ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME
(Academic Session : 2020 - 2021)

JEE(Advanced)
MAJOR TEST # 05
28-02-2021

JEE(Main + Advanced) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Full Syllabus

PART-1 : PHYSICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	C	B	B	C	A,B,D	A,B,C,D	B,C,D	A,B,C,D
	Q.	11	12								
	A.	A,B,C,D	A,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	25.00	5.17 to 5.18	170.00	80.00	2.00	42.00				

SOLUTION

SECTION-I

1. Ans. (B)

Sol. $\because \tau = I\alpha \quad \therefore fx = \left(\frac{m\ell^2}{3}\right)\alpha = \frac{3Fx}{m\ell^2}$

Acceleration of centre of mass a = $\left(\frac{\ell}{2}\right)\alpha$

$= \frac{3Fx}{2m\ell}$

2. Ans. (C)

Sol. Optical path difference between (OPD) P & Q

$(O.P.D.) = 2.25\,\lambda_0 \times 1 + (3.5\,\lambda_0) \times 2 + 3\lambda_0 \times 3 = 18.25$

λ_0 and $\Delta\phi = \frac{2\pi}{\lambda_0} \times \Delta x = \frac{\pi}{2}$

3. Ans. (C)

Sol. $F = YA\alpha\Delta\theta = (2 \times 10^{11})\,(100 \times 10^{-4})\,(1.2 \times 10^{-5})\,(110 - 10) = 2.4 \times 10^6$ N

4. Ans. (B)

Sol. For compound pendulum

$T = 2\pi\sqrt{\frac{I_0}{mgr_{cm}}}$

I_0 – M.I. about point for suspension

r_{cm} = distacne of center of mass from point of suspension

$= 2\pi\sqrt{\frac{mR^2 3\pi}{2mg \times 4R}} \qquad = 2\pi\sqrt{\frac{3\pi R}{8g}}$

5. Ans. (B)

Sol. $\lambda \rightarrow 3V_0$

$2\lambda \rightarrow V_0$

$\lambda_0 = ?$

(Thereshold wavelength)

Einstain s photoelectric equition,

$h\nu = \phi + KE\ max$ ...(i)

Case-I : $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + 3eV_0$ ...(ii)

Case-II : $\frac{hc}{2\lambda} = \frac{hc}{\lambda_0} + eV_0$ ...(iii)

(ii) – 3 × (iii)

$\Rightarrow \frac{hc}{\lambda} - \frac{3hc}{2\lambda} = \frac{hc}{\lambda_0} - \frac{3hc}{\lambda_0}$

$\Rightarrow \frac{hc}{\lambda}\left(1 - \frac{3}{2}\right) = \frac{1}{\lambda_0}(1-3)$

$\Rightarrow \lambda_0 = \lambda\frac{(-2)}{(-1/2)} = 4\lambda$

$\therefore \lambda_0 = 4\lambda$

6. Ans. (C)

Sol. $y = \frac{a}{2}\,[\cos(2\omega t - 2kx) + 1]$

7. **Ans. (A, B, D)**

8. **Ans. (A,B,C,D)**

Sol. $I = \sum mr^2 = mx_1^2 + mx_2^2 = m4a(y_1 + y_2)$

$P.E. = mgh = mg(y_1) + mg(y_2) = mg(y_1 + y_2)$

From energy conservation,

$mgy_1 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy_1}$

$KE_{rotational} = \frac{1}{2}I\omega^2 = 2ma(y_1 + y_2)\omega^2$

9. **Ans. (B,C,D)**

Sol. Required heat — Available heat

10 g ice (0°C) — 5 g steam (100°C)

↓ 800 cal — ↓ 2700 cal

10 g water (0°C) — 5 g water (100°C)

↓ 1000 cal

10 g water (100°C)

So available heat is more than required heat therefore final temperature will be 100°C.

