ENTHUSIAST & LEADER COURSE | **ALL INDIA OPEN TEST # 01**

TEST TYPE : MAJOR | **PATTERN : JEE (Advanced)**

TARGET : JEE (Advanced) 2015 | **Date : 08 - 02 - 2015**

PAPER-1

PART-1 : PHYSICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	A	C	B	B	D	D	C	D
	Q.	11	12	13	14	15	16				
	A.	D	B	A	C	C	B				
SECTION-IV	Q.	1	2	3	4						
	A.	4	4	6	3						

SOLUTION

SECTION-I

1. Ans. (A)

Sol. $E = \frac{d\phi}{dt} = \frac{Bd(b\ell)}{dt} = Bbv = B \times 2 \times 10^{-2} \times 20$

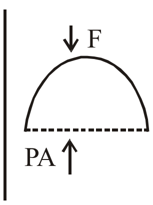
$= 0.40B$

$\Delta t = \frac{1 \times 10^{-2}}{20} = 5 \times 10^{-4}$ sec $= 500\ \mu$ sec

$t = \frac{6 \times 10^{-2}}{20} = 3 \times 10^{-3}$ sec $= 3000\ \mu$ sec

2. Ans. (A)

Sol.



$PA - F = F_B = \frac{2\pi}{3} r^3 \rho_1 g$

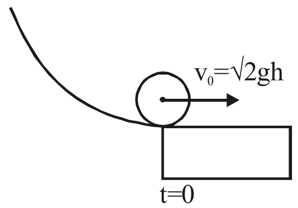
$(P_0 + \rho_1 gh)\ \pi r^2 - F$

$= \frac{2\pi}{3} r^3 \rho_1 g$

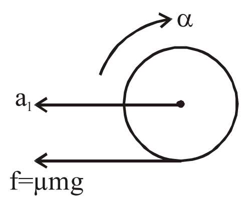
$F = P_0 \pi r^2 + \left(h - \frac{2}{3} r\right) \pi r^2 \rho_1 g$

3. Ans. (A)

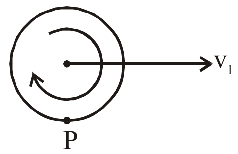
Sol.

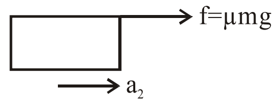


$\frac{mR^2}{2} \alpha = \mu mgR \Rightarrow \alpha R = 2\mu g$

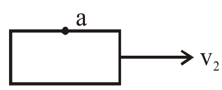


$a_1 = \mu g$





$a_2 = \frac{\mu mg}{M}$



When rolling $\Rightarrow v_p = v_a$

$\Rightarrow v_0 - \mu gt - (\alpha \cdot t \cdot R) = \left(\frac{\mu mg}{M}\right) t$

$v_0 = 3\mu gt + \frac{\mu mg}{M} t \Rightarrow t = \frac{v_0}{\mu g\left(3 + \frac{m}{M}\right)}$

Time after which rolling starts.

In frame of plane

$$v_0 t - \frac{1}{2}\mu g\left(1+\frac{m}{M}\right)t^2$$

$$= \frac{v_0^2}{\mu g\left(3+\frac{m}{M}\right)} - \frac{1}{2}\left(1+\frac{m}{M}\right)\frac{v_0^2}{\mu g\left(3+\frac{m}{M}\right)^2}$$

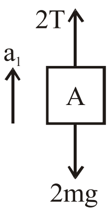
$$= \frac{2gh}{\mu g\left(3+\frac{m}{M}\right)} - \frac{\left(1+\frac{m}{M}\right)\cdot 2gh}{2\mu g\left(3+\frac{m}{M}\right)}$$

$$L = \frac{7}{8}$$

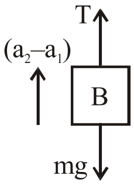
4. Ans. (C)

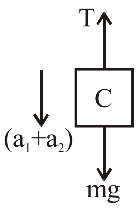
5. Ans. (B)

Sol.



a_2 = w.r.t pully moving with a_1





$2T - 2mg = 2ma_1$ ...(i)

$T - mg = m(a_2 - a_1)$ ... (ii)

