

**LEADER TEST SERIES / JOINT PACKAGE COURSE**
**TARGET : JEE (Main + Advanced) 2019**

 Test Type : **ALL INDIA OPEN TEST (MAJOR)** Test Pattern : JEE-Advanced

**TEST # 12**
**TEST DATE : 28 - 04 - 2019**
**PAPER-2**
**PART-1 : PHYSICS**
**ANSWER KEY**

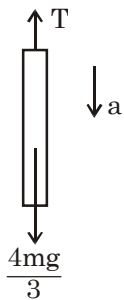
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	B	A	C	C	C	B,D	B,D	C,D
	Q.	11	12	13	14	15	16	17	18		
A.	A,C	A,D	A,C	A,D	C	B	D	A			

**SOLUTION**
**SECTION-I**

1. Ans. (C)

2. Ans. (C)

$$\text{Sol. } a = \frac{2m - m}{3m} g = \frac{g}{3}$$

 Now apply Newton's 2<sup>nd</sup> law on lower 2/3<sup>rd</sup> part of rod.


$$\text{We get } \frac{4mg}{3} - T = \frac{4m}{3} a = \frac{4m}{3} \times \frac{g}{3}$$

$$T = \frac{4mg}{3} - \frac{4mg}{9}$$

$$T = \frac{8mg}{9}$$

3. Ans. (B)

4. Ans. (A)

$$\text{Sol. } R_{AB} = \int_0^{\ell} \frac{\rho_0}{A} \left(1 - \frac{x}{\ell}\right) dx = \frac{\rho_0}{2A} \ell$$

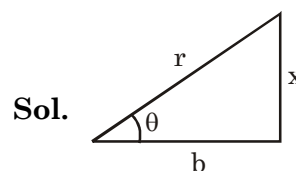
$$R_{BC} = \int_0^{2\ell/3} \frac{\rho_0}{A} \left(1 - \frac{x}{\ell}\right) dx = \frac{5\rho_0 \ell}{18A}$$

$$\& R_{AC} = \frac{\rho_0 \ell}{2A} - \frac{5\rho_0 \ell}{18A} = \frac{2\rho_0 \ell}{9A}$$

$$\text{at null point ; } \frac{X}{R} = \frac{R_{AC}}{R_{CB}} = \frac{2}{9} \times \frac{18}{5} = \frac{4}{5}$$

$$\therefore X = \frac{4R}{5}$$

5. Ans. (C)

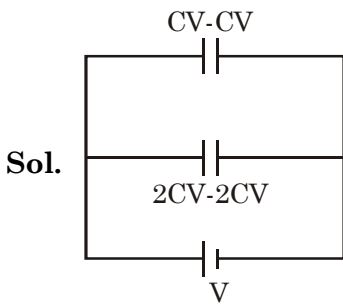


Sol.

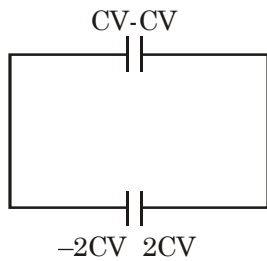
$$x = b \tan \theta$$

$$v = \left( \frac{dx}{dt} \right) = b \sec^2 \theta \left| \frac{d\theta}{dt} \right|$$

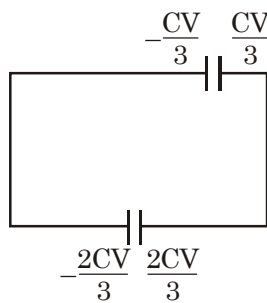
6. Ans. (C)



Now, polarity change



↓



$$V_{\text{common}} = \frac{-CV + 2CV}{3C}$$

$$= \frac{V}{3}$$

$$\text{Charge flow} \Rightarrow 2CV - \frac{2CV}{3} \Rightarrow \frac{4CV}{3}$$

Energy loss

$$= \frac{1}{2}CV^2 + \frac{1}{2}2CV^2 - \frac{1}{2}C\left(\frac{V}{3}\right)^2 - \frac{1}{2}2C\left(\frac{V}{3}\right)^2$$