Mass of steam condensed

$= \frac{800 + 1000}{540} = \frac{10}{3}$ g

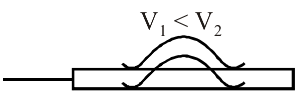
Total mass of water = 10 + $\frac{10}{3} = \frac{40}{3}$

$= 13\frac{1}{3}$ g,

Total mass of steam = 5 – $\frac{10}{3} = \frac{5}{3} = 1\frac{2}{3}$ g

10. **Ans. (A,B,C,D)**

Sol.



$A_t = \frac{V_2 + V_2}{V_2 + V_1}$ $A_i > A_i$

$V_2 > V_1$

$A_r = \frac{V_2 - V_2}{V_2 - V_1} \times A_i < A_i \Rightarrow \lambda_2 f > \lambda_1 f \Rightarrow \lambda_2 > \lambda_1$

11. **Ans. (A,B,C,D)**

Sol. $\phi_{net} = \frac{q_{in}}{\in_0}$

12. **Ans. (A,C)**

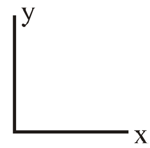
Sol. Magnetic field inside the cavity remains constant

$\vec{B}_{cavity} = \frac{\mu_0 \vec{J} \times \vec{\ell}}{2}$

SECTION-II

1. **Ans. 25.00**

Sol. $\vec{V}_I - \vec{V}_L = m^2(\vec{V}_0 - \vec{V}_L)$



$\vec{V}_I - 5\hat{i} = \left(\frac{f}{f+u}\right)^2 (10\hat{i} - 5\hat{i})$

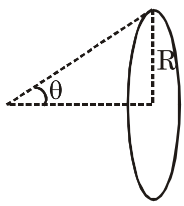
$\vec{V}_I - 5\hat{i} = \left(\frac{20}{20-30}\right)^2 (5\hat{i})$

$\vec{V}_I = 25\hat{i}$

$\Rightarrow |\vec{V}_I|$ = 25 cm/sec

2. **Ans. 5.17 to 5.18**

Sol.



$\frac{\phi}{\phi_T} = \frac{1}{4} = \frac{2\pi(1-\cos\theta)}{4\pi}$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \frac{R}{x_1} = \sqrt{3}$

$x_1 = \frac{R}{\sqrt{3}} = 10\sqrt{3}$ cm = 17.32 cm

$\frac{\phi}{\phi_T} = \frac{1}{5} = \frac{2\pi}{4\pi}(1 - \cos\theta')$

$\cos\theta' = \frac{3}{5} \Rightarrow \theta' = 53°$

$\frac{R}{x_2} = \tan 53° = \frac{4}{3}$

$x_2 = \frac{3}{4} \times R = 22.5$ cm

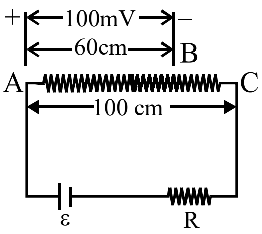
$\Delta x = 5.18$ cm

3. Ans. 170.00

Sol. Following the theory of potentiometer,

$$V_{AB} = iR_{AB} = \left(\frac{\varepsilon}{R + R_{AB}}\right) R_{AB},$$

$\varepsilon = 3V$, $R_{AB} = 10\Omega$, $V_{AB} = 100 \times 10^{-3}$ V



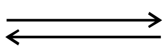
and $R_{AC} = \frac{AB}{AC} R_{AB} = \frac{60}{100} \times 10 = 6\Omega$

We have $100 \times 10^{-3} = \left(\frac{3}{R + 10}\right) \times 6$

or, R = 170 ohm

4. Ans. 80.00

Sol. $\Delta t = \frac{L}{v} = 0.1$



$v = 40 \text{ m/s} = \sqrt{\frac{T}{\mu}}$

$T = \frac{0.2}{4} \times 16000 = 80$ N

5. Ans. 2.00

Sol. $KE_{max} = (5 - \phi)$ eV

when these electrons are accelerated through 5V,

they will reach the anode with maximum energy = $(5 - \phi + 5)$eV

$\therefore 10 - \phi = 8$

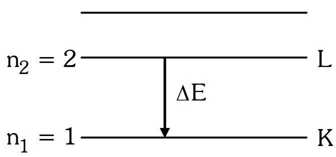
$\phi = 2$eV Ans.

