(1001CJA102120055) Test Pattern

ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2020 - 2021)

JEE(Main)
FULL SYLLABUS
03-03-2021

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	C	B	C	C	B	D	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	B	A	B	A	B	C	A	A	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	30.00	0.28	350.00	105.00	30.00	1.00	1.00	4.00	357.00	4.00

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	B	A	D	D	D	C	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	D	C	B	D	A	C	D	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	68.14 TO 68.15	4.80	4.00	7.00	1.00	4.00	0.25	6.00	0.00	2.00

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	B	D	A	A	D	C	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	C	A	A	B	B	A	C	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	–0.08	1.15	0.00	3.00	4.00	0.25	0.33	1.57	4.00	1.00

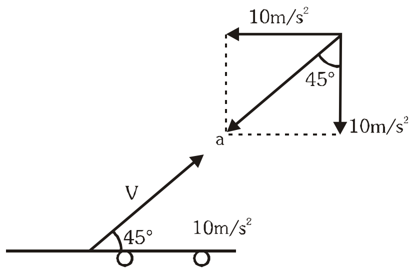
JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A)

Sol. w.r.t. flat car, velocity of projection makes angle 45° with east. In order to catch the ball without moving trajectory has to be a straight line. So acceleration wrt flat car should be just opposite to projection velocity.



2. Ans. (B)

Sol. Let speed of ring be v then $v\cos\theta = u$

$$\Rightarrow v = \frac{u}{\cos\theta}$$

3. Ans. (C)

Sol. $mgR + \frac{1}{2}mv'^2 = \frac{1}{2}m(v')^2$

$$\Rightarrow \frac{mv'^2}{R} = 3g$$

4. Ans. (B)

Sol. Velocity after collision $mu = 2mv_0$

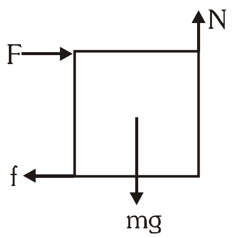
$\Rightarrow v_0 = u/2$

So final kinetic energy $= \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{K}{2}$

loss in kinetic energy $K - \frac{K}{2} = \frac{K}{2}$

5. Ans. (C)

Sol. $FL \geq mg\left(\frac{L}{2}\right) \Rightarrow F \geq \frac{mg}{2}$

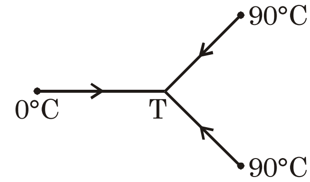


6. Ans. (C)

Sol. $\frac{MgL}{2} = \frac{2ML^2\alpha}{3}$

7. Ans. (B)

Sol.



Let temperature of junction is T

$$\frac{kA(90-T)}{\ell} + \frac{kA(90-T)}{\ell} + \frac{kA(0-T)}{\ell} = 0$$

T = 60°C

8. Ans. (D)

Sol. According to Newton's law of cooling

$$\frac{\theta_2 - \theta_1}{t} = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_s\right)$$

Since the temperature decreases from 60°C to 40°C in 7 minutes

$$\frac{60-40}{7} = K\left(\frac{60+40}{2} - 10\right)$$

$$\Rightarrow \frac{20}{7} = K(50-10) \Rightarrow K = \frac{1}{14}$$

If the temp. of object becoms θ' in next 7 minutes then $\frac{40-\theta'}{7} = \frac{1}{14}\left(\frac{40+\theta'}{2} - 10\right)$

$$\Rightarrow 40 - \theta' = \frac{1}{4}(40 + \theta' - 20)$$

$$\Rightarrow 160 - 4\theta' = 20 + \theta'$$

$$\Rightarrow 5\theta' = 140 \Rightarrow \theta' = 28°C$$

OR

According to Newton's law of cooling

$$-\frac{d\theta}{dt} = K(\theta - \theta_0) \text{ or } \quad dt = -\frac{1}{K}\frac{d\theta}{(\theta-\theta_0)}$$

$$\therefore \int_0^t dt = -\frac{1}{K}\int_{\theta_1}^{\theta_2}\frac{d\theta}{(\theta-\theta_0)}$$

$$\Rightarrow t = \frac{1}{K}\log_e\left\{\frac{(\theta_1-\theta_0)}{(\theta_2-\theta_0)}\right\}$$

As per the question $7 = \frac{1}{K}\log_e\left\{\frac{60-10}{40-10}\right\}$

Also $7 = \frac{1}{K}\log_e\left\{\frac{40-10}{\theta-10}\right\}$

from above equations we have

$$\log_e\left(\frac{50}{30}\right) = \log_e\left(\frac{30}{\theta-10}\right)$$

$$\therefore \frac{5}{3} = \frac{30}{\theta-10} \Rightarrow 5 - 50 = 90$$

$$\Rightarrow q = \frac{100}{5} = 28°C$$

9. Ans. (A)

Sol. $\eta = 1 - \frac{273}{1092} = \frac{3}{4}$

$$\text{C.O.P.} = \frac{1-\eta}{\eta} = \frac{1}{3} = \frac{Q_2}{w}$$

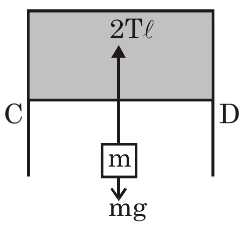
$$\Rightarrow Q_2 = \frac{1}{3}w = \frac{1}{3}\times 1260 \text{ watt}$$

= 420 watt

$$\therefore \frac{dm}{dt}\times 80 \times 4.2 = 420$$

10. Ans. (B)

Sol.