$2mg - T = m(a_1 + a_2)$ ... (iii)

$$2T = \frac{10mg}{4}N$$

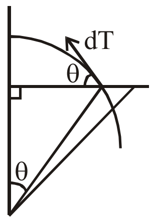
reading = 5 kg

6. Ans. (B)

Sol. $dT = (dm)g \sin\theta$

$dT_x = (dm)g \sin\theta \cos\theta$

$$= \frac{m}{\ell} Rg \sin\theta\cos\theta d\theta$$



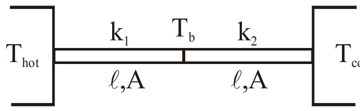
$$T = \int dT_x = \frac{mgR}{2\ell}\int_0^x \sin 2\theta d\theta$$

$$T = \frac{mg}{2\alpha}\left(\sin^2\alpha\right)$$

7. Ans. (D)

8. Ans. (D)

Sol.



$$\Rightarrow R = \frac{\ell}{kA}$$

$\frac{dq}{dt}$ is same in both

If $k_1 > k_2 \Rightarrow R_1 < R_2 \Rightarrow$ temperature drop across rod (1) is less than that drop across rod (2)

$(T_h - T_b) < (T_b - T_c)$

$\Rightarrow T_b$ is closer to T_h

If $k_2 > k_1 \Rightarrow R_1 > R_2 \Rightarrow$ temperature drop across rod (1) is more than that across rod (2)

$\therefore (T_h - T_b) > (T_b - T_c)$

$\Rightarrow T_b$ is closer to T_c

9. Ans. (C)

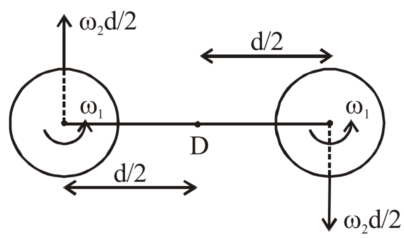
Sol. Due to angular momentum conservation there is no need of rotation of rod

Total energy of spring = Rotational kinetic energy of two disks

$$\Rightarrow \frac{1}{2}k\left(\Delta\ell\right)^2 = \left[\frac{1}{2}\frac{mR^2}{2}\omega^2\right]\times 2$$

10. Ans. (D)

Sol.



ω_1 is angular velocity of disk

ω_2 is angular velocity of rod

Angular momentum conservation

$$\left(m\times\frac{\omega_2 d}{2}\times\frac{d}{2}\right)\times 2=\left(\frac{mR^2}{2}\omega_1\right)\times 2$$

$$\Rightarrow \omega_2 = 2\omega_1\times\frac{R^2}{d^2}$$

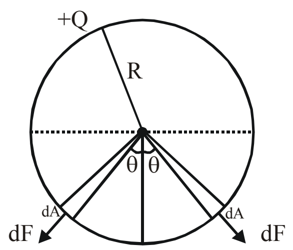
From energy conservation

$$\frac{1}{2}k\times(\Delta\ell)^2=\left(\frac{1}{2}m\times v_{cm}^2+\frac{1}{2}1_{cm}\omega_1^2\right)\times 2$$

$$=\left[\frac{1}{2}\times m\times\left(\omega_2\times\frac{d}{2}\right)^2+\frac{1}{2}\times\frac{mR^2}{2}\times\omega_1^2\right]\times 2$$

11. Ans. (D)

Sol.



$dF = P \times dA\cos\theta$

$F = P.\ A_{cross}$

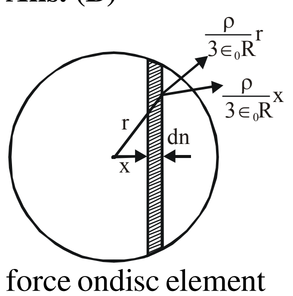
$$F=\frac{\sigma^2}{2\in_0}\times\pi R^2$$

$$F=\frac{Q^2}{(4\pi R)^2}\times\frac{1}{2\in_0}\times\pi R^2$$

$$F=\frac{Q^2}{32\pi\in_0 R^2}$$

12. Ans. (B)

Sol.