Current is less than saturation current because if slowest electron also reached the plate it would have 5eV energy at the anode, but there it is given that the minimum energy is 6eV

6. Ans. 42.00

Sol. $\Delta E = h\nu = Rhc(Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For K-series b = 1



$\nu = Rc(Z - 1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Substituting the values

$$4.2 \times 10^{18} = (1.1 \times 10^7)(3 \times 10^8)(Z - 1)^2 \times \left(\frac{1}{1} - \frac{1}{4}\right)$$

$\therefore (Z - 1)^2 = 1697$

or $Z - 1 \approx 41$ or $Z = 42$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	D	B	C	C	A,D	A,B,C,D	A,B,C,D	A,C,D
	Q.	11	12								
	A.	A,B,D	A,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	0.50	600.00	0.00	2.00	4.00	5.00				

SOLUTION

SECTION–I

1. Ans. (A)

Sol. $2H_2 \quad + \quad O_2 \rightarrow 2H_2O$

mol 2 1

mass 2 × 2 1 × 32 g

10 g 80 g

mol 5 2.5 mol $5\times\frac{50}{100}\times 18$

2. Ans. (A)

Sol. $\frac{\lambda_1}{\lambda_2}=\frac{(T_{\lambda_2})_2}{(T_{\lambda_2})_1}=\frac{20}{60}=\frac{1}{3}$

energy per atom of A = $20 \times \frac{1}{4}+40\times\frac{3}{4}$

= 35 MeV

3. Ans. (D)

Sol. x is PH_3

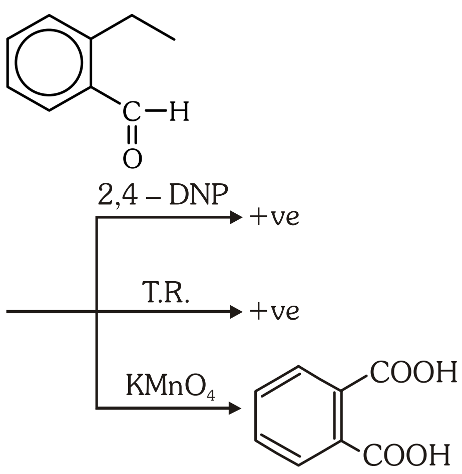
PH_3 explodes in contact with traces of oxidants like HNO_3.

4. Ans. (B)

5. Ans. (C)

6. Ans. (C)

Sol.



7. Ans. (A,D)

Sol. **(A)** Fact

(B) Probability of finding an electron is nearly 90% in an orbital

(C) No of angular nodes are l

(D) For 1s $|\Psi|^2$ is maximum at nucleus

8. Ans. (A,B,C,D)

9. Ans. (A,B,C,D)

Sol. Uub → 112

Group number → 112 – 100 = 12 (d-block)

7^{th} period element

All elements beyond uranium are transuranic elements

10. Ans. (A,C,D)

11. Ans. (A,B,D)

Sol. Natural rubber → Natural, Addition, Homopolymer.

Starch → Natural, Condensation, Homopolymer.

Insulin → Natural, Condensation, Co–polymer.

Dacron → Synthetic, Condensation, Co–polymer.

12. Ans. (A,C,D)

SECTION-II

1. Ans. (0.50)

Sol. $H_2 + H_2O + \frac{1}{2} O_2 \rightarrow 2H^+ + 2OH^-$

$\Delta G^o = -256.5 + 2 \times 80 = -96.5$ kJ

$-\Delta G^o = nFE^o$

$+96.5 \times 1000 = 2 \times 96500 \times E^o$

$E^o = 0.5$ Volt.

