(0000CJA102119027) Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Advanced)
AIOT
09-02-2020

JEE(Main+Advanced) : LEADER & ENTHUSIAST COURSE

ANSWER KEY — PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	A	D	A,B,C,D	B,C,D	B,C	A,C,D	B,C,D	B
	Q.	11	12								
	A.	A,B,C	A,B,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	1.00	1.60	0.07 to 0.08	12.00	2.23 to 2.24	24.00				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	A	A	C	A,C	A,C,D	A,B,C,D	A,B,C	C,D	B,D
	Q.	11	12								
	A.	A,B,C	B,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	374.15	0.06 or 0.07	158.00	6.00	2.00	5.00				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	B	B	B,C	A,B	B,C	B,C,D	B,C,D	A,B,C,D
	Q.	11	12								
	A.	B,C	B,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	0.37	0.66 or 0.67	1.73	1.50	4.00	1.70				

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ANSWER KEY — PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,B,D	B,D	B,D	A,B,D	C,D	B,D	A,B,D	B	D
	Q.	11	12								
	A.	C	D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	1.10 to 1.15	5.00	960.00	22.22	0.40	7.35 to 7.38				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	D	A,B,C	A,B,D	A,C,D	C	A,C,D	D	B
	Q.	11	12								
	A.	A	B								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	5.50	–4.20	1650.00	5.00	3.00	3.00				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	C	A,B,D	A,C	A,C	B,C,D	A,C	B,C	A	B
	Q.	11	12								
	A.	A	B								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	4.00	5.00	15.00	5.00	7.00	6.00				

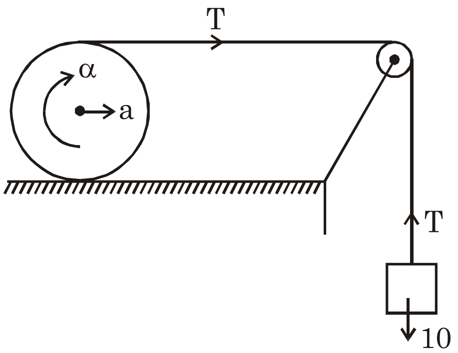
PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (C)

Sol. Smooth



$10 - T = 1a_1 = a + R\alpha$

$TR = \frac{1}{2} \times 2R^2\alpha = R\alpha$

$T = 2a$

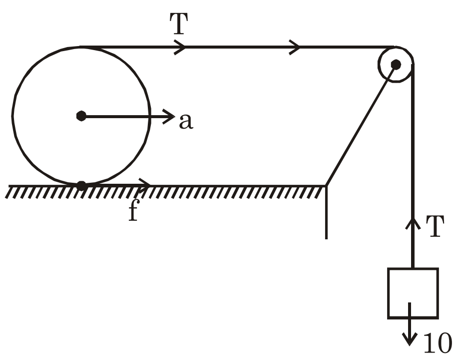
$\Rightarrow \; 10 - T = \frac{T}{2} + T$

$10 = 2.5\,T$

$\Rightarrow \; T = 4$

$a = 2\text{ m/s}^2 \Rightarrow \ell = \frac{1}{2} \times 2t_1^2 \quad t_1 = \sqrt{\ell}$

Case (ii)



$10 - T' = 1a'_1 = a' + R\alpha' = 2R\alpha'$

$T' \times 2R = \frac{3}{2} \times 2 \times R^2\alpha'$

$R\alpha' = \frac{2T'}{3}$

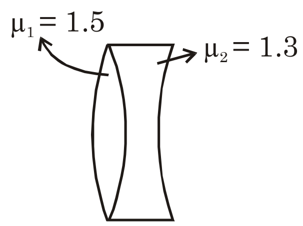
$10 - T' = 2 \times \frac{2T'}{3}$

$10 = \frac{7T'}{3} \Rightarrow T' = \frac{30}{7} \Rightarrow a' = \frac{20}{7}$

$\ell = \frac{1}{2} \times \frac{20}{7} \times t_2^2 \Rightarrow t_2 = \sqrt{\frac{7\ell}{10}}$

2. Ans. (B)

Sol.



$F_{eq} \Rightarrow$ +ve converging.

$F_{eq} \Rightarrow$ –ve diverging.

$$\frac{1}{F_{eq}} = \frac{1}{F_1} + \frac{1}{F_2}$$

$$\frac{1}{F_1} = \left[\frac{\mu_1}{\mu_m} - 1\right]\left[\frac{2}{R}\right] \text{ and } \frac{1}{F_2} = \left[\frac{\mu_2}{\mu_m} - 1\right]\left[\frac{-2}{R}\right]$$

$$\frac{1}{F_{eq}} = \frac{2}{R}\left[\left(\frac{\mu_1}{\mu_m} - 1\right) - \left(\frac{\mu_2}{\mu_m} - 1\right)\right]$$

$$\frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{\mu_1}{\mu_m} - \frac{\mu_2}{\mu_m}\right]$$

(A) $\frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.4} - \frac{1.3}{1.4}\right], \; \frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.6} - \frac{1.3}{1.6}\right]$

Converging Converging

(B) $\frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.2} - \frac{1.3}{1.2}\right], \; \frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.4} - \frac{1.3}{1.4}\right]$

Converging Converging

(C) $\frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.6} - \frac{1.3}{1.6}\right]$, $\frac{1}{F_{eq}} = \frac{2}{R}\left[\frac{1.5}{1.2} - \frac{1.3}{1.2}\right]$

Converging Converging

3. Ans. (A)

Sol. $\Delta W_{AB} + \Delta U_{AB} = 0$

$$\Delta U_{AB} = -\frac{1}{2} \times 3 \times 10^5 (10^{-3}) = -150J$$

$\Delta Q_{ACB} = W + \Delta V$

$= -150 + 200 = 50$ J

4. Ans. (D)

Sol. $q = 10\mu C = 10^{-5}C$

$m = 10^{-3}$ kg

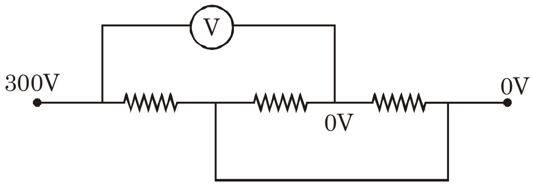
$$\vec{v} = 10^6\left[\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\vec{F} = q[\vec{v} \times \vec{B}]$$

5. Ans. (A,B,C,D)

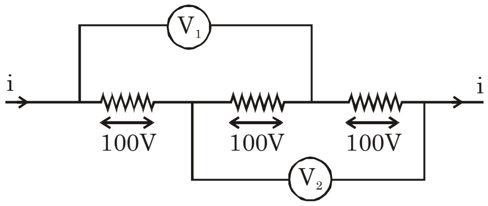
Sol. Ideal voltmeter ($R_V = \infty$)
Ideal ammeter ($R_A = 0$)

(A)



Reading = 300V.