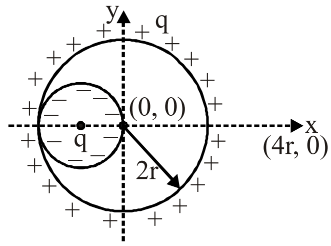


For equilibrium, $2T\ell = mg$

$$m = \frac{2T\ell}{g}$$

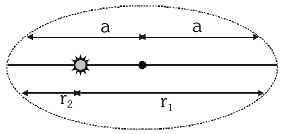
11. Ans. (A)

Sol. $\vec{E} = \frac{kq}{(4r)^2} = \frac{kq}{16r^2}$



12. Ans. (B)

Sol.



$$2a = r_1 + r_2 \Rightarrow a = \frac{r_1 + r_2}{2};$$

$$T \propto a^{3/2} \Rightarrow T \propto (r_1 + r_2)^{3/2}$$

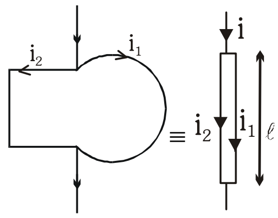
13. Ans. (A)

Sol. $(20 + 10)\frac{1\times10^{-3}}{2} = s\times\left(10 - \frac{1\times10^{-3}}{2}\right)$

$$\Rightarrow s \simeq 1.5 m\Omega$$

14. Ans. (B)

Sol.



$F = Bi_1\ell + Bi_2\ell = B\ell(i_1 + i_2) = Bi\ell$

15. Ans. (A)

Sol. $\phi = \int_0^{R/2} B(2\pi r)dr = B_0 t 2\pi\left(\frac{r^3}{3}\right)_0^{R/2}$

$$= B_0(2\pi)\frac{1}{3}\left(\frac{R^3}{8}\right)t$$

$$\because E\left(2\pi\left(\frac{R}{2}\right)\right) = \frac{-d\phi}{dt} = -B_0(2\pi)\frac{R^3}{24}$$

$$\Rightarrow E = \frac{B_0R^2}{12}$$

16. Ans. (B)

Sol. p.f. = $\dfrac{R}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$

17. Ans. (C)

Sol. From Bohr model $\dfrac{1}{\lambda}=R\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)$

$\dfrac{1}{\lambda_1}=R\left(\dfrac{1}{2^2}-\dfrac{1}{3^2}\right)=\dfrac{5}{36}R$ ...... (i)

and $\dfrac{1}{\lambda_2}=R\left(\dfrac{1}{1^2}-\dfrac{1}{2^2}\right)=\dfrac{3}{4}R$ ...... (ii)

Dividing eq. (i) and (ii), we get $\dfrac{\lambda_1}{\lambda_2}=\dfrac{27}{5}$

18. Ans. (A)

Sol. Q value of the reaction

$Q = (2 \times 4 \times 7.06 - 7 \times 5.6) = 17.28$ MeV ...(i)

$K_P + Q = 2K_\alpha$ ....(ii)

$\sqrt{2m_PK_P}=2\sqrt{2m_\alpha K_\alpha}\cos\alpha$

$K_P = K_\alpha$ ....(iii)

So $K_P = 17.28$ MeV

19. Ans. (A)

Sol. $\dfrac{1}{f_{eq}}=\dfrac{1}{f_m}-\dfrac{2}{f_L}$

$\dfrac{1}{f_L}=(\mu-1)\left[\dfrac{+1}{R}\right]=\dfrac{1}{2R}$

$\therefore\ \dfrac{1}{f_q}=-\dfrac{1}{R}\Rightarrow f_{eq}=-R$ [concave mirror]

∴ Image of object kept at 2R will form at 2R

20. Ans. (B)

Sol. $i=\dfrac{\varepsilon}{504}$

$v = \varepsilon - 4i$

$=\varepsilon-\dfrac{\varepsilon}{504}\times 4$

$v=\varepsilon\left(1-\dfrac{1}{126}\right)=\varepsilon\dfrac{125}{126}$

$\Delta v=\dfrac{\varepsilon}{126}$

SECTION-II

1. Ans. 30.00

Sol. $\tau = MB\sin\theta$

$\sin\theta=\dfrac{\tau}{MB}=\dfrac{1.2\times10^{-3}}{60\times40\times10^{-6}}=\dfrac{1}{2}\Rightarrow\theta=30°$

2. Ans. 0.28

Sol. We assume that b is small compared to $\sqrt{\dfrac{k}{m}}$ and we take $T=2\pi\sqrt{(m/k)}=1\,s$. It is given that at t = 4T, the amplitude falls to 3A/4, i.e.

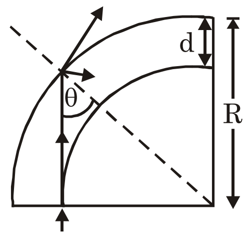
$e^{-bt/2m} = 3/4$

$-2bT/m = \ln(3/4)$

or b = 0.28 kg/s.

3. Ans. 350.00

Sol.



A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta=\dfrac{R-d}{R}$ and by n sin θ > 1 sin 90°.

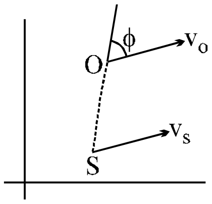
Then $\dfrac{n(R-d)}{R}>1\Rightarrow nR-nd>R$

$\Rightarrow nR-R>nd \qquad R>\dfrac{nd}{n-1}$.