2. Ans. (600.00)

Sol. $\frac{\Delta P}{P_s}=\frac{m}{1000}\times M_A$

$P_s = \frac{\Delta P\times 1000}{M_A\times m}=\frac{0.6\times 1000}{18\times\frac{1}{18}}$ = 600 mm Hg

3. Ans. (0.00)

4. Ans. (2.00)

5. Ans. (4.00)

6. Ans. (5.00)

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	D	A	A	C	A	A,B,C	A,B,D	A,C,D	A,C,D
	Q.	11	12								
	A.	B,C,D	C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	2.00	2.00	1.00	1.00	2.00	24.00				

SOLUTION

SECTION–I

1. Ans. (A)

Sol. $D=\begin{vmatrix} n & n^2 & n^3 \\ n^2 & n^3 & n^5 \\ 1 & 2 & 3 \end{vmatrix}$

$\Rightarrow\ M_{11}=3n^3-2n^5,\ M_{13}=2n^2-n^3\ \&\ C_{33}=0$

$$\Rightarrow\ \lim_{n\to\infty}\frac{M_{11}+C_{33}}{(M_{13})^2}=\frac{3n^3-2n^5}{(2n^2-n^3)^2}$$

$$=\frac{n^5\left(\frac{3}{n^2}-2\right)}{n^6\left(\frac{2}{n^4}-1\right)^2}=\frac{1\left(\frac{3}{n^2}-2\right)}{n\left(\frac{2}{n^4}-1\right)}=0$$

2. Ans. (D)

Sol. $f'(x)=\sin x\cos x\,(x-2)(x-3)$

$f'(x)>0$ in $(3.5, 4.5)$

3. Ans. (A)

Sol. $f'(x)$

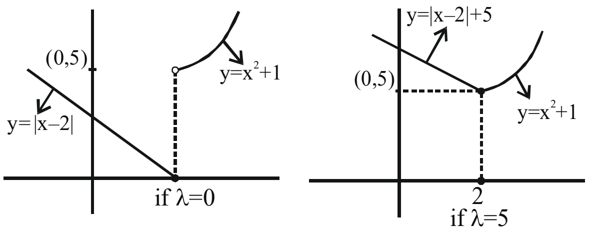
$$=\frac{(1+x)^{0.7}}{(1+x^{0.7})}\times\frac{1}{(1+x^{0.3})}(x^{0.3}\underbrace{(1-(1+x)^{0.7})}_{-ve}+x-\underbrace{(1+x)^{0.7}}_{-ve})$$

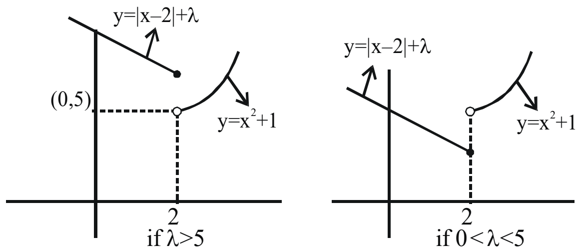
$f'(x)<0\ \Rightarrow$ function is decreasing & having the range $\left[\frac{1}{2^{0.3}},1\right]$

$f(x)=\frac{1}{2}$ is not possible.

4. Ans. (A)

There are different possible figures for different values of λ.





Since $f(x)$ is local minimum at x = 2

$$\Rightarrow\ \left.\begin{matrix} f(2)<f(1-h) \\ f(2)<f(1+h) \end{matrix}\right\}$$

for a very small (+)ve h.

clearly $\lambda\le 5$

5. Ans. (C)

Sol. $\ell n(\ell nx+\ell ny)=4\ell nx.\ \ell ny$ ...(1)

differentiable w.r.t x,

$$\Rightarrow\ \frac{1}{(\ell nx+\ell ny)}\left[\frac{1}{x}+\frac{1}{y}y'\right]=4\left[\frac{\ell ny}{x}+\frac{\ell nx}{y}.y'\right]\ ..(2)$$

from (1), at x = e.

