrt{2a^2+2b^2-c^2}=\frac12\sqrt{26-7}=\frac{\sqrt{19}}{2}\ \therefore\ (C)$$

12. Ans. (A,B,C)

$$y=\sqrt{\cos x+y}$$

squaring, $y^2-y-\cos x=0$ ...(1)

$$y=\frac{1\pm\sqrt{1+4\cos x}}{2}\ \text{rejecting (–) sign(as } y>0)$$

$$\therefore\ y=\frac{1+\sqrt{1+4\cos x}}{2}$$

$$\frac{dy}{dx}=\frac{-4\sin x}{2.2\sqrt{1+4\cos x}}=\frac{-\sin x}{\sqrt{1+4\cos x}}$$

$$\text{again (1)}\Rightarrow 2y\frac{dy}{dx}-\frac{dy}{dx}=-\sin x$$

$$\Rightarrow\frac{dy}{dx}=\frac{\sin x}{1-2y}=\frac{y\sin x}{y-2y^2}$$

$$=\frac{-y\sin x}{y^2+\left(y^2-y\right)}=-\frac{y\sin x}{y^2+\cos x}\ \text{(\& using (1))}$$

13. Ans. (A,B)

Clearly $\lim_{x\to\alpha^+}\dfrac{\sin\left(\left[x^3\right]-[x]^3\right)}{(x-\alpha)^3}=0$

$\therefore$ for existence of limit L.H.L = 0.

$\Rightarrow \alpha^3 - 1 - (\alpha - 1)^3 = 0$

$\Rightarrow \alpha^3 - 1 - (\alpha^3 - 3\alpha^2 + 3\alpha - 1) = 0$

$3\alpha^2 - 3\alpha = 0 \Rightarrow \alpha = 0,1$

Paragraph for Question 14 to 16

Consider

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{f(x)+f(h)}{1-f(x)f(h)}-f(x)}{h}$$

$f'(x)=\lim_{h\to 0}\dfrac{f(h)}{h}(1+f^2(x))$

where $f(0)=0$

$f'(x)=f'(0)(1+f^2(x))$

$\tan^{-1}(f(x))=\lambda x+c$

$f(x)=\tan(\lambda x+c)$

$f(0)=0 \quad \Rightarrow \quad c=0$

$f(x)=\tan\lambda x$

14. Ans. (B)

If $\lambda=3$, then $\lim_{x\to 0}\dfrac{\tan(3x)}{\tan(x)}=3$

15. Ans. (A)

$f(x)=\tan x$

$f(\tan^{-1}x)=x$

function is one-one

16. Ans. (A)

$f(x)=\tan 2x$

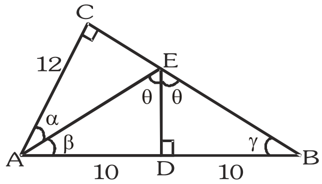
$f\left(\dfrac{1}{2}\right)=\tan 1$

$f\left(\dfrac{1}{4}\right)=\tan\dfrac{1}{2}$

$f(1)=\tan 2=$ negative

$f\left(\dfrac{1}{2}\right)>f\left(\dfrac{1}{4}\right)>f(1)$

Paragraph for Question 17 & 18



$BC^2 = AB^2 - AC^2 = 400 - 144 = 256$

$\Rightarrow \quad BC = 16$

$\Rightarrow \quad \tan\gamma=\dfrac{AC}{BC}=\dfrac{12}{16}=\dfrac{3}{4}$

$\Delta AED \cong \Delta BED$ by SAS $\Rightarrow AE = EB$ & $\angle\beta = \angle\gamma$

$\Rightarrow \quad \dfrac{ED}{DB}=\tan\gamma=\dfrac{3}{4}$

$\Rightarrow \quad ED=\dfrac{15}{2}$

Also $\cos\gamma=\dfrac{4}{5}=\dfrac{BD}{BE} \quad \Rightarrow \quad BE=\dfrac{25}{2}$

$\Rightarrow \quad CE=\dfrac{7}{2}$

17. Ans.(B)

Area of $\Delta AEC=\dfrac{1}{2}AC\times CE=\dfrac{1}{2}\times 12\times\dfrac{7}{2}=21$ sq. units

18. Ans.(A)

$\tan\gamma\,\tan(60^\circ-\gamma)\,\tan(60^\circ+\gamma)$

$$=\tan 3\gamma=\frac{3\tan\gamma-\tan^3\gamma}{1-3\tan^2\gamma}=\frac{\frac{9}{4}-\frac{27}{64}}{1-\frac{27}{16}}=-\frac{117}{44}$$