$R_{min}=\dfrac{1.40(100\times10^{-6}m)}{0.40}=350\times10^{-6}m$

4. Ans. 105.00

Sol. $\overrightarrow{SO} = 3\hat{i} + 4\hat{j}$



Component of velocity along SO

$$v_s = 15(2\hat{i} + \hat{j})\cdot\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 30$$

$$v_0 = 5\hat{i} + 15\hat{j}\cdot\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 15$$

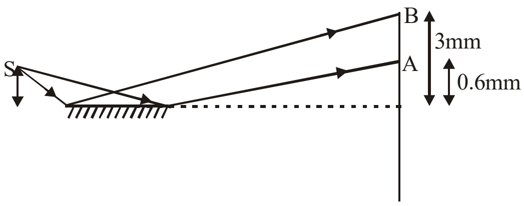
$$f = \frac{330-15}{330-30} \times 100 = 105\text{Hz}$$

5. Ans. 30.00

Sol. $v_{max} = a\omega = 3 \times 10 = 30$

6. Ans. 1.00

Sol.



At A

$$\frac{(0.6)(1.2)\times 10^{-6}}{6} = n_1\left(4\times 10^{-7}\right)$$

$\Rightarrow n_1 = 0.3$

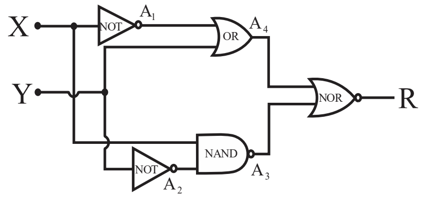
At B

$$\frac{(3)(1.2)\times 10^{-6}}{6} = n_2\left(4\times 10^{-7}\right) \Rightarrow n_2 = 1.5$$

So, there will be two maximas in the pattern.

7. Ans. 1.00

Sol. The given circuit can be drawn as shown in the figure given below



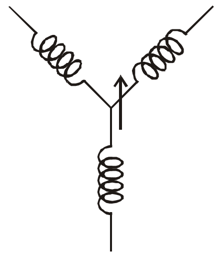
Truth table for this given logic gate is

Given inputs in the options		A_1	A_2	A_3	A_4	R
X	Y					
0	0	1	1	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	0
0	1	1	0	1	1	0

So to get output R = 1, inputs must be X = 1 and Y = 0

8. Ans. 4.00

Sol. at GP



$F_{restoring} = 2k_1 x\cos^2\theta + k_2 x$

So, $TP = 2\pi\sqrt{\dfrac{m}{2k_1\cos^2\theta + k_2}}$

9. Ans. 357.00

Sol. $Q_1 = m$ (2.1 kJ/kg°C) (9°C)

$Q_2 = 19Q_1 = m$(2.1 kJ/kg°C) (1°C)

$19[(18.9)m] = (2.1 + L_f)m$

$359.10 = 2.1 + L_f$

$L_f = 359.1 - 2.1 = 357.$ kJ/kg

10. Ans. 4.00

Sol. $V_R = 3V$

$V_{2R} = 6V$

$C(V_0 - 6) + 2C(V_0 - 9) + 3C(V_0 - 0) = 0$

$\Rightarrow V_0 = 4$ V

$\therefore Q_C = 4\ \mu C$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans.(A)

Sol. $2\pi r = n\lambda \Rightarrow \underbrace{2\pi r}_{x} = 4\lambda \Rightarrow \lambda = \frac{x}{4}$

2. Ans.(A)

Sol. Let mol. formula be $C_xH_{2x}O_x$

$0.0833 \times 2x = 1 \Rightarrow x = 6$

3. Ans. (B)

Sol. $= \left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$

$$= \left(P + \frac{a(1/3)^2}{V^2}\right)\left(V - \frac{b}{3}\right) = \frac{RT}{3}$$

$$= \left(P + \frac{a}{9V^2}\right)\left(V - \frac{b}{3}\right) = \frac{RT}{3}$$

4. Ans.(A)

Sol. $\Delta G° = -2.303\ RT \log K_P$

$$= -2.303 \times 8.314 \times 298 \times \log\left(\frac{(P_{SO_3})^2}{(P_{SO_2})^2(P_{O_2})}\right)$$

$$= -2.303 \times 8.314 \times 298 \times \log\left(\frac{(0.01)^2}{(0.00001)^2(0.01)}\right)$$

$= -45.65$ kJ

5. Ans. (D)

Sol. $Ag_{(s)} \longrightarrow Ag^+_{(a)}$

$$\left[Ag^+\right]^2\left[CrO_4^{2-}\right] = K_{sp}$$

$$(2s)^2(s) = K_{sp}$$

$$\Rightarrow s = \left(\frac{K_{sp}}{4}\right)^{\frac{1}{3}}$$

$$\left[Ag^+\right]_{(a)} = 2s = 2\left(\frac{K_{sp}}{4}\right)^{\frac{1}{3}}$$

$Ag^+_{(c)} \longrightarrow Ag_{(s)}$

$Ag^+_{(c)} \longrightarrow Ag^+_{(s)}$

$$E_{cell} = E^0_{Cell} - \frac{0.06}{1}\log\frac{[Ag^+_{(a)}]}{[Ag^+_{(c)}]}$$

$$0.162 = -0.06\ \log\frac{(2k_{sp})^{1/3}}{(0.1)}$$

$$2 \times 10^{-3} = \frac{(2k_{sp})^{1/3}}{(0.1)}$$

$8 \times 10^{-9} \times 10^{-3} = 2\ K_{sp} \Rightarrow K_{sp} = 4 \times 10^{-12}$

6. Ans.(D)

Sol. Composition in vapour phase and liquid phase remains same.