(B)

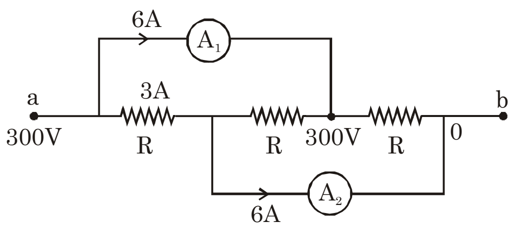


$$i = \frac{300}{300} = 1A$$

$V_1 = 100 + 100 = 200V$

$V_2 = 100 + 100 = 200V$

(C)

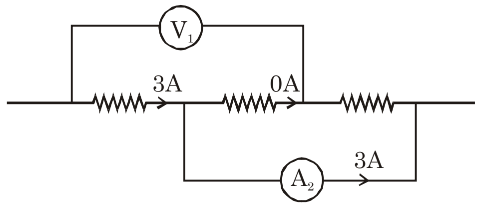


All resistance are in parallel combination.

$$i = \frac{300}{100/3} = 9A$$

Reading of ammeter is 6A

(D)



$$i = \frac{300}{100} = 3A$$

6. Ans. (B,C,D)

7. Ans. (B,C)

Sol. $Q = CB\ell v$

$\ell = x \tan\alpha$

$\Rightarrow Q = CB\ell vx \tan\alpha$

$i = CB\ell v^2 \tan\alpha$

$F = Bi\ell = Bi \times x \tan\alpha = Bi(vt \tan\alpha + x_0)$

$\tau = 0$

$$U = \frac{Q^2}{2C} = \frac{\left(CB\ell v\left(x_0 + vt\right)\tan\alpha\right)^2}{2C}$$

8. Ans. (A,C,D)

Sol. ${}_{Z_1}A^{A_1} \longrightarrow {}_{Z_1-1}C^{A_1-8}$ and

${}_{Z_2}B^{A_2} \longrightarrow {}_{Z_2+3}C^{A_2-4}$

$\therefore Z_1 - 1 = Z_2 + 3 \quad \Rightarrow Z_1 - Z_2 = 4$

and $A_1 - 8 = A_2 - 4 \quad \Rightarrow A_1 - A_2 = 4$

At time t

$N_A = 4N_0 e^{-\lambda_1 t}$ and $N_B = N_0 e^{-\lambda_2 t}$

when $N_A = N_B \Rightarrow 4e^{-\lambda_1 t} = e^{-\lambda_2 t}$

$$\Rightarrow t = \frac{2\ln 2}{\lambda_1 - \lambda_2} = 4hr$$

$N_C = 4N_0(1 - e^{-\lambda_1 t}) + N_0(1 - e^{-\lambda_2 t})$

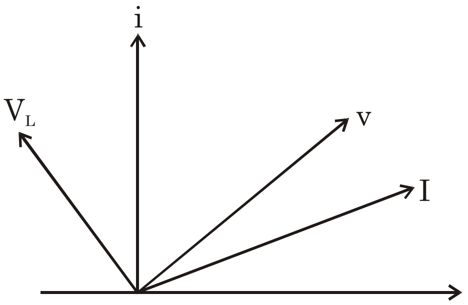
putting the value to t = 4hr, we get

$$N_c = \frac{9N_0}{2}$$

9. Ans. (B,C,D)

10. Ans. (B)

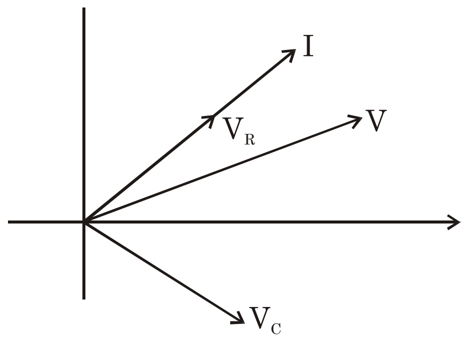
Sol. (i) when box contains inductor, phasor diagram will be



Hence V_L is leading with V of source, So A is not possible.

and V_R is lagging behind V of source, So B is possible.

(ii) When box containr capacitor, phasor diagram will be



Hence V_R is leading with V of source, So C is not possible.

And V_C is lagging behind V, So D is also not possible

11. Ans. (A,B,C)

Sol. $V = \int \vec{F}.d\vec{r} = \frac{kr^2}{2}$

$\therefore \frac{mv^2}{r} = kr \Rightarrow v^2 \propto r^2 \Rightarrow v \propto r$ or $v = cr$

also $mvr = \frac{nh}{2\pi} \Rightarrow cr^2 \propto n \Rightarrow r \propto \sqrt{n}$ and also

$v \propto r \propto \sqrt{n}$

$\omega = v/r \propto n^\circ$

TE = KE + PE

$\propto \frac{1}{2}mV^2 + \frac{1}{2}Kr^2$

$TE \propto r^2 \propto (\sqrt{n})^2 \propto n$

$\therefore TE \propto n$

12. Ans. (A,B,D)

Sol. From graph clealry there are 10 intersection points including the intersection of lines of A & B also; So there are total 10 collisions.

Now by exhchanging the slope of line at each intersection we can easily find that final velocity of A,C & D are

$V_A = +1.5$ m/s

$V_C = 0$

$V_D = -\frac{10}{9}$ m/s

SECTION-II

1. Ans. 1.00

Sol. Fringe width,

$\beta = \frac{D\lambda}{d} = \frac{1}{3}$mm

At a point 1mm above maxima of order-1 = 4 th order maxima will be formed

$\therefore$ Intensity at this point = 1 W/m^2

2. Ans. 1.60

Sol. $f_1 = f_0$

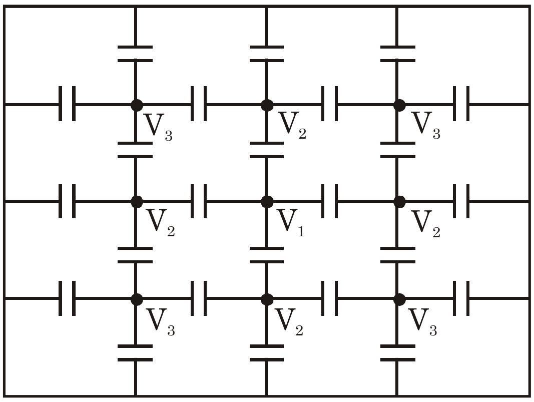
$f_2 = \frac{c+v}{c-v} \times f_0$

$f_b = 256\left(\frac{2v}{c-v}\right) = 1.6\text{Hz}$

3. Ans. 0.07 to 0.08

4. Ans. 12.00

Sol.



$2(V_3 - 0) + 2(V_3 - V_2) = 0$

$\Rightarrow 2V_3 = V_2$

$4(V_1 - V_2) = q$

$V_1 - V_2 = \frac{q}{4}$

$(V_2 - 0) + (V_2 - V_1) + 2(V_2 - V_3) = 0$

$4V_2 = V_1 + 2V_3$

$\Rightarrow 3V_2 = V_1 \Rightarrow \frac{2V_1}{3} = \frac{q}{4}$

$\Rightarrow V_1 = \frac{3q}{8} = 3V$

$V_2 = \frac{V_1}{3} = \frac{q}{8} = 1V$

$V_3 = \frac{V_2}{2} = \frac{q}{16} = \frac{1}{2}V$

$U = \frac{1}{2} \times C \times V_3^2 \times 8 + \frac{1}{2} \times C \times V_2^2 \times 4 +$

$\frac{1}{2}C(V_1 - V_2)^2 \times 4$

$+ \frac{1}{2} \times C \times (V_2 - V_3)^2 \times 8$

$= \frac{1}{2} \times \frac{1}{4} \times 8 + \frac{1}{2} \times 1^2 \times 4^2 + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times \frac{1}{4} \times 8$

$= 12\ \mu J$

5. Ans. 2.23 to 2.24

Sol. $\rho = \rho_0\left(1 - \frac{r^2}{a^2}\right)$

By Gauss law :

$E(4\pi r^2) = \frac{1}{\varepsilon_0}\int_0^r \rho(4\pi r^2)dr$

$Er^2 = \frac{\rho_0}{\varepsilon_0}\int_0^r \left(1 - \frac{r^2}{a^2}\right)r^2 dr$