$\ell n(1+\ell ny)=4\ \ell ny$

$\Rightarrow\ 1+\ell ny=y^4$

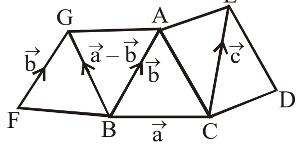
$\Rightarrow\ y=1$

$$\Rightarrow\ \text{from(2)},\ \frac{1}{(1+0)}\left[\frac{1}{e}+y'\right]=4\left[\frac{0}{e}+\frac{1}{1}.y'\right]$$

$$\Rightarrow\ \frac{1}{e}+y'=4y'$$

$$\Rightarrow\ 3y'=\frac{1}{e}\ \Rightarrow\ y'=\frac{1}{3e}$$

6. Ans. (A)



Sol. $\overrightarrow{GA}=2\vec b-\vec a$

$\overrightarrow{CA}=\vec b-\vec a$

$\overrightarrow{AE}=\overrightarrow{AC}+\overrightarrow{CE}=\vec a-\vec b+\vec c$

$\overrightarrow{GE}=\overrightarrow{GA}+\overrightarrow{AE}$

$=2\vec b-\vec a+\vec a-\vec b+\vec c$

$=\vec b+\vec c$

$\Rightarrow\ \therefore\ \alpha=0,\ \beta=1,\ \gamma=1$

7. Ans. (A,B,C)

Sol. $\sum_{r=1}^{n}\left(r.\sum_{p=1}^{r}(\omega^{P-1})\right)-155\omega$

$=\sum_{r=1}^{n}\left(r.\left(\omega^0+\omega^1+\omega^2+\omega^2+.....+\omega^{r-1}\right)\right)-155\omega$

$= 1.\omega^0 + 2(\omega^0 + \omega^0) + 3(\omega^0 + \omega^1 + \omega^2) + 4(\omega^0 + \omega^1 + \omega^2 + \omega^3) ..... n(\omega^0 + \omega^1 + \omega^2 + ...+ \omega^{4-1}) - 155\omega$

$= 1 + 2(-\omega^2) + 3(0) + 4(1) + 5(-\omega^2) + ...$
upto n terms $- 155\omega$
will be real if n = 29, 30, 31

8. Ans. (A,B,D)

Sol. Equations are $(ax - b)(x - 1) = 0$
& $(bx - c)(x + 1) = 0$

$\Rightarrow$ Roots are $\frac{b}{a},1$ and $\frac{c}{b},-1$

Case-I : $\frac{b}{a}=\frac{c}{b} \Rightarrow b^2=ac$

Case-II : $\frac{b}{a}=-1$ & $\frac{c}{b}\neq 1 \Rightarrow -a=b\neq c$

Case-III : $\frac{c}{b}=1$ & $\frac{b}{a}\neq -1 \Rightarrow b=c\neq -a$

9. Ans. (A,C,D)

Sol. $A^T + B^2 = A$ ....(i)
Taking transpose
$A + (B^T)^2 = A^T$ ....(ii)
Adding (i) & (ii)
$B^2 + (B^T)^2 = A^T$ ....(iii)
$\Rightarrow B^2 = -(B^T)^2$
taking det of both sides
$|B|^2 = -|B^T|^2$
$\Rightarrow |B|^2 = -|B|^2 \Rightarrow |B| = 0$
$\det(A - A^T) = \det(B^2) = 0$
Similarly $\det(B^2 - (B^T)^2) = \det((A - A^T) - (A^T - A))$
$= \det(2(A - A^T)) = 2^3\det(A - A^T) = 0$

10. Ans. (A,C,D)

Sol. $f(x) = x\sin x - \cos x$ for $x \in A$

where $A=\left\{x : x\log\left(\frac{2x}{\pi}\right)\leq 0\right\}$

$\Rightarrow x\log\left(\frac{2x}{\pi}\right)\leq 0,\ x>0$

$\Rightarrow 0<\frac{2x}{\pi}\leq 1 \Rightarrow x\in\left(0,\frac{\pi}{2}\right]$

Now, $f'(x)=x\cos x+2\sin x>0$ for $x\in\left(0,\frac{\pi}{2}\right]$

$\Rightarrow$ Range of $f(x)$ is $\left(-1,\frac{\pi}{2}\right]$

$\int_0^{\pi/2}(x\sin x-\cos x)dx=\left[-x\cos x\right]_0^{\pi/2}=0$

$\because |f(x)| = 0.5$

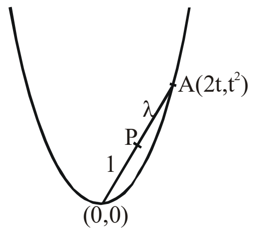
$\Rightarrow f(x) = \frac{1}{2},-\frac{1}{2}$