7. Ans.(D)

Sol. Max. conc. of I^- required for negatively charge colloid.

$[Ag^+] < [I^-]$

8. Ans.(C)

Sol. pH = 2 (highly acidic) → carboxyl group 'O' is protonated

pH = 7 (neutral) → since carboxylic acid is more acidic than phenol.

∴ it loses H^+.

pH = 12 (highly basic) → Both carboxylic acid and phenol lose their H^+.

9. Ans. (B)

Sol. They are not a mirror image of each other

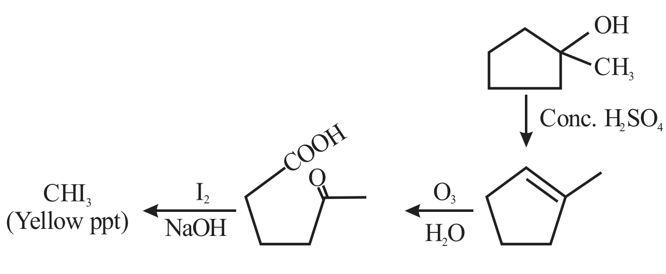
10. Ans. (A)

11. Ans. (C)

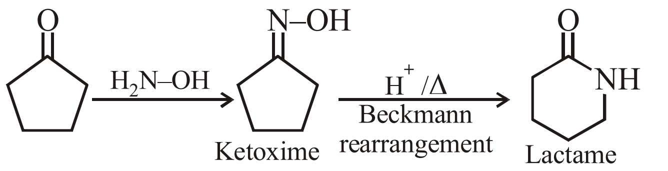
12. Ans. D

Sol. A does not change colour with $K_2Cr_2O_7$ i.e. it is a 3 ° alcohol.

Alkene formed after dehydration oxidative ozonolysis give methyl ketone.

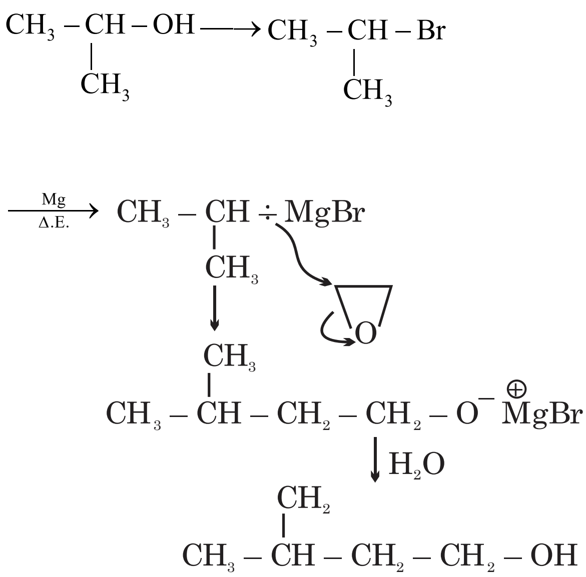


13. Ans. (D)



14. Ans. (C)

Sol.



15. Ans. (B)

16. Ans. (D)

Sol.

Atomic Number	Ionization Enthalpy (kJ/mol)		
	I_1	I_2	I_3
n	1579	3374	6043
n+1	2075	3952	6125
n+2	493	4562	6907
n+3	739	1451	7731

By observing the values of I_1, I_2 & I_3 for atomic number (n + 2), it is observed that $I_2 >> I_1$.

This indicates that number of valence shell electrons is 1 and atomic number (n+2) should be an alkali metal.

Also for atomic number (n + 3), $I_3 >> I_2$.

This indicates that it will be an alkaline earth metal which suggests that atomic number (n + 1) should be a noble gas & atomic number (n) should belong to Halogen family. Since n < 10; hence n = 9 (F atom)

Note: n = 1 (H atom) cannot be the answer because it does not have I_2 & I_3 values.

17. Ans.(A)

Sol. In alkalimetals down the group hardness decreases due to decrease in metallic bond strength.

18. Ans. (C)

Sol. SF_4 has triagonal bipyramidal geometry.

19. Ans. (D)

Sol. Np: Rn $5f^4 6d^1 7s^2$

Np show maximum number of oxidation state from +3 to +7

20. Ans.(B)

Sol. A, C, D are used to remove permanent as well as temporary hardness.

SECTION-II

1. Ans.(68.14 to 68.15)

Sol. $\alpha = \dfrac{M_0 - M}{(n-1)\cdot M} = \dfrac{208.5-124}{(2-1)\cdot 124} = 0.681$

2. Ans.(4.80)

Sol. In final solution [HA] = [A^-]

3. Ans.(4.00)

Sol. $\ell n\dfrac{K_2}{K_1} = \ell n\dfrac{t_1}{t_2} = \dfrac{E_a}{R}\left(\dfrac{1}{T_1}-\dfrac{1}{T_2}\right)$

$\ell n\dfrac{1}{3} = \dfrac{E_a}{R}\left(\dfrac{1}{300}-\dfrac{1}{280}\right)$ and

$\ell n\dfrac{16}{t} = \dfrac{E_a}{R}\left(\dfrac{1}{300}-\dfrac{1}{330}\right)$

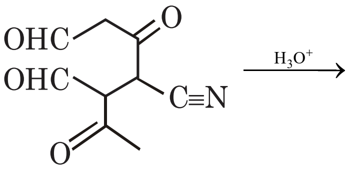
t = 4 hrs.