$\frac{\rho_0}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$

for maximum E

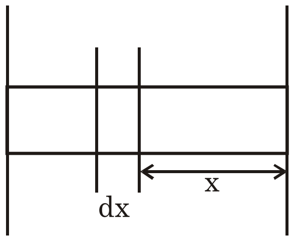
$\frac{dE}{dr} = 0$ $\qquad \frac{1}{3} - \frac{3r^2}{5a^2} = 0$

$\Rightarrow \frac{3r^2}{5a^2} = \frac{1}{3} \Rightarrow r = \sqrt{\frac{5a^2}{9}}$

$\Rightarrow r = \sqrt{\frac{5}{9}}a = 2.23$ fm

6. Ans. 24.00

Sol. In steady state,



$d\ell = dx\ \alpha\Delta T$

$T = ax + b$

$x = 0, T = 0$

$x = 1m, T = 100°C$

$T = 100\ x$

$\int d\ell = \int 100x \times dx \times 10^{-5}$

$\Delta\ell = 10^{-3} \times \frac{1}{2}$

$F = \frac{YA}{\ell}\Delta\ell$

$= \frac{2 \times 10^{11} \times 4 \times 10^{-6} \times 10^{-3}}{1} \times \frac{1}{2} = 400$ N

$2\mu N = mg$

$m = \frac{2 \times 0.3 \times 400}{10} = 24kg$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans.(B)**
2. **Ans.(A)**
3. **Ans.(A)**
4. **Ans.(C)**

Solubility, S = $\frac{5520\times10^{-6}}{276}=2\times10^{-5}M$

K_{sp} = $4S^3 = 4 \times (2 \times 10^{-5})^3 = 3.2 \times 10^{-14}$

5. **Ans.(A,C)**
6. **Ans.(A,C,D)**
7. **Ans.(A,B,C,D)**

Tanning of leather, delta formation and cottrell precipitation are applications and examples of colloids.

8. **Ans.(A,B,C)**

$\Lambda_m^0(CH_3COOH) = \lambda_m^0(H^+) + \lambda_m^0(CH_3COO^-)$

$= 3.474 \times 10^{-2} + 1.351 + 10^{-2}$

$= 4.825 \times 10^{-2}$ ohm^{-1}m^{2}mol^{-1}

$k = G \times G^* = \frac{1}{2000}\times20=0.01\,ohm^{-1}m^{-1}$

$\Lambda_m=\frac{k}{C}=\frac{0.01ohm^{-1}m^{-1}}{0.1\times10^{-3}molm^{-3}}$

$= 1 \times 10^{-4}$ ohm^{-1}m^{2}mol^{-1}

Now, $\alpha = \frac{\Lambda_m}{\Lambda_m^0}=\frac{1\times10^{-4}}{4.825\times10^{-2}}$

$\therefore [H^+] = C\alpha = 0.1 \times \frac{10^{-2}}{4.825}=2.07\times10^{-4}M$

9. **Ans.(C,D)**
10. **Ans.(B,D)**
11. **Ans.(A,B,C)**
12. **Ans.(B,D)**

SECTION-II

1. **Ans.(374.15)**

$$\Delta T_b = K_b.m = 0.52 \times \frac{\frac{11.7}{58.5}\times2+\frac{34.2}{34.2}}{0.260}=1K$$

$\therefore$ B.P. = 373.15 + 1 = 374.15 K

2. **Ans.(0.06 or 0.07)**

	A(g) →	2B(g) + C(l)
t = 0	0.08 bar	0
t = 10 min	(0.08 – x) bar	2x bar
t = ∞	(≈ 0)	0.16 bar

but at t = ∞, total pressure is 0.165 bar. hence (0.165 – 0.160) = 0.005 bar is the vapour pressure of C(l).

Now, at t = 10 min, (0.08 – x) + 2x + 0.005 = 0.125

$\therefore$ x = 0.04

Hence,

k =

$$\frac{1}{t}.\ln\frac{P_A^0}{P_A}=\frac{1}{10}.\ln\frac{0.08}{0.08-0.04}=0.06931\,min^{-1}$$

3. **Ans.(158.00)**

$\Delta U = n.C_{v,m}.(T_2 - T_1) = 1 \times 3.2R \times (800 - 300)$

= 3.2 × 0.08 × 500 = 128 L-atm/mol.

Now, $\Delta H = \Delta U + \Delta(PV)$ = 128 + (10 × 5 – 10 × 2)

= 158 L-atm

4. **Ans.(6.00)**

Sol. (1,2,3,5,8,9).

5. **Ans.(2.00)**
6. **Ans.(5.00)**

PART-3 : MATHEMATICS — SOLUTION

SECTION–I

1. Ans. (B)

Sol. $\frac{\sin y}{\cos 3y}=\frac{1}{2}\frac{2\sin y\cos y}{\cos 3y.\cos y}=\frac{1}{2}\left[\frac{\sin 2y}{\cos 3y\cos y}\right]$

$=\frac{1}{2}\left[\frac{\sin(3y-y)}{\cos 3y\cos y}\right]=\frac{1}{2}[\tan 3y-\tan y]$

$\therefore\ S=\frac{1}{2}$ [tan3x – tanx + tan3^2x – tan3x +....+tan3^nx – tan3^{n-1}x]

$=\frac{1}{2}\left[\tan 3^n x-\tan x\right]$

2. Ans. (D)

Sol. Diagonals of rhombus are angle bisector of adjacent sides.

Equation of bisector of the sides are

$\frac{ax+y-16}{\sqrt{1+a^2}}=\pm\left(\frac{x+ay-6}{\sqrt{1+a^2}}\right)$

$\Rightarrow$ ax + y – 16 = ±(x + ay – 6)

passes through (3,5)

$\Rightarrow$ 3a + 5 – 16 = ±(3 + 5a – 6)

$\Rightarrow\ a=-4,\frac{7}{4}$

$\therefore$ sum of values $=-\frac{9}{4}$

3. Ans. (B)

Sol. $\because$ $x^3+y^3+(-1)^3=3(x)(y)(-1)$

$\Rightarrow$ x + y + –1 = 0 or x = y = –1

$\because$ (x,y) ≠ (–1,–1)

$\therefore$ locus of P is x + y – 1 = 0

$\therefore$ Equation of family of circles touching x + y – 1 = 0 at (–1,2) is

$(x+1)^2+(y-2)^2+\lambda(x+y-1)=0$

$\because$ It passes through (1,–2)

$\Rightarrow (1+1)^2+(y-2)^2+\lambda(1-2-1)=0$

$\Rightarrow 20-2\lambda=0\Rightarrow\lambda=10$

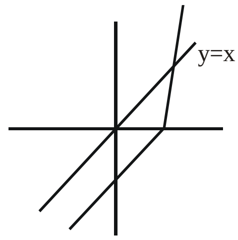
$\therefore$ Circle is $x^2+y^2+12x+6y-5=0$

4. Ans. (B)

Sol. $f(x)=2x-1+\left|x-\frac{1}{2}\right|$

$=\begin{cases}x-\frac{1}{2} & x<\frac{1}{2}\\ 3x-\frac{3}{2} & x\geq\frac{1}{2}\end{cases}$

As f(x) is incresing, solution of equation $f(x)=f^{-1}(x)$ is same as f(x) = x.



$\therefore$ No. of solutions of $f(x)=f^{-1}(x)$ is one.