$$= \frac{4}{3}CV^2$$

7. Ans. (C)

Sol.  $E_1 = E_4 = E_5 = 0$

$$E_2 = E_3 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

8. Ans. (B, D)

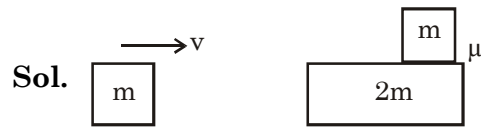
Sol.  $T_A > T$

Loss by radiation > absorption by radiation

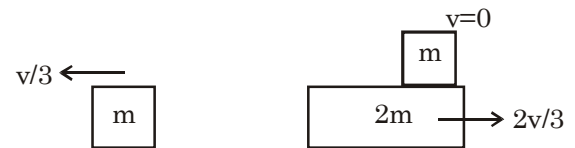
$T_B < T$

Loss by radiation < absorption by radiation

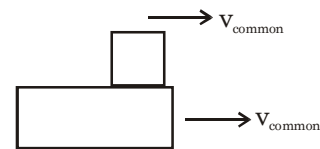
9. Ans. (B,D)



After collision



After some time



$$v_{\text{common}} = \frac{2m\left(\frac{2v}{3}\right)}{3m}$$

$$\Rightarrow \frac{4v}{9}$$

$$w_f = K_f - K_i = \frac{4}{27}mv^2$$

10. Ans. (C, D)

Sol.  $\vec{F} = q(\vec{v} \times \vec{B})$  and  $\vec{v} \parallel \vec{B}$  so,  $F = 0$

11. Ans. (A,C)

Sol. 1-2 → Isobaric expansion

2-3 → Isothermal compression

3-4 → Isobaric compression

4-1 → Isothermal expansion

↓

Draw all graph accordingly

12. Ans. (A,D)

Sol. Electric field at P → towards left

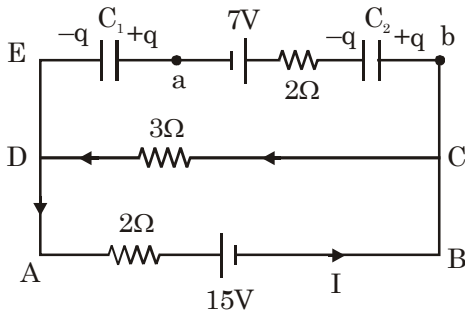
Force experience by  $-q$  → towards right

→ Potential energy will decrease

13. Ans. (A,C)

14. Ans. (A,D)

Sol. At steady state  
 $I(3) + I(2) = 15$



$$I = 3$$

Using KVL in loop CDEab

$$\frac{q}{11} + \frac{q}{5} = 7 + 3 \times 3 = 16$$

$$q = 55 \mu\text{C}$$

Thus

$$V_a - 7 + \frac{q}{5} = v_b$$

$$\Rightarrow V_a - V_b = 7 - \frac{q}{5} = 7 - \frac{55}{5} = \frac{55}{5} = -4\text{V}$$

p.d. across  $C_1$

$$\frac{q}{11} = \frac{55}{11} = 5\text{V}$$

p.d. across  $C_2$

$$\frac{q}{5} = 11\text{V}$$

$$\text{p.d. across terminal} = 15 - I(2) = 15 - 3 \times 2 = 9\text{V}$$

15. Ans. (C)

Sol.  $\vec{M} = -\frac{ge\vec{L}}{2m}$

$$\tau = \vec{M} \times \vec{B}$$

$$\vec{\omega}_p \times \vec{L} = \vec{M} \times \vec{B}$$

$$\Rightarrow \omega_p L = MB$$

$$\Rightarrow \omega_p L = \frac{geL}{2m} \cdot B$$

$$\Rightarrow \omega_p = \frac{geLB}{2mL} = \frac{geB}{2m}$$

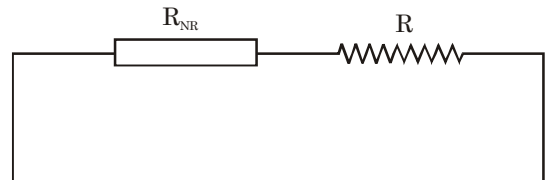
16. Ans. (B)

Sol.  $\omega = \omega_p$

$$\frac{eB}{m} = \frac{geB}{2m}$$

$$\Rightarrow g = 2$$

17. Ans. (D)



Sol.