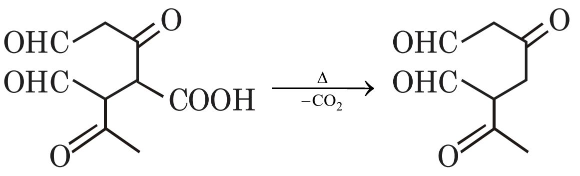
4. Ans.(7.00)

Sol. (i), (ii), (iii), (iv), (v), (viii), (ix) $\Rightarrow$ 7

5. Ans. (1.00)

Sol.





6. Ans. (4.00)

Sol.

$$H_2N-\overset{*}{C}H(R)-\overset{O}{\overset{||}{C}}-OH + H_2N-\overset{*}{C}H(R')-COOH$$

C^* is chiral carbon tetrapeptide has four amino acids joined by three peptide linkage.

—COOH group is on alanine part, thus it is at fixed C-terminal position in each combination.

Glycine is optically inactive thus it cannot be on the N—terminal side. Thus possible combinations are

Phe-Gly-Val-Ala, Phe-Val-Gly-Ala,
Val-Gly-Phe-Ala, Val-Phe-Gly-Ala

7. Ans.(0.25)

Sol. $FeCr_2O_4 + NaOH + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + H_2O$

CrO_4^{2-} has 4 equal Cr–O bond length

8. Ans.(6.00)

9. Ans.(0.00)

Sol. PbS $\longrightarrow$ PbO (on roasting). There is no difference in O.S.

10. Ans. (2.00)

Sol. $Pb(NO_3)_2$ & $AgNO_3$

PART-3 : MATHEMATICS SOLUTION

1. Ans. C

Sol. $\sim(p \to q) = p \wedge \sim q$

2. Ans. B

Sol. $\bar{x} = 72$

(number of boys)$n_1 = 70$
(number of girls)$n_2 = 30$

$\bar{x}_1 = 75; \bar{x}_2 = ?$

$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 72 = \frac{70 \times 75 + \bar{x}_2 \times 30}{70 + 30}$

$\Rightarrow \bar{x}_2 = 65$

3. Ans. B

Sol. Given, $iz^3 + z^2 - z + i = 0$

Dividing both side by i and using $\frac{1}{i} = -i$, we have

$z^3 - iz^2 + iz + 1 = 0$
$\Rightarrow z^2(z - i) + i(z - i) = 0 \quad (\because i^2 = -1)$
$\Rightarrow (z - i)(z^2 + i) = 0$
$\therefore z = i$ or $z^2 = -i$
$\therefore |z| = |i| = 1$ or $z^2 = -i$
$|z^2| = |-i| \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1, -1$
$\therefore |z| = 1$

4. Ans. D

Sol. The given relation may be written in set builder form as
R = {(a, b) : a – b divided by n and a, b ∈ Z}
As a – a = 0 and 0 divided by n
∴ (a, a) ∈ R
∴ R is reflexive
Let a, b ∈ Z such that (a, b) ∈ R
Then (a, b)∈ R ⇒ a – b divided by n.
a – b = nk for some integer k
⇒ b – a = n(– k)
∴ (a, b) ∈ R ⇒ (b, a) ∈ R
∴ R is symmetric
Now, (a, b), (b, c)∈ R
Now, $a - b = nc_1$ and $b - c = nc_2$ for some integers c_1 and c_2.
∴ $(a - b) + (b - c) = n(c_1 + c_2)$
⇒ $a - c = nk$, where $k = c_1 + c_2$, an integer.
⇒ (a, c) ∈ R.
∴ (a, b), (b, c) ∈ R ⇒ (a, c) ∈ R
∴ R is transitive and hence R is an equivalence relation.

5. Ans. A

Sol. $r_1 = 2r_2 = 3r_2 = k$

$r_1 = k;\ r_2 = \frac{k}{2},\ r_3 = \frac{k}{3}$

$k = \frac{\Delta}{s-a};\ \frac{k}{2} = \frac{\Delta}{s-b};\ \frac{k}{3} = \frac{\Delta}{s-c}$

$s - a = \frac{\Delta}{k};\ \frac{s-b}{2} = \frac{\Delta}{k};\ \frac{s-c}{3} = \frac{\Delta}{k}$ .....(i)

$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

$\frac{1}{k} + \frac{2}{k} + \frac{3}{k} = \frac{1}{r} \Rightarrow \frac{1}{r} = \frac{6}{k} \Rightarrow r = \frac{k}{6}$

$r = \frac{\Delta}{s} = \frac{k}{6} = \frac{\Delta}{s};\ \frac{s}{6} = \frac{\Delta}{k}$ from (i)

$s - a = \frac{s}{6} \Rightarrow \frac{5s}{6} = a$

$\frac{s-b}{2} = \frac{s}{6};\ 6s - 6b = 2s;\ 4s = 6b;\ s = \frac{3b}{2};$

$b = \frac{2s}{3}$

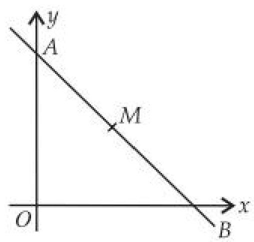
$\frac{s-c}{3} = \frac{s}{6};\ 6s - 6c = 3s;\ 3s = 6c \Rightarrow c = \frac{s}{2}$

$a : b : c \equiv \frac{5s}{6} : \frac{2s}{3} : \frac{s}{2} \equiv 5 : 4 : 3$ **Ans.**