5. Ans. (B,C)

Sol. Using LMVT, for x ∈ (–4,4),

$\frac{f'(x)-f'(-4)}{x+4}\geq-3$

$\Rightarrow f(x)\geq-2-3x$ ...(i)

Also $\frac{f'(4)-f'(x)}{4-x}\geq-3$

$\Rightarrow f'(x)\leq-2-3x$ ...(ii)

from (i) and (ii)

$f'(x)=-2-3x\Rightarrow f(x)$ has maximum at $x=-\frac{2}{3}$

$f(x)=-2x-\frac{3x^2}{2}$ [$\because f(0)=0$]

$g'(x)=f(x)=-2x-\frac{3x^2}{2}$

∀ x ∈ [0,4) g'(x) < 0 ⇒ g(x) is decreasing

Also, $g''(x)=f'(x)=-2-3x<0$ ∀ x ∈ (0,4)

⇒ Graph of g(x) is concave down in (0,4)

6. Ans. (A,B)

Sol. $\int \frac{x^4+1}{x^6+1}dx = \int \frac{x^4-x^2+1+x^2}{(x^2+1)(x^4-x^2+1)}dx$

$= \int \frac{dx}{x^2+1} + \int \frac{x^2}{(x^3)^2+1}dx$

$= \tan^{-1}x + \frac{1}{3}\tan^{-1}(x^3) + c$

$\int \frac{(x^2+1)^2-2x^2}{(x^2+1)(x^4-x^2+1)}dx = \int \frac{x^2+1}{x^4-x^2+1}dx - 2\int \frac{x^2}{(x^3)^2+1}dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+1}dx - \frac{2}{3}\int \frac{d(x^3)}{(x^3)^2+1}$

$= \tan^{-1}\left(x-\frac{1}{x}\right) - \frac{2}{3}\tan^{-1}(x^3) + c$

7. Ans. (B,C)

Sol. Clearly if $z = c \Rightarrow \rho = 0$ and $t \in R$
if $z - c \neq 0$.

$|z-c| = |\rho|\left|\frac{1+it}{1-it}\right| \Rightarrow \rho = \pm|z-c|$

Also. $\frac{z-c}{\rho} = \frac{1+it}{1-it} \Rightarrow t = i\left(\frac{\rho-z+c}{\rho+z-c}\right)$

$\Rightarrow t = i\left(\frac{1-\frac{z-c}{\rho}}{1+\frac{z-c}{\rho}}\right) = i\left(\frac{1-\omega}{1+\omega}\right)$

where $\omega = \frac{z-c}{\rho} = \frac{z-c}{\pm|z-c|} \Rightarrow |\omega| = 1$

$= i\left(\frac{(1-\omega)(1+\bar{\omega})}{(1+\omega)(1+\bar{\omega})}\right) = i\left(\frac{1-\omega\bar{\omega}-\omega+\bar{\omega}}{1+\omega\bar{\omega}+\omega+\bar{\omega}}\right)$

$= i\left(\frac{-2i\,\text{Im}(\omega)}{2+2\text{Re}(\omega)}\right) = \frac{\text{Im}(\omega)}{1+\text{Re}(\omega)}$

$\Rightarrow t = \frac{\text{Im}(z-c)}{\pm|z-c| + \text{Re}(z-c)}$

8. Ans. (B,C,D)

Sol. $S_n = \sum_{k=1}^{\infty} k^n r^k$

$\frac{d}{dr}(S_n) = \sum_{k=1}^{\infty} k^n.k.r^{k-1} = \frac{1}{r}\sum_{k=1}^{\infty} k^{n+1}r^n$

$\Rightarrow \frac{d}{dr}(S_n) = \frac{1}{r}S_{n+1}$

$\Rightarrow S_{n+1} = r\frac{d}{dr}(S_n)$

$\Rightarrow S_{n+1} = r\frac{d}{dr}(S_n)$

Now. $S_0 = \sum_{k=1}^{\infty} r^k = \frac{r}{1-r}$

$S_1 = r\frac{d}{dr}(S_0) = \frac{r}{(1-r)^2}$

$S_2 = r\frac{d}{dr}(S_1) = r\frac{d}{dr}\left(\frac{r}{(1-r)^2}\right)$

$= r\left[\frac{(1-r)^2+2r(1-r)}{(1-r)^4}\right] = \frac{r}{(1-r)^2} + \frac{2r^2}{(1-r)^3}$

$= \frac{1}{r}\left[S_0^2 + 2S_0^3\right]$

9. Ans. (B,C,D)

Sol. Let events A,B,C correspond to prize being behind selected, opened and remaining door respectively. Let H_B denote event that host opens door B.
Probability that he sticks with his choice

$= P\left(\frac{A}{H_B}\right)$ and he switches $= P\left(\frac{C}{H_B}\right)$

Now. $P(A) = P(B) = P(C) = \frac{1}{3}$

$$P\left(\frac{H_B}{A}\right)=\frac{1}{2},\ P\left(\frac{H_B}{B}\right)=0,\ P\left(\frac{H_B}{C}\right)=1$$

$$P(H_B)=P(A).P\left(\frac{H_B}{A}\right)+P(B).P\left(\frac{H_B}{B}\right)+P(C).P\left(\frac{H_B}{C}\right)$$

$$=\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 0+\frac{1}{3}\times 1=\frac{1}{2}$$

$$P\left(\frac{A}{H_B}\right)=\frac{P\left(\frac{H_B}{A}\right).P(A)}{P(H_B)}=\frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{2}}=\frac{1}{3}$$

$$P\left(\frac{C}{H_B}\right)=\frac{P(C).P\left(\frac{H_B}{C}\right)}{P(H_B)}=\frac{2}{3}$$

Clearly it is advantageous to switch.

10. Ans. (A,B,C,D)

Sol. Adding equation (i) and (ii)

$2z = a + b \Rightarrow z=\frac{a+b}{2}$.

Adding (i) and (iii)

$2y = a + c \Rightarrow y=\frac{a+c}{2}$

Adding (ii) and (iii)

$2x = b + c \Rightarrow x=\frac{b+c}{2}$

$\Rightarrow x + y + z = a + b + c.$

11. Ans. (B,C)

Sol. Tangents to hyperbola $\frac{x^2}{2}-\frac{y^2}{3}=1$

$y=mx+\sqrt{2m^2-3}$

passes through (α,β)

$\Rightarrow (\beta - m\alpha)^2 = 2m^2 - 3$

$\Rightarrow m^2(\alpha^2 - 2) - 2\alpha\beta m + \beta^2 + 3 = 0$

tanA.tanB = product of roots of above equation $=\frac{\beta^2+3}{\alpha^2-2}=k$

if $k = 2$, $\frac{\beta^2+3}{\alpha^2-2}=2\Rightarrow\beta^2+3=2\alpha^2-4$

$\Rightarrow \beta^2 = 2\alpha^2 - 7$

if $k = 3$ $\quad \beta^2 + 3 = 3\alpha^2 - 6$

$\Rightarrow \beta^2 = 3a^2 = 9$

12. Ans. (B,D)

Sol. $(a + b)^{2n} + (a - b)^{2n} = 2\sum_{r=0}^{n}\left({}^{2n}C_{2r}\right)a^{2n-2r}b^{2r}$

put $a = 1$, $b=\sqrt{3}$

$$2\sum_{r=0}^{n}\left({}^{2n}C_r\right)\left(\sqrt{3}\right)^{2r}=\left(\sqrt{3}+1\right)^{2n}+\left(\sqrt{3}-1\right)^{2n}$$

$$=\left(4+2\sqrt{3}\right)^n+\left(4-2\sqrt{3}\right)^n$$

$$=2^n\left(\left(2+\sqrt{3}\right)^n+\left(2-\sqrt{3}\right)^n\right)$$

$$\Rightarrow\sum_{r=0}^{n}\left({}^{2n}C_r\right)\left(\sqrt{3}\right)^{2r}=2^{n-1}\left(\left(2+\sqrt{3}\right)^n+\left(2-\sqrt{3}\right)^n\right)$$

$$\Rightarrow T_n=\left(2+\sqrt{3}\right)^n+\left(2-\sqrt{3}\right)^n$$

$$\Rightarrow T_{n+2}-\left[\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)\right]\ T_{n+1}+\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)T_n=0$$