By KVL

$$iR_0 \left[ \left( \frac{i}{i_0} \right)^2 - 1 \right] + iR = 0$$

$$\Rightarrow \frac{R_0 i^2}{i_0^2} - R_0 + R = 0$$

$$\Rightarrow \frac{i^2}{i_0^2} [R_0] = R_0 - R$$

Since  $R_0 > R$

$$i^2 = i_0^2 \left[ \frac{R_0 - R}{R_0} \right]$$

$$i = i_0 \sqrt{\frac{R_0 - R}{R_0}}$$

18. Ans. (A)

Sol.  $\frac{i^2}{i_0^2} [R_0] = R_0 - R$

when  $R_0 < R$ ,  $i \neq 0$ , LHS & RHS are of different sign, which is not possible.

So,  $i = 0$

$\Rightarrow$  V across R will be zero.

**PART-2 : CHEMISTRY**
**ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	D	A	C	B	A	A,D	B,C	A,C,D
	Q.	11	12	13	14	15	16	17	18		
A.	A,C	A,B,C,D	A,B	A,B,D	C	D	D	C			

**SOLUTION**
**SECTION-I**

 1. **Ans.(D)**

 2. **Ans.(A)**

**Sol.**  $\Delta U = 1 \times 20 \times 100 = 2000 \text{ J/mol}$   
 $\Delta H = \Delta U + (50-10) \times 100 = 2000 + 4000$   
 $= 6000 \text{ J/mol} = 6 \text{ kJ/mole}$

 3. **Ans.(D)**

**Sol.** Due to inert pair effect lower oxidation state in 15<sup>th</sup> group stabilizes down the group. Hence  $\text{Bi}^{3+}$  is more stable than  $\text{Bi}^{5+}$ .  $\text{Sb}^{5+}$  is more stable than  $\text{Sb}^{3+}$  hence  $\text{Sb}^{3+}$  is reducing agent.

 4. **Ans.(A)**

**Sol.** Chlorides, Bromides & Iodides due to their large size have high covalent nature. However in fluorides, Lattice energy is more dominating factor. Hence follows a decreasing trend  
 $\text{LiF} > \text{NaF} > \text{KF} > \text{RbF} > \text{CsF}$

 5. **Ans.(C)**

 6. **Ans.(B)**

 7. **Ans.(A)**

 8. **Ans.(A,D)**

**Sol.**  $d = \frac{M}{V_m} \Rightarrow 0.25 = \frac{2}{V_m}$   
 $\Rightarrow V_m = 8 \text{ lit}$

$$\frac{PV_m}{P(V_m - b)} = \frac{ZRT}{RT} \Rightarrow \frac{V_m}{V_m - b} = Z$$

$$Z = 1.06$$

 9. **Ans.(B,C)**

**Sol.**  $k = c \times \frac{\ell}{A} = \frac{1}{200} \times \frac{4}{10} = 2 \times 10^{-3} \Omega^{-1} \text{cm}^{-1}$

$$\lambda_m = k \times \frac{1000}{M} = 2 \times 10^{-3} \times \frac{1000}{0.1} = 20 \Omega^{-1} \text{cm}^2 / \text{mol}$$

$$\alpha = \frac{\lambda_m}{\lambda_m^\circ} = \frac{20}{500} = 0.04$$



$$\begin{array}{ccc} 0.1 & & \\ 0.1(1-\alpha) & 0.1\alpha & 0.1\alpha \end{array}$$

$$k_b = 0.1 \frac{\alpha^2}{1-\alpha} \approx 0.1 \times 16 \times 10^{-4} = 1.6 \times 10^{-4}$$

$$[\text{OH}^-] = 0.1 \times 0.04 = 4 \times 10^{-3}$$

$$\text{pOH} = 3 - \log 4 \Rightarrow \text{pH} = 14 - 3 + \log 4 = 11.6$$

$$\pi = iCRT = 1.04 \times 0.1 \times 0.08 \times 400$$

$$\pi = 3.328 \text{ atm}$$

 10. **Ans.(A,C,D)**

**Sol.** Composition of FeS is always higher in matte as compared to blister copper (98.5% pure). If  $\text{O}_2$  is used, copper cannot be obtained from its oxide.

In electrolysis of  $\text{CuSO}_4$  (aq.), on passing  $\text{H}_2$  direction of reaction

$2\text{H}_{(\text{aq})}^+ + 2\text{e}^- \longrightarrow \text{H}_{2(\text{g})}$  can be reversed which in turn will facilitate reduction of Cu in place of hydrogen.

$\text{Cu}_2\text{O}$  can be reduced by coke.

Hence answer is A,C,D.