6. Ans. A

Sol. Coordinates of $A \equiv \left(0, \frac{p}{\sin\alpha}\right)$

Coordinates of $B \equiv \left(\frac{p}{\cos\alpha}, 0\right)$



$\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$

Now, let $x = \frac{p}{2\cos\alpha}$ ....(i)

$y = \frac{p}{2\sin\alpha}$ ....(ii)

Since, $\cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow \left(\frac{p}{2x}\right)^2 + \left(\frac{p}{2y}\right)^2 = 1$ [From (i) and (ii)]

$\Rightarrow \frac{p^2}{4}\left[\frac{x^2+y^2}{x^2y^2}\right] = 1 \Rightarrow x^{-2} + y^{-2} = 4p^{-2}$

7. Ans. (D)

Equation of chord of the circle $x^2 + y^2 = 25$, whose midpoint is P(h, k)

$T = S_1$

$xh + yk - 25 = h^2 + k^2 - 25$

$\Rightarrow xh + yk - h^2 - k^2 = 0$ ....(1)

this chord is tangent to the circle

$x^2 + y^2 - 20x - 20y + 175 = 0$

Length of perpendicular fromcentre = Radius

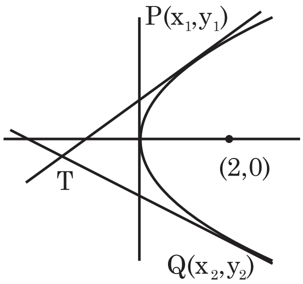
$$\left|\frac{10h+10k-h^2-k^2}{\sqrt{h^2+k^2}}\right| = 5$$

$\Rightarrow (10h+10k-h^2-k^2)^2 = 25(h^2+k^2)$

$(10x + 10y - x^2 - y^2) = 25(x^2 + y^2)$

8. Ans. C

Sol.



$x_1 + x_2 = 17$

$2t_1^2 + 2t_2^2 = 17$

$2(t_1^2 + t_2^2) = 17$

$(t_1 + t_2)^2 - 2t_1t_2 = 17/2$

$x_1x_2 = 11$

$at_1^2\ at_2^2 = 11$ (a = 2)

$4t_1^2\ t_2^2 = 11$

$t_1^2\ t_2^2 = 11/4$

$T = [at_1t_2, a(t_1 + t_2)]$

$T = [2t_1t_2, 2(t_1 + t_2)]$

S : focus (2, 0)

$TS = \sqrt{(2t_1t_2 - 2)^2 + 4(t_1+t_2)^2}$

$= \sqrt{4(t_1t_2 - 1)^2 + 4(t_1+t_2)^2}$

$= \sqrt{4[(t_1+t_2)^2 + (t_1t_2-1)^2]}$

$= 2\sqrt{(t_1+t_2)^2 + (t_1t_2)^2 + 1 - 2t_1t_2}$

$= 2\sqrt{\frac{17}{2} + 1 + \frac{11}{4}} = 2\sqrt{\frac{34+4+11}{4}} = \sqrt{49} = 7$

9. Ans. D

Sol. Tangent at (3, – 9/2)

$\frac{x(3)}{a^2} + \frac{y(-9)}{2b^2} = 1 \Rightarrow x - 2y = 12$

Compare: $\frac{3}{a^2} = \frac{9}{2b^2(2)} = \frac{1}{12}$

$a^2 = 36;\ b^2 = 27$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 27}{6} = 9$

10. Ans. D

Sol. $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow 1(3\beta - 4\alpha) - 1(4\beta - 4) + 1(4\alpha - 3) = 0$

$\Rightarrow 3\beta - 4\alpha - 4\beta + 4 + 4\alpha - 3 = 0$

$\Rightarrow -\beta + 1 = 0 \Rightarrow \beta = 1$

Now, $|\vec{c}| = \sqrt{3}$ (given)

$\sqrt{1+\alpha^2+\beta^2} = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3 \Rightarrow 1 + \alpha^2 + 1 = 3 \Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

11. Ans. B

Sol. If the planes $x - cy - bz = 0$, $-cx + y - az = 0$, $-bx - ay + z = 0$ pass through a line, then determined formed by coefficients of un-knowns is equal to zero.

$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ca + b) = 0$

$\Rightarrow 1 - a^2 - c^2 - abc - abc - b^2 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc - 1 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

12. Ans. B

Sol. Given, α, β are the roots of equation

$x^2 - (a - 2)x - a - 1 = 0$

$\therefore \alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6 = a^2 - 2a + 1 + 5$

$= (a - 1)^2 + 5 \geq 5$

$\therefore \alpha^2 + \beta^2$ is least if $(a - 1)^2 = 0 \Rightarrow a = 1$.

13. Ans. C

Sol. Let common difference be d.

$a_p = a_1 + (p-1)d$, $a_q = a_1 + (q-1)d$,

$a_r = a_1 + (r-1)d$

As a_p, a_q, a_r are in G.P.