force ondisc element

$$dF=\rho.\pi\left(\sqrt{R^2-x^2}\right)^2(dx)\left(\frac{\rho}{3\in_0}\right)x$$

$$dF=\frac{\pi}{3\in_0}\rho^2\left[\left(R^2-x^2\right)xdx\right]$$

$$dF=\frac{-\pi}{6\in_0}\rho\int_0^R\left[\left(R^2-x^2\right)d\left(R^2-x^2\right)\right]$$

13. Ans. (A)

Sol. $B=\frac{\mu_0 i}{2R}\Rightarrow\frac{\mu_0 q}{2RT},\ T=\frac{2\pi r}{v}$

$$B=\frac{\mu_0 e\times v}{2\times 2\pi r\times r}=\frac{\mu_0 ev}{4\pi r^2}$$

$$=\frac{\mu_0 e\left[\frac{ke^2}{n\hbar}\right]}{4\pi\left[\frac{n^4\hbar^4}{m^2k^2e^4}\right]}=\frac{\mu_0 e^7 m^2 k^3}{n^5\hbar^5\times 4\pi}$$

$$\Delta E=2\mu_s B=\frac{2e\hbar}{2m}\times\left[\frac{\mu_0 e^7 m^2}{(4\pi\in_0)^3 n^5\hbar^2 4\pi}\right]$$

$$=\frac{\mu_0 e^8 m}{256\pi^4\in_0^3 n^5\hbar^4}$$

14. Ans. (C)

Sol. $E=\frac{hc}{\lambda}$

$$\Delta E=-\frac{hc}{\lambda^2}\Delta\lambda$$

$$2\mu_S B=\frac{hc}{\lambda^2}\Delta\lambda$$

$$\left(\frac{eh}{2\pi m}\right)B=\frac{hc}{\lambda^2}\Delta\lambda$$

$$\Delta\lambda=\frac{eB\lambda^2}{2\pi mc}$$

15. Ans. (C)

Sol. Variation of temperature in troposphere is adibatic in given paragraph.

so PV^γ = constant

Bulk modulus β is γP

16. Ans. (B)

Sol. $TP^{\frac{1-\gamma}{\gamma}}=\text{constant}$

$$\frac{dT}{dy}P^{\frac{1-\gamma}{\gamma}}+T^{\frac{1-\gamma}{\gamma}}P^{\frac{1-\gamma}{\gamma}-1}\frac{dP}{dy}=0$$

$$\frac{dT}{dy}P^{\frac{1-\gamma}{\gamma}}=\left(-T^{\frac{1-\gamma}{\gamma}}P^{\frac{1-\gamma}{\gamma}-1}\right)\frac{dP}{dy}$$

$$\frac{dT}{dy}=\left(\frac{-T^{\frac{1-\gamma}{\gamma}}P^{\frac{1-\gamma}{\gamma}-1}}{P^{\frac{1-\gamma}{\gamma}}}\right)\frac{dP}{dy}$$

$$\frac{dT}{dy}=\left(\frac{-T}{P}\frac{1-\gamma}{\gamma}\right)\frac{dP}{dy}$$

SECTION-IV

1. Ans. 4

Sol. From LMC

$Mv = Mv_1 + mv_2$

& from e = 1

$v = v_2 - v_1$

$\frac{v_1}{v_2} = \frac{1}{4}$

for both time of flight is same then

$x = v_1 \times t$

$y = v_2 t \Rightarrow \frac{y}{x} = 4$

2. Ans. 4

3. Ans. 6

Sol. $1\ell = 10^3(cc)^3$

$1\ell = 10^{-3}\ m^3$

Rate of water flowing = $\frac{1000}{7}\ell/\min$

$= \frac{1}{7}m^3/\min\ \{\rho = 1000\ kg/m^3\}$

Rate of water flow = $\frac{1000}{7}kg/\min$

Power of source = $P \times 10^6$ watt/sec

Let for a 60 sec power supply

Energy of source = $P \times 10^6 \times 60$

using energy conservation

$P \times 10^6 \times 60$

$= \frac{1000}{7} \times 1 \times 4200(60) + \frac{1000}{7} \times 1 \times 2268 \times 10^3$