 11. **Ans.(A,C)**

**Sol.**  $\text{Cl}_2 + \text{Slaked lime} \xrightarrow{\text{(dry)}} \text{Ca(OCl)}_2 + \text{CaCl}_2 + 2\text{H}_2\text{O}$

 12. **Ans. (A,B,C,D)**

 13. **Ans.(A,B)**

 14. **Ans. (A,B,D)**

 15. **Ans.(C)**

 16. **Ans.(D)**

**Sol.**  $\text{Ag}^+ (0.05\text{M}) + \text{X}^- (0.02\text{M}) \longrightarrow \text{AgX}_{(\text{s})}$

$$Q = \frac{1}{10^{-3}} = 10^3$$

$$E_{\text{cell}}^0 = -0.06 \log K_{\text{sp}}$$

$$E_{\text{cell}} = -0.06 \log K_{\text{sp}} - 0.06 \log Q$$

$$= -0.06[\log K_{\text{sp}} + \log 10^3]$$

$$\frac{-E_{\text{cell}}}{0.06} - 3 = \log K_{\text{sp}}$$

$$\frac{-0.66}{0.06} - 3 = \log k_{\text{sp}} \Rightarrow \log k_{\text{sp}} = -14$$

$$k_{\text{sp}} = 10^{-14}$$

 17. **Ans.(D)**

**Sol.** Cu will be precipitated as  $\text{Cu}_2\text{S}$  (Black)

Cd will be precipitated as CdS (Yellow)

 18. **Ans.(C)**

**PART-3 : MATHEMATICS**

**ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	D	D	C	C	B	A,B,C	A,D	A,C
	Q.	11	12	13	14	15	16	17	18		
A.	A,B,C,D	A,D	A,B,D	A,B,D	D	A	A	C			

**SOLUTION**

**SECTION-I**

1. **Ans. (C)**

**Sol.**  $P = \{a, a+1, a+2, \dots, a+m-1\}$

$$\frac{m}{2} \{2a + m - 1\} = 2m \Rightarrow 2a + m = 5 \quad \dots(i)$$

$$Q = \{b, b+1, b+2, \dots, b+2m-1\}$$

$$\frac{2m}{2} \{2b + 2m - 1\} = m \Rightarrow b + m = 1 \quad \dots(ii)$$

$$(b + 2m - 1) - (a + m - 1) = 1009$$

$$b - a + m = 1009 \quad \dots(iii)$$

$$1 - m - \frac{5}{2} + \frac{m}{2} + m = 1009$$

$$\frac{m}{2} = 1010.5 \Rightarrow m = 2021$$

2. **Ans. (B)**

**Sol.**  $P_1 : x + y + z + 3 = 0$

$$P_2 : x - y + z = 2$$

Let equation of plane be  $P_1 + \lambda P_2 = 0$

$$(x + y + z + 3) + \lambda(x - y + z - 2) = 0$$

$$\begin{matrix} 2 & 1 \\ \bullet & \bullet \\ (3,0,2) & (1,2,0) & (0,3,-1) \end{matrix}$$

(1,2,0) satisfy the plane and  $\lambda = 2$

$$3x - y + 3z = 1$$

3. **Ans. (D)**

**Sol.**  $(2\operatorname{cosec} x - 1)^{1/3} + (\operatorname{cosec} x - 1)^{1/3} = 1$

cube both sides

$$(3 \operatorname{cosec} x - 2) + 3(2 \operatorname{cosec} x - 1)^{1/3}(\operatorname{cosec} x - 1)^{1/3} = 1$$

$$(2 \operatorname{cosec} x - 1)^{1/3}(\operatorname{cosec} x - 1)^{1/3} = (1 - \operatorname{cosec} x)$$

$$(2\operatorname{cosec} x - 1)^{1/3} = -(\operatorname{cosec} x - 1)^{2/3}$$

$$\Rightarrow \operatorname{cosec} x = 0 \text{ (neglect)}$$

or

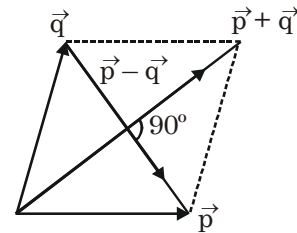
$$\operatorname{cosec} x = 1 \Rightarrow \sin x = 1, \quad x = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow 1 \text{ solution in } (0, 2\pi)$$