$\therefore \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q}$ (by law of proportions)

or $\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_1 + (q-1)d - a_1 - (r-1)d}{a_1 + (p-1)d - a_1 - (q-1)d} = \frac{q-r}{p-q}$

or $\frac{a_q}{a_p} = \frac{q-r}{p-q} = \frac{r-q}{q-p}$

14. Ans. A

Sol. Coefficient of x^{15} in $(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^3$

$\left(\frac{x(1-x^6)}{1-x}\right)^3$

$x^3(1-x)^{-3}(1-x^6)^3$

$x^3(1 - 3x^6 + 3x^{12})(1-x)^{-3}$, neglect x^{18}

$(x^3 - 3x^9 + 3x^{15})(1-x)^{-3}$

coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r\, x^r$

$x^3 \times {}^{3+r-1}C_r \cdot x^r - 3 \cdot x^9 \times {}^{3+r-1}C_r \cdot x^r + 3 \cdot x^{15} \times {}^{3+r-1}C_r \cdot x^r$

coefficient of x^{12}

coefficient of x^6 coefficient of x^0

${}^{3+12-1}C_{12}$ $-3 \cdot {}^{3+6-1}C_6$

${}^{3+0-1}C_0$

${}^{14}C_{12}$ $-3 \cdot {}^{8}C_6$

$+ 3$

$= 10$

15. Ans. A

Sol. We have $T_{r+1} = {}^{21}C_r \left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r} \left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$

$= {}^{21}C_r (a)^{7-\frac{r}{2}} (b)^{\frac{2}{3}r - \frac{7}{2}}$

Since the power of a and b are same,

$\therefore 7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$

16. Ans. B

Sol. The total number of selections of two numbers x and y from the number 0 to 10 = 11 × 11 = 121

Now, if $|x - y| > 5$, then the possible values for (x, y) are

(0, 6), (0, 7), (0, 8), (0, 9), (0, 10), (1, 7), (1, 8) (1, 9), (1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10), (6, 0), (7, 0), (8, 0), (9, 0), (10, 0), (7, 1), (8, 1), (9, 1), (8, 2), (10, 1), (9, 2), (10, 2), (9, 3) (10, 3), (10, 4)

So, there are 30 pairs of values of x and y.

Hence, the required probability = $\frac{30}{121}$

17. Ans. B

Sol. We have $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$

$\Rightarrow x\sin\phi = \tan\theta - x\cos\phi\tan\theta$

$\Rightarrow x = \frac{\tan\theta}{\sin\phi + \cos\phi\tan\theta}$

$= \frac{\sin\theta}{\cos\theta\sin\phi + \cos\phi\sin\theta} = \frac{\sin\theta}{\sin(\theta+\phi)}$

Similarly, $y = \frac{\sin\phi}{\sin(\theta+\phi)}$; $\therefore \frac{x}{y} = \frac{\sin\theta}{\sin\phi}$

18. Ans. A

Sol. $\sin 2B = \frac{3}{2}\sin 2A$

$3\sin^2 A = \cos 2B$

$\tan 2B = \frac{3}{2}\frac{\sin 2A}{3\sin^2 A} = \frac{3 \cdot 2\sin A\cos A}{2 \cdot 3\sin^2 A}$ = cot

A = $\tan\left(\frac{\pi}{2} - A\right)$

$2B = \frac{\pi}{2} - A \Rightarrow B = \frac{\pi}{4} - \frac{A}{2}$

19. Ans. C

Sol. $\int \frac{\sin(5x-3x)}{\sin 5x \sin 3x} dx$

$= \int \frac{\sin 5x\cos 3x - \cos 5x\sin 3x}{\sin 5x\sin 3x} dx$

$= \int \cot 3x\, dx - \int \cot 5x\, dx$

$= \frac{1}{3} \ln \sin 3x - \frac{1}{5} \ln \sin 5x + C$

20. Ans. B

Sol. $\frac{dy}{dx} = \frac{x}{1+x^2} - \frac{xy}{1+x^2} \Rightarrow \frac{dy}{dx} + \frac{xy}{1+x^2} = \frac{x}{1+x^2}$

I.F. = $e^{\int \frac{x}{1+x^2}dx} = e^{\frac{1}{2}\ln(1+x^2)} = e^{\ln(1+x^2)^{1/2}} = (1 + x^2)^{1/2}$

solution given by $y(1 + x^2)^{1/2}$

$= \int \frac{x}{(1+x^2)} \times (1+x^2)^{1/2}\, dx$

$y(1 + x^2)^{1/2} = \int \frac{x}{\sqrt{1+x^2}} dx$

$y(1 + x^2)^{1/2} = \sqrt{1+x^2} + C$

at $x = 0$, $y = \frac{4}{3}$; $\frac{4}{3} = C + 1 \Rightarrow C = \frac{1}{3}$

$y\sqrt{1+x^2} = \sqrt{1+x^2} + \frac{1}{3}$

at $x = \sqrt{8}$, $y(3) = 3 + \frac{1}{3}$

$\Rightarrow y \times 3 = \frac{10}{3} \Rightarrow y = \frac{10}{9}$

$y\left(\sqrt{8}\right) = \frac{10}{9}$

Hence $y\left(\sqrt{8}\right) + \frac{8}{9} = 2$ **Ans.**

SECTION-II

1. Ans. – 0.08

Sol. $f^{-1}(x) = \frac{1}{g(x)} \Rightarrow x = f\left(\frac{1}{g(x)}\right)$

$\Rightarrow 1 = f'\left(\frac{1}{g(x)}\right) \times \frac{-1}{(g(x))^2} \cdot g'(x)$

at $x = 4$, $1 = f'\left(\frac{1}{g(4)}\right) \times \frac{-1}{(g(4))^2} \cdot g'(4)$