$\Rightarrow T_{n+2} = 4T_{n+1} - T_n$

$T_1 = {}^2C_0 + 3x\ {}^2C_2 = 4$

$2T_2 = {}^4C_0 + 3.{}^4C_2 + 3^2.{}^4C_4$

$\Rightarrow 2T_2 = 1 + 18 + 9 \Rightarrow T_2 = 14$

$T_3 = 4 \times T_2 - T_1 = 4 \times 14 - 4 = 52$

$T_4 = 4 \times T_3 - T_2 = 4 \times 52 - 14 = 194$

SECTION–II

1. Ans. 0.37

Sol. $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\left(\frac{\tan x}{x-\tan x}\right)}$

$$=e^{\lim_{x\to0}\left(\frac{\tan x}{x-\tan x}\right)\left(\frac{\tan x-x}{x}\right)}=e^{-\lim_{x\to0}\frac{\tan x}{x}}=e^{-1}$$

$$=\frac{1}{e}=0.37$$

2. Ans. 0.66 or 0.67

Sol. $\int_0^1 f(x)dx=\sum_{n=1}^{\infty}\int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}}f(x)dx=\sum_{n=1}^{\infty}\int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}}\frac{1}{2^{n-1}}dx$

$$\int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}}f(x)dx=\frac{1}{2^{n-1}}\left[\frac{1}{2^{n-1}}-\frac{1}{2^n}\right]=\frac{1}{2^{2n-1}}$$

$$\therefore\int_0^1 f(x)dx=\sum_{n=1}^{\infty}\frac{1}{2^{2n-1}}=\frac{\frac{1}{2}}{1-\frac{1}{4}}=\frac{2}{3}$$

3. Ans. 1.73

Sol. $(\tan^{-1}y - x)dy = (1 + y^2)dx$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

IF $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$$x.e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2}.e^{\tan^{-1}y}dy$$

$$I = \int \frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}dy$$

Let $\tan^{-1}y = t$

$$\frac{1}{1+y^2}dy = dt$$

$$\int t\, e^t dt = te^t - e^t + c = e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$$

$$\therefore\ x.e^{\tan^{-1}y} = e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$$

$\Rightarrow x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$

passes through (0, tan1)

$0 = 1 - 1 + c \Rightarrow c = 0$

$\therefore x = \tan^{-1}y - 1 \Rightarrow \tan^{-1}y = (x + 1)$

$\Rightarrow y = \tan(x + 1)$

for $x = \frac{\pi}{3} - 1$

$$y = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} = 1.732$$

4. Ans. 1.50

Sol. $\vec{c}\times\vec{a} = \vec{b} \Rightarrow |\vec{c}\times\vec{a}| = |\vec{b}|$

$$\Rightarrow |\vec{c}||\vec{a}|\sin\theta = 3 \Rightarrow |\vec{c}| = \frac{3}{2\sin\theta}$$

$$|\vec{c}-\vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a}$$

$$= |\vec{c}|^2 + 4 - 2|\vec{c}||\vec{a}|\cos\theta$$

$$= \frac{9}{4}\operatorname{cosec}^2\theta + 4 - 6\cot\theta$$

$$= \frac{9}{4} + \left(\frac{3}{2}\cot\theta - 2\right)^2$$

$$\Rightarrow |\vec{c}-\vec{a}|^2 \ge \frac{9}{4} \Rightarrow |\vec{c}-\vec{a}| \ge \frac{3}{2}$$

$\therefore$ Least value = 1.50

5. Ans. 4.00

Sol. Given plane can be

$(2x + 3y - z + 1) + \lambda(x + y - 2z + 3) = 0$

$\Rightarrow x(2 + \lambda) + y(3 + \lambda) - z(1 + 2\lambda) + 1 + 3\lambda = 0$

It is perpendicular to $3x - y - 2z - 4 = 0$

$\Rightarrow 3(2 + \lambda) - (3 + \lambda) + 2(1 + 2\lambda) = 0$

$\Rightarrow 6\lambda + 5 = 0 \Rightarrow \lambda = -\frac{5}{6}$

$\therefore$ Plane is $7x + 13y + 4z - 9 = 0$

$\therefore \ell = 7 \quad m = 4 \quad n = -9$

$$\frac{\ell - n}{m} = \frac{16}{4} = 4.00$$

6. Ans. 1.70

Sol. 2,3,..........,n – 1 n.

1,3,..........,n.

....

1,2,3,..........,n – 1.

In each sequence missing number can be accomodated in (n –1) ways, but in every consecutive pair one sequence is recounted.

$\therefore f(n) = n(n - 1) - (n - 1) + 1 = n^2 - 2n + 2$

$f(4) = 16 - 8 + 2 = 17$

$f(5) = 5^2 - 10 + 2 = 17$

$$\therefore \frac{f(5)}{f(4)} = 1.70$$

JEE(Main+Advanced) : LEADER & ENTHUSIAST COURSE

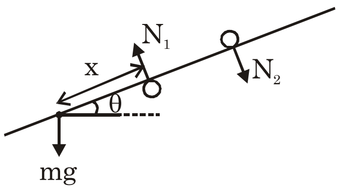
PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A, C)

Sol. Velocity will be maximum where a = 0



$mg \cos \theta = N_2 \times 10$

$mg \sin \theta = \mu(N_1 + N_2)$

$mg\cos\theta + N_2 = N_1$

$mg\sin\theta = \mu(mg\cos\theta + 2N_2)$

$$mg\sin\theta = \mu \; mg \cos \theta \left(1 + \frac{2 \times x}{10}\right)$$

$$1 = \frac{4}{5}\left(1 + \frac{2x}{10}\right)$$

$$x = \frac{5}{4}$$

Initially N_2 will be zero as torque about the first peg is zero.

2. Ans. (A,B,D)

Sol. Power leaving the system = P

$P = \sigma(4\pi C^2)T_C^4$

Net power leaving B = P

$P = \sigma(4\pi b^2)T_B^4 - \sigma(4\pi b^2)T_C^4$

3. Ans. (B,D)

Sol. As 3H inductor is short circuited $\frac{di}{dt}$ across it will be zero and it will retain its energy.

4. Ans. (B,D)

Sol. $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\ell T}{m}}$

$$T = \frac{\ell}{V} = \sqrt{\frac{M\ell}{T}}$$

5. Ans. (A,B,D)

Sol. Curve is symmetric near minimum deviation.

$$\mu = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \mu = \sqrt{2}$$

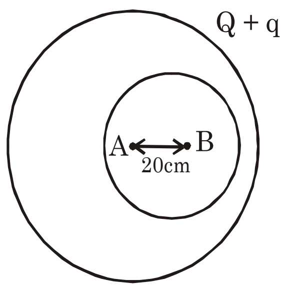
Critical angle is 45°. If $\angle r_1 < 15°$ then TIR will happen on second face.

6. Ans. (C, D)

Sol. Since there is no parallax image formed in the same place so MI = 2f. We need position of M & I.

7. Ans. (B,D)

Sol.