$\Rightarrow f(x)=\frac{1}{2}$ has one solution ($\because f(x)$ is $\uparrow$)

$\Rightarrow f(x)=-\frac{1}{2}$ has one solution

11. Ans. (B,C,D)

Sol. Let $\frac{a}{b}=\frac{1}{\lambda}(\lambda>0)$



$P\left(\frac{2t}{1+\lambda},\frac{t^2}{1+\lambda}\right)$

$\Rightarrow \frac{t^4}{(1+\lambda)^2}=\frac{2kt}{(1+\lambda)}$

$\Rightarrow \frac{t^3}{2k}=1+\lambda \Rightarrow \lambda=\frac{t^3-2k}{2k}$

$\Rightarrow \frac{t^3-2k}{2k}>0\Rightarrow\frac{2k-t^3}{2k}<0 \Rightarrow k\in\left(0,\frac{t^3}{2}\right)$

$\because \beta < 16 \Rightarrow t^2 < 16 \Rightarrow t^3 < 64$

$k \in (0,32) \Rightarrow$ Sum of all integral values of k is $\frac{32\times 31}{2}=496$

if k = 4 then $\lambda=\frac{t^3-8}{8} \Rightarrow t^3 > 8 \Rightarrow t > 2$
$\Rightarrow 4 < \beta < 16$

if $\beta = 9 \Rightarrow t = 3$ $\lambda=\frac{27-4}{4}=\frac{23}{4}$

12. Ans. (C,D)

Sol. Option (A) $f(x) = 0$ & $g(x) = 2\tan^{-1}x$

Option (B) $f(x) = \cos^{-1}x + \pi - \cos^{-1}x$

$\Rightarrow \quad f(x) = \pi$

$g(x) = 2\cos^{-1}x$

Option (C) $f(x) = \cos x + \cos x = 2\cos x = g(x)$

Option (D) $|\cos^{-1}x| = \cos^{-1}x$

$\Rightarrow \quad f(x) = 2\cos^{-1}x = g(x)$

SECTION-II

1. Ans. 2.00

Sol. Let $\operatorname{cosec}^{-1}x = \theta \Rightarrow \operatorname{cosec}\theta = \frac{x}{1}$

$$\sec\theta = \frac{x}{\sqrt{x^2-1}}$$

$$f(x) = \frac{x}{\sqrt{x^2-1}}$$

$$\lim_{x\to 2}\frac{\frac{x}{f(x)}-\sqrt{3}}{(x-2)} = \lim_{x\to 2}\frac{\sqrt{x^2-1}-\sqrt{3}}{(x-2)}$$

$$\lim_{x\to 2}\frac{(x^2-4)}{(x-2)(\sqrt{x^2-1}+\sqrt{3})} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

2. Ans. 2.00

Sol. $\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\pi}{4}\right)$

$px + qy = -r$

$$\Rightarrow \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{ap} = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{bq} = -\frac{1}{\sqrt{2}r}$$

$$\therefore \quad \frac{a^2p^2+b^2q^2}{r^2} = 2$$

3. Ans. 1.00

Sol. $A = \sum_{r=1}^{3}\log|\tan(60^\circ-\alpha_r)|$

$B = \sum_{r=1}^{3}\log|\tan(60^\circ+\alpha_r)|$

$\Rightarrow \quad A + B = \log|\tan(60-\alpha_1)\tan(60+\alpha_1)|$
$+ \log|\tan(60-\alpha_2)\tan(60+\alpha_2)|$
$+ \log|\tan(60-\alpha_3)\tan(60+\alpha_3)|$

$$= \log\left|\frac{\tan 3\alpha_1}{\tan\alpha_1}\right| + \log\left|\frac{\tan 3\alpha_2}{\tan\alpha_2}\right| + \log\left|\frac{\tan 3\alpha_3}{\tan\alpha_3}\right|$$