$$\Rightarrow k \text{ solutions in } (-k\pi, k\pi) \Rightarrow k = 16$$

4. **Ans. (D)**

**Sol.**  $\vec{p} + \vec{q} = \vec{r} \Rightarrow \vec{p} \wedge \vec{q} = 120^\circ$  and  $\vec{r}$  is along angle bisector of  $\vec{p}$  and  $\vec{q}$



$$\Rightarrow |\vec{p}| = |\vec{q}| \Rightarrow \text{rhombus}$$

$$\Rightarrow \vec{p} + \vec{q} \perp \vec{p} - \vec{q}$$

$$\Rightarrow \vec{r} \perp \vec{p} - \vec{q}$$

$$|\vec{p} - \vec{q}| = \sqrt{3} \Rightarrow |\vec{p} - \vec{q} + \vec{r}| = 2$$

5. **Ans. (C)**

$$\begin{aligned} \text{Sol. } f'(x) &= \frac{1}{1 + \sqrt{1-x^2}} \frac{(-2x)}{2\sqrt{1-x^2}} + \frac{2x}{2\sqrt{1-x^2}} - \frac{1}{x} \\ &= \frac{-\sqrt{1-x^2}}{x} \end{aligned}$$

$$y - (\ln(1 + \sqrt{1-h^2}) - \sqrt{1-h^2} - \ln h)$$

$$= \frac{-\sqrt{1-h^2}}{h} (x-h)$$

$$B(0, \ln(1 + \sqrt{1-h^2}) - \ln h)$$

$$A(h, \ln(1 + \sqrt{1-h^2}) - \sqrt{1-h^2} - \ln h)$$

$$AB = d = 1$$

6. **Ans. (C)**

$$\text{Sol. } \frac{(4 - 4 \cos 2\theta)t^2 + (1 + \cos 2\theta) - 2t \sin 2\theta}{(4 - 4 \cos 2\theta)t^2 + (1 + \cos 2\theta) + 2t \sin 2\theta}$$

$$\frac{4.2 \sin^2 \theta t^2 + 2 \cos^2 \theta - 4 \sin \theta \cos \theta t}{4.2 \sin^2 \theta t^2 + 2 \cos^2 \theta + 4 \cos \theta \sin \theta t}$$

$$\frac{4 \sin^2 \theta t^2}{\cos^2 \theta} - \frac{2 \sin \theta}{\cos \theta} t + 1$$

$$\frac{4 \sin^2 \theta t^2}{\cos^2 \theta} + \frac{2 \sin \theta}{\cos \theta} t + 1$$

$$\text{put } 2 \tan \theta t = x$$

$$\frac{x^2 - x + 1}{x^2 + x + 1}$$

$$\text{it's range } \left[ \frac{1}{3}, 3 \right]$$

7. Ans. (B)

Sol. For consistency 
$$\begin{vmatrix} a & 1 & b \\ b & 1 & 2a \\ a & b & 3ab \end{vmatrix} = 0$$

$$a^2b - 3ab^2 + 2a^2 + b^3 - ab = 0$$

$$b^3 - 3ab^2 + (a^2 - a)b + 2a^2 = 0$$

$$b = 0 : a = 0$$

$$b = 1; 1 - 3a + a^2 - a + 2a^2 = 0$$

$$3a^2 - 4a + 1 = 0$$

$$(3a - 1)(a - 1) = 0 \Rightarrow a = 1 \text{ (reject) or } a = \frac{1}{3}$$

$$b = 2 : 8 - 12a + 2a^2 - 2a + 2a^2 = 0$$

$$4a^2 - 14a + 8 = 0$$

$$2a^2 - 7a + 4 = 0$$

$\Rightarrow$  not integral solution

only one ordered pair (0,0)

8. Ans. (A,B,C)

Sol. 
$$f(x) = \frac{d}{dx} \left( \tan^{-1} \frac{x}{\sqrt{x^2+1}+1} \right)$$

Let  $x = \tan\theta \Rightarrow \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan^{-1} \left( \frac{\tan\theta}{\sec\theta+1} \right) = \tan^{-1} \left( \tan \frac{\theta}{2} \right)$$

$$\Rightarrow \frac{\theta}{2} = \frac{\tan^{-1} x}{2} \Rightarrow f(x) = \frac{1}{2(1+x^2)}$$

$$g(x) = \frac{1}{f(x)} + \frac{1}{f(x^2)} = 2(1+x^2) + 2(1+x^4)$$

$$= 2(x^4 + x^2 + 2)$$

Range :  $[4, \infty)$

Let  $g^{-1}(x) = h(x) \Rightarrow h(g(x)) = x$

$$h'(g(x)) = \frac{1}{g'(x)}$$

put  $x = 1$

$$(g^{-1})'(8) = \frac{1}{g'(1)} = \frac{1}{12}$$

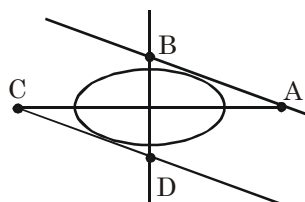
9. Ans. (A,D)