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The charge distribution on the inner surface of the sphere will be non-uniform, while charge distribution on the outer surface will be uniform.

$$\therefore\ V_B = \frac{k(Q+q)}{R} + \frac{k(-q)}{r} + \frac{kq}{r'}$$

$$= \frac{9\times10^2\times9\times10^{-6}}{1}$$

$$+\frac{k(-5)\times10^{-6}}{0.5} + \frac{k(5\times10^{-6})}{0.2} = 216\text{ kV}$$

Also outside the sphere field is due to charge on the outer surface which doesnot change on changing the position of charge in the cavity. Hence field outside the sphere will not change.

8. Ans. (A,B,D)

Sol. Just before point A,

$N_1 = mg$

Just after point A,

$$N_2 - mg = \frac{mv^2}{R} \Rightarrow N_2 = mg + \frac{mv^2}{R}$$

$$\therefore\ N_1 = 40\ \&\ N_2 = 40 + \frac{4\times16}{1} = 104$$

Applying work energy theorem,

$$-mgh = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

$$\Rightarrow -4\times10\times\frac{1}{2} = \frac{1}{2}\times4\times V'^2 - \frac{1}{2}\times4\times16$$

$$\therefore\ V' = \sqrt{6}\ \text{m/s}$$

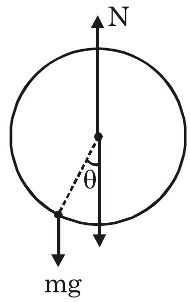
From geometry, $R - R\cos\theta = \frac{R}{2}$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

Note : At maximum height, horizontal component of velocity will remain non-zero.

9. Ans. (B)

Sol.



Writing torque equation about centre point,

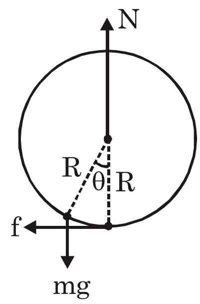
$\tau = mgR\sin\theta$

$$\therefore\ \alpha = \frac{mg\,R\sin\theta}{mR^2 + mR^2}$$

$$\Rightarrow \alpha = \frac{g}{2R}\theta \quad (\theta \text{ is very small})$$

10. Ans. (D)

Sol. Writing torque equation of out lower-most point,



$\tau = mg\,R\sin\theta$

$I \approx 2\,mR^2$

11. Ans. (C)

Sol. Intensity of light is given by $I = \frac{\dot{n}hC}{A\lambda}$

Also when position of jockey are interchanged, polarity of the plates changes.

12. Ans. (D)

Sol. Since $V_0 >>> \phi$ (work function)

$\Rightarrow$ all the electrons ejected due to PEE reaches the +ve plate.

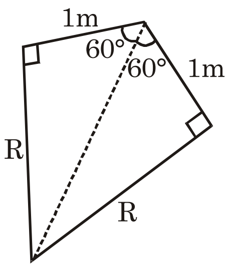
On increasing V_0, K.E. of is reaching at +ve plate increases thus decreasing the cut off wavelength of X-rays.

Changing the plate material will change atomic no. thus nature of characteristic X-ray.

SECTION-II

1. Ans. 1.10 to 1.15

Sol.



$$\frac{\pi}{3} = \omega t$$

$$\tan 60 = \frac{R}{1} = \sqrt{3} = \frac{mv}{qB}$$

$$B = \frac{9\times10^{-31}}{1.6\times10^{-19}} \times \frac{\sqrt{3}}{2} = \frac{9}{3.2}\times10^{-12}$$

$$= 4\pi \times 10^{-5} \times 200\ i$$

$$i = \frac{9}{3.2\times8\pi}\times10^{-7}$$

2. Ans. 5.00

Sol. $v\cos\theta = 5$

$v\cos(53 - \theta) = 3$

$$v\left[\frac{3}{5}\cos\theta + \frac{4}{5}\times\sin\theta\right] = 3$$

$$\frac{4}{5}v\sin\theta = 0$$

$\theta = 0^o$

$\Rightarrow v = 5$

3. Ans. 960.00

Sol. Net work done is zero.

$$(\rho - 800) \times gV \times 1.5 = (1200 - \rho)gV \times 1$$

$$\rho = \frac{4800}{5} = 960 kg/m^3$$

4. Ans. 22.22

Sol. $$\lambda_1 = \frac{h}{m(v_1 - v_2)} = 25\mu$$

$$\lambda_2 = \frac{h}{m(v_1 + v_2)} = 20\mu m$$

$$\Rightarrow \frac{v_1 + v_2}{v_1 - v_2} = \frac{5}{4} \Rightarrow v_2 = \frac{v_1}{9}$$

again

$$\lambda_1 = \frac{h}{m\left(v_1 - \frac{v_1}{9}\right)} = 25\mu m \Rightarrow \frac{h}{mv_1} = \frac{25\times8}{9}\mu m$$

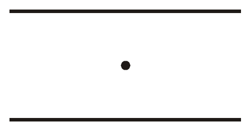
$$\lambda = \frac{h}{mv_1} = \frac{200}{9}\mu m = 22.22\mu m$$

5. Ans. 0.40

Sol. $$-1 \times 0.04 \times \frac{3}{4} + \frac{1}{2} \times 2 \times v^2 = 0$$

6. Ans. 7.35 to 7.38

Sol.



$$v = v_0 e^{-t/RC} = 20e^{-\frac{t}{2}}$$

$$qE = mg$$

$$q \times \frac{20}{d} = mg$$

$$mg - qE = ma$$

$$mg\,(1 - e^{-t/2}) = ma = \frac{dv}{dt}$$

$$v = 10\ [t + 2e^{-t/2} - 2]$$

$$v = \frac{20}{e} \text{ at } t = 2$$

PART–2 : CHEMISTRY — SOLUTION

SECTION-I

1. **Ans.(B)**

2. **Ans.(B)**

3. **Ans.(D)**

4. **Ans.(A,B,C)**

For liquid → gas, ΔH_{sys} + ve, ΔS_{sys} + ve

As it is equilibrium condition, process is thermodynamically reversible and hence, $\Delta G_{sys} = 0$, $\Delta S_{univ} = 0$

5. **Ans.(A,B,D)**

$$Z_A = 8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

$$Z_B = 1 + 12 \times \frac{1}{4} = 4$$

Hence, simplest formula is $A_4B_4 \equiv AB$, and there is 4 AB formula units per unit cell. As 'B' atoms are smaller than 'A' atoms, they are not in contact but as $\frac{r_B}{r_A} = (\sqrt{2}-1)$ = ideal value of octahedral void, 'B' atoms are in contact.

6. **Ans.(A,C,D)**

7. **Ans.(C)**

8. **Ans.(A,C,D)**

9. **Ans.(D)**

10. **Ans.(B)**

11. **Ans.(A)**

12. **Ans.(B)**

SECTION-II

1. **Ans.(5.50)**

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

or, $$\left(35.5+\frac{a\times5^2}{5^2}\right)(5-5\times0.1)=5\times0.082\times450$$

$\therefore$ a = 5.50 atm L^2 mol^{-2}

2. **Ans.(–4.20)**

left electrode : $H_2(g) + 2A^-(aq.) \rightleftharpoons 2HA(aq.) + 2e^-$

Right electrode : $2HB(aq.) + 2e^-(aq.) \rightleftharpoons H_2(g) + 2B^-(aq.)$

Net cell reaction : $2HB(aq.) + 2A^-(aq.) \rightleftharpoons 2HA(aq.) + 2B^-(aq.)$

$$K_{eq.} = \frac{[HA]^2[B^-]^2}{[HB]^2[A^-]^2} = \frac{K_a^2(HB)}{K_a^2(HA)}$$

$$\text{Now, } E^0_{cell} = \frac{0.06}{n}.\log K_{eq.} = \frac{0.06}{2}.\log\frac{K_a^2(HB)}{K_a^2(HA)}$$

$$= \frac{0.06}{1}.\log\frac{4\times10^{-6}}{2\times10^{-5}} = -0.042V$$

3. **Ans.(1650.00)**

$$\Delta t = \frac{t_{1/2}}{\ln 2}.\ln\frac{r_A}{r_B} = \frac{5775}{\ln 2}\times\ln\frac{5000}{4000} = 1650 \text{ yrs.}$$

4. **Ans.(5.00)**

Angular nodes = 2 = l ⇒ d - orbital ⇒ five in number.