($\because \tan(60-\theta)\tan(60+\theta)\tan\theta = \tan 3\theta$)

$$= \log\left|\frac{\tan 3\alpha_1.\tan 3\alpha_2.\tan 3\alpha_3}{\tan\alpha_1\tan\alpha_2\tan\alpha_3}\right|$$

$$= \log\left|\frac{\tan\theta.\tan\frac{\theta}{3}.\tan\frac{\theta}{9}}{\tan\frac{\theta}{3}.\tan\frac{\theta}{9}.\tan\frac{\theta}{27}}\right| = \log\left|\frac{\tan\theta}{\tan\frac{\theta}{27}}\right|$$

$$= \log\left|\frac{\tan\left(\frac{9\pi}{4}\right)}{\tan\left(\frac{9\pi/4}{27}\right)}\right|$$

$$= \log\left|\frac{1}{\tan\frac{\pi}{12}}\right| = \log(2+\sqrt{3}) = 1$$

($\because$ base of logarithm is $2+\sqrt{3}$)

4. Ans. 1.00

Sol. Clearly, a = 1, b = 2, c = 3.

The second line can be written as

$$\frac{x-a}{a} = \frac{y}{0} = \frac{z}{c} \quad ....(1)$$

Now equation of plane through the line

$\frac{y}{b}+\frac{z}{c}=1$, $x = 0$ is $\left(\frac{y}{b}+\frac{z}{c}-1\right)+\lambda x = 0$

or $\quad \lambda x + \frac{y}{b}+\frac{z}{c} = 1 \quad ....(2)$

$\because$ plane (2) is parallel to the line (1)

$\Rightarrow \quad \lambda.a + \frac{1}{6}.0 + \frac{1}{c}.c = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{a}$

$\therefore$ equation of plane Π is

$$-\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{or} \quad -\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$$

$\therefore$ A(–1, 0, 0), B(0, 2, 0), C(0, 0, 3)

$\therefore$ volume of tetrahedron

$$OABC = \left|\frac{1}{6}[-\hat{i}\ \ 2\hat{j}\ \ 3\hat{k}]\right| = 6.\frac{1}{6}[\hat{i}\ \hat{j}\ \hat{k}] = 1$$

5. Ans. 2.00

Sol. $\int_{-1}^{1}(ax^2+bx+c)dx$

$$=p\left(\frac{a}{4}-\frac{b}{2}+c\right)+q(c)+r\left(\frac{a}{4}+\frac{b}{2}+c\right)$$

$\frac{p+r}{4}=\frac{2}{3}$ $\Rightarrow$ $p+r=\frac{8}{3}$

$q - r = 0$ $\Rightarrow$ $p = r$

$p + q + r = 2$

$p = r = \frac{4}{3}$

$q=-\frac{2}{3}$

6. Ans. 24.00

Sol. Given $f(x) = 2x^3 - 3(\lambda + 1)x^2 + 6(2\lambda - 1)x + \mu$

$f'(x) = 6[x^2 - (\lambda + 1)x + (2\lambda + 1)]$

$\because$ $f(x)$ has a positive point of local maxima, therefore the equation $f'(x) = 0$ must have both roots positive and distinct.

$\therefore$ $D > 0 \Rightarrow (\lambda + 1)^2 - 4(2\lambda + 1) > 0$

$\Rightarrow (\lambda - 3)^2 > 12 \Rightarrow |\lambda+3| > \sqrt{12}$

$\Rightarrow \lambda \in \left(-\infty, 3-2\sqrt{3}\right) \cup \left(3+2\sqrt{3}, \infty\right)$ ...(1)

Also, $f'(0) > 0 \Rightarrow \lambda > -\frac{1}{2}$ ...(2)

and $\frac{\lambda+1}{2} > 0 \Rightarrow \lambda > -1$ ...(3)

& $\lambda \in (-10,10)$ ...(4)

From intersection of (1), (2), (3), (4) the integral values of λ are 7,8,9.

$\therefore$ sum = 24.