$g(4) = \frac{1}{f^{-1}(4)}$

$f^{-1}(4) = \frac{1}{g(4)} = 3$

$1 = f'(3) \times \frac{-1}{(1/9)^2} \cdot g'(4)$

$1 = f'(3)(-9)\, g'(4)$

$1 = \frac{4}{3}(-9)g'(4) = -12g'(4) \Rightarrow g'(4) = \frac{-1}{12}$

2. Ans. 1.15

Sol. $\lim\limits_{x\to\infty} \frac{x^{\frac{1}{2}}\left[2 + \frac{3}{x^{1/6}} + \frac{4}{x^{1/4}} + ...\right]}{x^{\frac{1}{2}}\left[\left(3 - \frac{4}{x}\right)^{1/2} + \left(3 - \frac{4}{x}\right)^{1/3} + ...\right]}$

$= \frac{2}{\sqrt{3}} = 1.15$ **Ans.**

3. Ans. 0.00

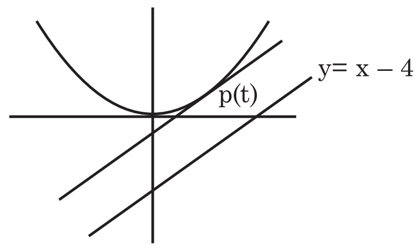
Sol. Applying L'Hospital's Rule

$\lim\limits_{h\to 0} \frac{f'(c+h) + f'(c-h)(-1)}{1}$

$f'(c) - f'(c) = 0$ **Ans.**

4. Ans. 3.00

Sol.



Minimum distance lie along normal

p(t) on parabola

$x^2 = 4y$

$(2t, t^2)$

Slope of tangent at p(t) = 1

equation of tangent to $x^2 = 4y$ at $(2t, t^2)$

$x(2t) = 2 \cdot 1(y + t^2)$

$tx = y + t^2$

slope = t = 1

p(2, 1)

sum of coordinate = 3

5. Ans. 4.00

Sol. Apply LMVT in [1, 2], $1 < c_1 < 2$

$$f'(c) = \frac{f(2) - f(1)}{2-1} \geq 2$$

$f(2) \geq 4$ .... (1)

apply LMVT in [2, 3], $2 < c_2 < 3$

$$f'(c_2) = \frac{f(3) - f(2)}{3-2} \geq 2$$

$7 - f(2) \geq 2 \Rightarrow f(2) \leq 5$ .... (2)

from (1) & (2) :

$4 \leq f(2) \leq 5$

6. Ans. 0.25

Sol. $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{\frac{\pi}{2} + \cos^{-1}x}{\pi + \cos^{-1}x + \cos^{-1}(1-x)} dx$

$(\sin^{-1}x + \cos^{-1}x = \pi/2)$

Apply $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{\frac{\pi}{2} + \cos^{-1}(1-x)}{\pi + \cos^{-1}(1-x) + \cos^{-1}x} dx$

Add $\quad 2I = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{\pi + \cos^{-1}(x) + \cos^{-1}(1-x)}{\pi + \cos^{-1}(x) + \cos^{-1}(1-x)} dx$

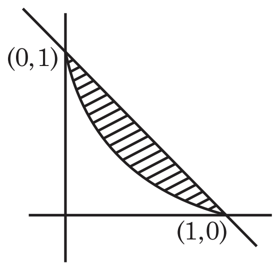
$= (x)_{1/4}^{3/4} = \frac{1}{2}$

$I = \frac{1}{4} = 0.25$ **Ans.**

7. Ans. 0.33

Sol. $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$

$\sqrt{y} = 1 - \sqrt{x}$



$A = \int_0^1 (1-x) - \left(1 + x - 2\sqrt{x}\right) dx$

$= \int_0^1 \left(-2x + 2\sqrt{x}\right) dx$

$= 2\int_0^1 \left(\sqrt{x} - x\right) dx = \frac{1}{3} = 0.33$

8. Ans. 1.57

Sol. $\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right) = \sin^{-1}\left(\sqrt{\frac{4-2\sqrt{3}}{8}}\right)$

$= \sin^{-1}\left(\sqrt{\frac{\left(\sqrt{3}-1\right)^2}{8}}\right) = \sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \frac{\pi}{12}$

$\cos^{-1}\left(\frac{\sqrt{12}}{4}\right) = \cos^{-1}\left(\frac{2\sqrt{3}}{4}\right) = \frac{\pi}{6}$

$\sec^{-1}\left(\sqrt{2}\right) = \frac{\pi}{4}$

$\cos^{-1}\left[\cot\left\{\frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4}\right\}\right] = \cos^{-1}\left[\cot\left\{\frac{\pi}{2}\right\}\right]$

$= \cos^{-1}(0) = \frac{\pi}{2}$

9. Ans. 4.00

Sol. $|\,(\text{adj. } A^{-1})^{-1}\,| = \frac{1}{|\text{adj. } A^{-1}|} = \frac{1}{|(\text{adj. } A)^{-1}|}$

$= |\text{ adj. } A\,| = |\,A\,|^{n-1}$ (n = order of matrix)

10. Ans. 1.00

Sol. $\begin{vmatrix} \alpha & \alpha+1 & \alpha+2 \\ \alpha+1 & \alpha+2 & \alpha \\ \alpha+2 & \alpha & \alpha+1 \end{vmatrix} = 0$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_2$

$\begin{vmatrix} \alpha & \alpha+1 & \alpha+2 \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = 0$

expand $\alpha(1-4) - (\alpha+1)(1+2) + (\alpha+2)(-2-1) = 0$

$-3\alpha - 3\alpha - 3 - 3\alpha - 6 = 0$

$-9\alpha - 9 = 0$

$9\alpha + 9 = 0$

$\alpha = -1$