5. **Ans.(3.00)**

6. **Ans.(3.00)**

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (A,C)

$$\operatorname{cosec}\left(\frac{\pi}{4}+x\right)+\operatorname{cosec}\left(\frac{\pi}{4}-x\right)=2\sqrt{2}$$

$$\frac{\sqrt{2}}{\sin x+\cos x}+\frac{\sqrt{2}}{\cos x-\sin x}=2\sqrt{2}$$

$$\frac{2\cos x}{\cos^2 x-\sin^2 x}=2$$

$\cos 2x = \cos x$

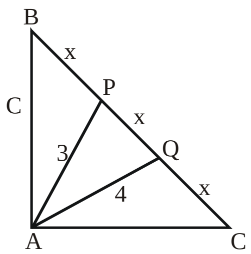
$$2\sin\frac{3x}{2}.\sin\frac{x}{2}=0$$

$$x = 2n\pi,\ \frac{2n\pi}{3}$$

$x \in [0, 4\pi]$ is 7

$x \in (0, 2\pi)$ is 2

2. Ans. (C)



$9 = c^2 + x^2 - 2cx\cos B$

$$\cos B=\frac{c}{3x}$$

$$9=c^2+x^2-2cx\left(\frac{c}{3x}\right)=\frac{c^2}{3}+x^2 \qquad ...(1)$$

Similarly in ΔACQ

$$16=\frac{b^2}{3}+x^2 \qquad ...(2)$$

Adding (1) + (2)

We get

$$25=\left(\frac{b^2+c^2}{3}\right)+2x^2$$

$25 = 5x^2$, $(b^2 + c^2 = a^2 = 9x^2) \Rightarrow x=\sqrt{5}$

$\ell(BC) = 3\sqrt{5}$

3. Ans. (A,B,D)

$ty = x + 2t^2 \xrightarrow[(6,8)]{} 8t = 6t2t^2$

$t^2 - 4t + 3 = 0 \Rightarrow t = 1,3 \qquad (2t^2,4t) \equiv Q \,\&\, R$

$Q = (2,4) \quad P = (6,8)$

$R \equiv (18,12)$

$$\text{area}=\frac{(64-48)^{3/2}}{4}=\frac{64}{4}=16$$

$PS : 2x - y - 4 = 0$

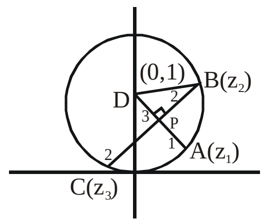
Equation of circle circumscribing the ΔPQR is normal at Q and R intersect at $(30,-24) = S$

Equation of circle on PS as diameter

$(x - 30)(x - 6) + (y + 24)(y - 8) = 0$

$x^2 + y^2 - 36x + 16y - 12 = 0$

4. Ans. (A,C)



$$\frac{3z_1+i}{4}=\frac{2z_2+2z_3}{4}$$

$AD = 1$

$$DP=\frac{3}{4}$$

$$r^2 = BP^2 + DP^2 \Rightarrow BP=1-\frac{9}{16}=\frac{\sqrt{7}}{4}$$

$$BC=\frac{\sqrt{7}}{2}$$

$$\text{Area of quad ABCD}=\frac{1}{2}\times\frac{\sqrt{7}}{2}\times 1=\frac{\sqrt{7}}{4}$$

5. Ans. (A,C)

$$\text{Let } S=\sum_{k=1}^{\infty}\frac{P_k}{3^k}=\frac{P_1}{3}+\frac{P_2}{3^2}+\frac{P_3}{3^3}+\frac{P_4}{3^4}+\frac{P_5}{3^5}$$

$$S=\frac{1}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{3}{3^4}+\frac{5}{3^5}+\frac{8}{3^6}+....+\frac{P_n}{3^n}$$

$$\frac{S}{3}=\frac{1}{3^2}+\frac{1}{3^3}+\frac{2}{3^4}+\frac{3}{3^5}+....+\frac{P_{n-1}}{3^n}+....$$

$$\frac{S}{9}=\frac{1}{3^3}+\frac{1}{3^4}+\frac{2}{3^5}+....+\frac{P_{n-2}}{3^n}$$

$$S-\left(\frac{S}{3}+\frac{S}{9}\right)=\frac{1}{3}\Rightarrow S=\frac{3}{5}$$

6. Ans. (B,C,D)

Let H = event that married man watches the show

W = event that married women watches the show

$P(H) = 0.4, \quad P(W) = 0.5, \quad P\left(\frac{H}{W}\right) = 0.7$

$P(H \cup W) = 0.35$

$P\left(\frac{W}{H}\right) = \frac{0.35}{0.4} = \frac{7}{8}$, $P(H \cup W) = 0.55$

7. Ans. (A,C)

Directly expand put –x = t

$\Delta = t(t^2 - ab) - a(bt - a^2) + b(b^2 - at)$

$\Delta = t^3 + a^3 + b^3 - 3abt = (t + a + b)$

$(t + a + b)(t^2 + a^2 + b^2 - at - bt - ab)$

$(a + b - x)(x^2 + x(a + b) + a^2 + b^2 - ab)$

If a = b $\quad \Delta = (2a - x)(x^2 + 2ax + a^2)$

8. Ans. (B,C)

$A.adj(2B) = 16I$

$A(4adjB) = 16I \Rightarrow A.adjB = 4I$

$\Rightarrow A|B|.B^{-1} = 4I$

$AB^{-1} = 2I \Rightarrow A = 2B$

$B.adjA = B.adj(2B) = 4B.adjB = 4|B|I_3 = 8I_3$

$A.adjB = 4I \Rightarrow A^{-1}A.adjB$

$\Rightarrow adjB = 4I \Rightarrow A^{-1}.A.adjB = 4A^{-1}$

$\Rightarrow adjB = 4A^{-1}$

$A^{-1}.adjB = A^{-1}.(4A^{-1}) = 4(A^{-1})^2$

$(A^{-1}(adjB))^{-1} = (4A^{-1}A^{-1})^{-1}$

$= \frac{1}{4}A^2 = \frac{1}{4}(2B)^2 = B^2$

Solution Q.9 & Q. 10

9. Ans. (A)

10. Ans. (B)

$$I = \int_{e^{-x}}^{1} (-\ell nt)^n\, dt$$

$-\ell nt = u \Rightarrow t = e^{-u} \Rightarrow dt = -e^{-u}du$

$$I = \int_0^x u^n e^{-u}\, du$$

Hence,

$$I_n = \int_0^\infty \underbrace{u^n}_{I}\, \underbrace{e^{-u}}_{II}\, du = \left|-u^n e^{-u}\right|_0^\infty + \int_0^\infty n u^{n-1} e^{-u}\, du$$

$\Rightarrow I_n = 0 + nI_{n-1} \Rightarrow I_n = n(n-1)I_{n-2}$

$\therefore I_n = n(n-1)(n-2).....2.I_1$

$$\Rightarrow I_1 = \int_0^\infty ue^{-u}\, du = 1$$

$\Rightarrow I_n = n!$

Let

$$J = \int_0^{1/2} \left(\frac{1}{4} - x^2\right)^4 dx = \int_0^{1/2} \left(\frac{1}{4} - \left(\frac{1}{2} - x\right)^2\right)^4 dx$$