SECTION – IV

1. Ans. 4

$(\sin^2x-\cos^2x)+\left(\dfrac{1}{\sin^2 x}-\dfrac{1}{\cos^2 x}\right)=0$

$(\sin^2x-\cos^2x)-\dfrac{(\sin^2x-\cos^2x)}{\sin^2x\cos^2x}=0$

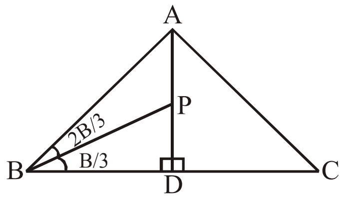
$\sin^2x=\cos^2x$ or $\sin^2x\cos^2x=1$

$\sin^2 2x = 4$ (Not Possible)

$\tan^2x = 1$

possible for four values in $[0, 2\pi]$

2. Ans. 2



$\angle ABP = \frac{2B}{3}$ & $\angle APB = 90° + \frac{B}{3}$

Use sine law in ΔAPB

$$\frac{C}{\sin\left(90°+\frac{B}{3}\right)} = \frac{AP}{\sin\left(\frac{2B}{3}\right)}$$

$$AP = 2C\sin\frac{B}{3}$$

$\Rightarrow \lambda = 2$

3. Ans. 1

$$\lim_{x\to 0}\frac{\sin(2\pi\sec x)}{\left(\frac{\ell n(x+1)}{x}\right)^2 .x^2} = \lim_{x\to 0}\frac{\sin(2\pi\sec x - 2\pi)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\pi(\sec x-1)}{x^2}$$

$$= \lim_{x\to 0}\frac{2\pi(\sec x-1)}{x^2} = \lim_{x\to 0}\frac{2\pi}{\cos x}\left(\frac{1-\cos x}{x^2}\right) = \pi$$

$\therefore$ k = 1

4. Ans. 6

$f(x) = x\{\sin^2 x\} + x^3$

Points of non-derivability when $\sin^2 x = 1$

$x = (2n+1)\pi/2$

$$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$$

5. Ans. 3

$$\lim_{x\to\frac{\pi}{8}} e^{\left(\tan^2 4x\right)\left(\sin 4x - 1\right)}$$

$$= \lim_{t\to 0} e^{\left(\tan\left(4\left(\frac{\pi}{8}+t\right)\right)\right)^2 . \left(\sin\left(4\left(\frac{\pi}{8}+t\right)\right)-1\right)}$$

$$= \lim_{t\to 0} e^{\cot^2 4t.(\cos 4t - 1)}$$

$= e^{-1/2}$

6. Ans. 5

$$\because \quad g'(y) = \frac{1}{f'(x)}$$

$$\Rightarrow \quad g''(y) = -\frac{1}{(f'(x))^2}.f''(x).\frac{dx}{dy}$$

$$\Rightarrow \quad g''(y) = -\frac{f''(x)}{(f'(x))^3}$$

at x = –1, y = –2

$$\Rightarrow \quad g''(-2) = -\frac{f''(-1)}{(f'(-1))^3}$$

$\because \quad f'(-1) = 3(-1)^2 + (1) = 4$

$f''(-1) = 6(-1) = -6$

$$\therefore \quad g''(-2) = -\frac{(-6)}{64} = \frac{3}{32} \quad \Rightarrow \quad b - 9a = 5$$

7. Ans. 4

$$\pi\cos x = \frac{\pi}{2} - \pi\sin x$$

$$\pi\cos x + \pi\sin x = \frac{\pi}{2}$$

$$\cos x + \sin x = \frac{1}{2}$$

$$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{2\sqrt{2}}$$

$$8\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = 4$$

8. Ans. 6

$$\cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$$

$$3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$$

$$\theta = (4n+1)\frac{\pi}{10}, (4n-1)\frac{\pi}{2}, n \in I$$

Values are $\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}, \frac{3\pi}{2}$

$\Rightarrow$ 6 solutions

9. Ans. 1

$$\lim_{x\to\infty} x^{\frac{7}{2}}\frac{2}{\left(\sqrt{x^7+1}+\sqrt{x^7+1}\right)} = \frac{2}{2} = 1$$

10. Ans. 1

$x = 0 \Rightarrow f(1) = 0$

$xf'(x) + f(x) + f'(x+1) = 1$

$x = 1 \Rightarrow f'(1) + f(1) + f'(2) = 1$

$\therefore f'(1) + f'(2) = 1$