Sol. Equation of tangents  $y = mx \pm \sqrt{a^2m^2 + b^2}$

$$A \left( -\frac{\sqrt{a^2m^2 + b^2}}{m}, 0 \right)$$

$$B \left( 0, \sqrt{a^2m^2 + b^2} \right)$$

$$C \left( \frac{\sqrt{a^2m^2 + b^2}}{m}, 0 \right)$$



$$D \left( 0, -\sqrt{a^2m^2 + b^2} \right)$$

$$OA \cdot OC = OB \cdot OD$$

$$-\frac{a^2m^2 + b^2}{m^2} = -\left(a^2m^2 + b^2\right) \Rightarrow m^2 = 1$$

$$m = \pm 1$$

10. Ans. (A,C)

Sol. Let

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; P \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -11 \\ 6 \end{bmatrix} \Rightarrow \begin{cases} 2a - 3b = -11 \\ 2c - 3d = 6 \end{cases}$$

$$\text{also } P \begin{bmatrix} -11 \\ -6 \end{bmatrix} = \begin{bmatrix} -7 \\ -12 \end{bmatrix} \Rightarrow \begin{cases} -11a - 6b = -7 \\ -11c - 6d = -12 \end{cases}$$

$$\Rightarrow a = -1, b = 3, c = 0, d = 2$$

11. Ans. (A,B,C,D)

Sol. X denotes the event that Z lies on b

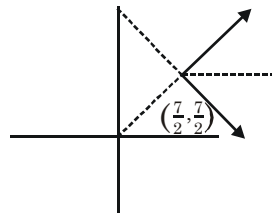
bisector of  $(0 + 0i)$  and  $(7 + 7i) \Rightarrow \alpha + \beta = 7$

$$X = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$P(X) = \frac{1}{6}$$

Y denotes the event that Z lies on a pair of rays starting from  $\frac{7}{2}(1+i)$  at an angle of

$$\frac{\pi}{4} \text{ or } -\frac{\pi}{4}$$



$$Y = \{(4,4), (5,5), (6,6), (4,3), (5,2), (6,1)\}$$

$$P(Y) = \frac{1}{6}; P(X \cap Y) = \frac{3}{36} \Rightarrow P\left(\frac{X}{Y}\right) = P\left(\frac{Y}{X}\right) = \frac{1}{2}$$

12. Ans. (A,D)

Sol. (A)  ${}^{40}C_{10} \cdot {}^{30}C_{10} \cdot {}^{20}C_{10} \cdot {}^{10}C_{10}$

(D)  ${}^{40}C_{20} \cdot 20! \cdot \underbrace{(2! \cdot 2! \cdot \dots \cdot 2!)}_{10 \text{ times}}$

13. Ans. (A,B,D)

$$2019 = 3^1 \times 673^1$$

$$\text{Number of divisors} = 2 \cdot 2 = 4$$

$$\sum_{k=1}^n d_k = (3^0 + 3^1)(673^0 + 673) = 4.674 = 2696$$

$$I = \int_{-3}^3 x \ln(d_1^x + d_2^x + d_3^x + d_4^x) dx$$

$$I = \int_{-3}^3 x \ln \left( \frac{2019^x}{2019^x} (d_1^x + d_2^x + d_3^x + d_4^x) \right) dx$$

$$I = \int_{-3}^3 x \ln \left( 2019^x \left( \left( \frac{d_1}{2019} \right)^x + \left( \frac{d_2}{2019} \right)^x + \left( \frac{d_3}{2019} \right)^x + \left( \frac{d_4}{2019} \right)^x \right) \right) dx$$

$$I = \int_{-3}^3 x^2 \ln 2019 + x \ln (d_4^{-x} + d_3^{-x} + d_2^{-x} + d_1^{-x}) dx$$

$$I = \int_{-3}^3 x^2 \ln 2019 dx + \int_{-3}^3 x \ln (d_1^{-x} + d_2^{-x} + d_3^{-x} + d_4^{-x}) dx$$