(Using King property)

$$\therefore J = \int_0^{1/2} (x - x^2)^4\, dx = \int_0^{1/2} x^4 (1-x)^4\, dx \quad ...(i)$$

$\Rightarrow J = K$, So $J - K = 0$

Put $x = 1 - y$

$$\text{Also, } J = -\int_1^{1/2} (1-y)^4 y^4\, dx$$

$$\Rightarrow J = \int_{1/2}^{1} (1-x)^4 x^4\, dx \quad ...(ii)$$

$\therefore$ (i) + (ii)

$$\Rightarrow J = \frac{1}{2}\int_0^1 x^4 (1-x)^4\, dx$$

(Using Queen property)

Put $x = \sin^2\theta$,

$$J = \int_0^{\pi/2} \sin^9\theta.\cos^9\theta\, d\theta$$

$$= \frac{(8\times6\times4\times2)(8\times6\times4\times2)}{(18\times16\times14\times12\times10\times8\times6\times4\times2)} = \frac{1}{1260}$$

Solution Q.11 & Q. 12

11. Ans. (A)

12. Ans. (B)

$L_2 : x - 3y - 4 = 0$ & $4y - z + 5 = 0$

$P \equiv (4, 0, 5) \qquad \overrightarrow{DR} = \vec{n} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$\vec{n} = 3\hat{i} + \hat{j} + 4\hat{k} \qquad \begin{pmatrix} \vec{n}_1 = \hat{i} - 3\hat{j} \\ \vec{n}_2 = 4\hat{j} - \hat{k} \end{pmatrix}$

$L_2 : \frac{x-4}{3} = \frac{y}{1} = \frac{z-5}{4} \quad ...(1)$

$P_1 : x + y - z = 5, \; L_2 : (3\lambda + 4, \lambda, 4\lambda + 5)$

$$\frac{x-(3\lambda+4)}{1}=\frac{y-\lambda}{1}=\frac{z-(4\lambda+5)}{-1}=-2\left(\frac{-6}{3}\right)=4$$

$(x, y, z) \equiv (3\lambda + 8, \lambda + 4, 4\lambda + 1)$

Image of line $L_2 : \frac{x-8}{3}=\frac{y-4}{1}=\frac{z-1}{4}$

$P_1 = 0; P_3 = 0 ; P_1 + \lambda P_3 = 0$

$(x + y - z - 5) + \lambda(x - 3y - 4) = 0$

$x(1 + \lambda) + y(1 - 3\lambda) - z - (5 + 4\lambda) = 0$

$p_1 = p_2$ from (4, 0, 0)

$$\Rightarrow \left|\frac{1}{\sqrt{(\lambda+1)^2+(1-3\lambda)^2+1}}\right|=\left|\frac{1}{\sqrt{3}}\right|$$

$$\lambda = 0, \frac{2}{5}$$

$P : 7x - y - 5z - 33 = 0$

$L_1 : x + y - z = 5, 2x - y + \lambda z = 3$

$$\text{Point} \equiv \left(\frac{8}{3}, \frac{7}{3}, 0\right)$$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 2 & -1 & \lambda\end{vmatrix}=(\lambda-1)\hat{i}-(\lambda+2)\hat{j}-3\hat{k}$$

If line L_1 and L_2 are coplanor

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{4}{3} & \frac{-7}{3} & 5\\ 3 & 1 & 4\\ (\lambda-1) & -(\lambda+2) & -3\end{vmatrix}=0$$

$$\lambda = -\frac{5}{4}$$

SECTION–II

1. Ans. 4.00

$-2 \le f'(x) \le 2$, Apply LMVT in $x \in [1, 2]$ and $x \in [2, 4]$

$f(2) - f(1) = f'(x) \Rightarrow f(2) - 2 \in [-2, 2]$

$f(2) \in [0, 4]$ ...(1)

$\frac{f(4)-f(2)}{2} \in [-2,2] \Rightarrow f(2) - f(4) \in [-4, 4]$

$f(2) \in [4, 12]$ ...(2)

So from (1) and (2), $f(2) = 4$

2. Ans. 5.00

$$f'(x)=\frac{(4a-2x)}{(4ax-x^2)\log a}\ge 0$$

$$f'(x)=\frac{(x-2a)}{x(x-4a)\log a}\ge 0$$

C-1 : $a \in (1, \infty) \Rightarrow \frac{(x-2a)}{x(x-4a)} \ge 0$

$-$ (0) $+$ (2a) $-$ (4a) $+$

$x \in (0, 2a)$ so always $\uparrow$

C-2 : $a \in (0, 1)$

$$\frac{(x-2a)}{x(x-4a)}<0$$

$-$ (0) $+$ (2a) $-$ (4a) $+$

$x \in (2a, 4a)$

$2a \le \frac{3}{2} \Rightarrow a \le \frac{3}{4}$ and $4a > 2 \Rightarrow a > \frac{1}{2}$

So $a \in \left(\frac{1}{2}, \frac{3}{4}\right] \cup [1,\infty)$

3. Ans. 15.00

$$I=\int\frac{(\sec x-\tan x)}{\sqrt{\sin^2 x-\sin x}}dx$$

$$I=\int\frac{(\sec x-\tan x)\sec x}{\sqrt{\tan^2 x-\sec x\tan x}}dx$$

Put $-\sec x + \tan x = t$

$= -\sec x \tan x + \sec^2 x = \frac{dt}{dx}$

also $(\tan x - \sec x)^2 = t^2$

$2\tan^2 x - 2\tan x \sec x = t^2 - 1$

$$I=\int\frac{dt}{\sqrt{\frac{t^2-1}{2}}}=\sqrt{2}\int\frac{dt}{\sqrt{t^2-1}}$$

$$I=\sqrt{2}\log_e\left|t+\sqrt{t^2-1}\right|$$

$$I=\sqrt{2}\log_e\left|(\tan x-\sec x)+\sqrt{2}\sqrt{\tan^2 x-\tan x\sec x}\right|+c$$

$k = \sqrt{2}$

$$f\left(\frac{4\pi}{3}\right)=\sqrt{3}+2$$

4. Ans. 5.00

$dy = \frac{(1-x)}{e^x}dx$ Integrate

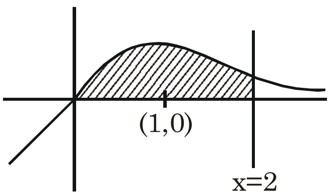
$y = x e^{-x} + c,\ f(0) = 0 \Rightarrow c = 0$

$y = xe^{-x}$

$\frac{d^2y}{dx^2} = (x-2)e^{-x}$

Point of inflection $\equiv \left(2, \frac{2}{e^2}\right)$

$A = \int_0^2 xe^{-x}\ dx$



$A = 1 - \frac{3}{e^2}$

$p + q = 5$

5. Ans. 7.00

$(x-c)^2 + (y-c)^2 = c^2,\ c > 0$

$p = r,\quad \left|\frac{7c-12}{5}\right| = c \Rightarrow c = 1, 6$

Sum = 7

6. Ans. 6.00

$f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$

$f'(0^+) = \lim_{h\to 0^+} \frac{h^{p-2}\sin\frac{1}{h} + h|\tan h|^{q-3}}{h}$

$f'(0^+) = \lim_{h\to 0^+} h^{p-3}\sin\frac{1}{h} + (\tan h)^{q-3}$

It exist only when $p > 3$ and $q \geq 3$, $p + q > 6$

$[p + q]_{min} = 6$