(apply King)

$$I = \int_{-3}^3 x^2 \ln 2019 dx + (-I)$$

$$2I = \int_{-3}^3 x^2 \ln 2019 dx$$

$$I = \int_0^3 x^2 \ln 2019 dx = \frac{x^3}{3} \ln 2019 \Big|_0^3 = 9 \ln 2019$$

14. Ans. (A,B,D)

$$f(x) = \ln^2 x - \frac{1}{6} \Rightarrow f(e) = \frac{5}{6}$$

$$f'(x) = \frac{2 \ln x}{x} \geq 0 \forall x \in [1, \infty)$$

$f(x)$  is strictly increasing function

$$f(1) = -\frac{1}{6} < 0$$

$$f''(x) = 2 \left[ \frac{1 - \ln x}{x^2} \right]$$

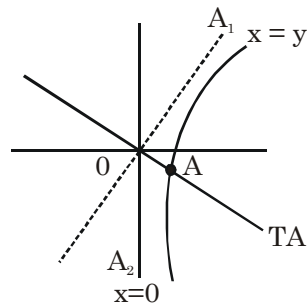
Equation of tangent :

$$y - \frac{5}{6} = \frac{2}{e} (x - e) \Rightarrow y = \frac{2x}{e} - \frac{7}{6}$$

$$\text{Area} = \int_1^e \left[ \left( \ln^2 x - \frac{1}{6} \right) - \left( \frac{2x}{e} - \frac{7}{6} \right) \right] dx = e + \frac{1}{e} - 3$$

**Paragraph for Question 15 to 16**

Sol.



$y = x - \frac{1}{x}$  is a hyperbola having asymptotes  $x^2 - xy = 0 \Rightarrow x = 0$  and  $x = y$

equation of TA :  $y = \tan \left( -\frac{\pi}{8} \right) x$

$$y = (1 - \sqrt{2})x$$

point A is intersection point of TA and H

$$(1 - \sqrt{2})x = x - \frac{1}{x} \Rightarrow x = \frac{1}{2^{1/4}}$$

$$\Rightarrow A \left( \frac{1}{2^{1/4}}, \frac{1 - \sqrt{2}}{2^{1/4}} \right)$$

$$OA = \sqrt{\frac{1}{\sqrt{2}} + \frac{3 - 2\sqrt{2}}{\sqrt{2}}} = \sqrt{2\sqrt{2} - 2}$$

Angle between asymptotes

$$\Rightarrow 2 \tan^{-1} \frac{b}{a} = \frac{3\pi}{4}$$

$$\frac{b}{a} = (\sqrt{2} + 1) \Rightarrow \frac{b^2}{a^2} = 3 + 2\sqrt{2} = e^2 - 1$$

$$e = \sqrt{4 + 2\sqrt{2}}$$

15. Ans. (D)

16. Ans. (A)

17. Ans. (A)

Sol. The number of sides in  $C_0$  is 5. Now each side



produce 11 sides

hence number of sides in  $C_1 = 5 \cdot 11$

number of sides in  $C_2 = 5 \cdot 11^2$

$\vdots \quad \quad \quad \vdots$

number of sides in  $C_n = 5 \cdot 11^n$

$$C_{11} = 5 \cdot 11^{11} = \frac{(11-1)11^{11}}{2} = \frac{11^{12} - 11^{11}}{2}$$

18. Ans. (C)

Sol. Side length of  $C_0$  is 1

Side length of  $C_1$  is  $\frac{1}{5}$

Side length of  $C_2$  is  $\frac{1}{5^2}$

$\vdots$

Hence  $P_0 = 5$

$$P_1 = 5 \cdot 11 \cdot \frac{1}{5}$$

$$P_2 = 5 \cdot 11^2 \cdot \frac{1}{5^2}$$

$\vdots$

$$P_n = 5 \cdot 11^n \cdot \frac{1}{5^n} = \frac{11^n}{5^{n-1}} \Rightarrow P_n = \left( \frac{11}{5} \right)^n \cdot 5$$

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{1}{P_k} = \lim_{n \rightarrow \infty} \sum_{k=0}^n \left( \frac{5}{11} \right)^k \cdot \frac{1}{5} = \frac{1}{5} \cdot \frac{1}{1 - \frac{5}{11}} = \frac{11}{30}$$