$P \times 60 = \frac{42 \times 6}{7} + \frac{2268}{7} = 36 + 324$

$P \times 60 = 360$

$P = 6$

4. Ans. 3

Sol. $mg - T = ma_{cm}$

$T = 2ma_{cm}$

$mg - 2ma_{cm} = ma_{cm}$

$mg = 3ma_{cm}$

$a_{cm} = g/3$

In centre of mass frame

$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k(2m+m)}{(2m \times m)}}$

$\omega = \sqrt{\frac{3k}{2m}}$

maximum acceleration of mass m = $\frac{g}{3}$

$\frac{g}{3} = \omega^2 x_1$

x_1 is maximum displacement of mass m

$x_1 = \frac{g}{3\omega^2}$

maximum acceleration of mass m = $\frac{2g}{3}$

$\frac{2g}{3} = \omega^2 \times 2$

x_2 is maximum displacement of mass 2m

$x_2 = \frac{2g}{3\omega^2}$

total displacement

$x = x_1 + x_2 = \frac{g}{3\omega^2} + \frac{2g}{3\omega^2}$

$= \frac{3g}{3\omega^2} = \frac{g}{\omega^2} = \frac{g}{3k} \times 2m$

$= 10 \times 2 \times 3 = 2$ mela

net separation between

then is = $\left(\frac{1}{100} + \frac{2}{100}\right)m = \frac{3}{100}m = 3cm$

PART-2 : CHEMISTRY — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	C	D	A	C	B	A	C	B	A
	Q.	11	12	13	14	15	16				
	A.	C	A	C	D	D	B				
SECTION-IV	Q.	1	2	3	4						
	A.	2	4	1	3						

SOLUTION

SECTION-I

1. **Ans. (C)**
$\Delta T_b = iK_b.m$
$\Delta T_b \propto i.m$

2. **Ans. (C)** ; 3. **Ans. (D)** ; 4. **Ans. (A)**

5. **Ans. (C)** ; 6. **Ans. (B)** ; 7. **Ans. (A)**

8. **Ans. (C)**

9. **Ans. (B)**
S_N1 is 1st order reaction
So, $T_{75\%} = 2 \times T_{50\%}$
$T_{50\%} = \frac{40}{2} = 20\,\text{min}$

10. **Ans. (A)**
$r = k\,[C_4H_9Cl]\,[OH^-]$

$$C_4H_9Cl + NaOH \xrightarrow{S_N2} C_4H_9OH$$

C_4H_9Cl	$NaOH$	C_4H_9OH
1	2	–
0.25	1.25	–

$r = 4 \times 10^{-4} \times 0.25 \times 1.25 = 1.25 \times 10^{-4}$

11. **Ans. (C)**
12. **Ans. (A)**
13. **Ans. (C)**
14. **Ans. (D)**
15. **Ans. (D)**
16. **Ans. (B)**

SECTION-IV

1. **Ans.2**
$w = -p_{ext}.\Delta V = -1\,\text{atm} \times (22-2)\,\text{litre} = -20\,\text{lit-atm}$
$= -2000\text{ J} = -2\text{ kJ}.$

2. **Ans.4**
$Q + 2H^+ + 2e^- \longrightarrow QH_2$
$E = E^o + \frac{0.06}{2}\log[H^+]^2$
$E = E^o - 0.06 \times P^H$
$pH = \frac{E^o - E}{0.06} = \frac{0.46-0.22}{0.06} = \frac{0.24}{0.06} = 4$

3. **Ans.1**
4. **Ans.3**

PART-3 : MATHEMATICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	C	B	D	C	A	A	C	B	A
	Q.	11	12	13	14	15	16				
	A.	B	D	B	B	B	B				
SECTION-IV	Q.	1	2	3	4						
	A.	5	1	7	9						

SOLUTION

SECTION-I

1. **Ans. (A)**

$$\int \frac{e^x + \sin x + 4\cos x + 3}{e^x + 5\sin x + 3\cos x + 6}dx$$

$$= \int \frac{\frac{1}{2}\left(e^x + 5\sin x + 3\cos x + 6\right) + \frac{1}{2}\left(e^x + 5\cos x - 3\sin x\right)}{\left(e^x + 5\sin x + 3\cos x + 6\right)}dx$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \frac{e^x + 5\cos x - 3\sin x}{e^x + 5\sin x + 3\cos x + 6}dx$$

$$= \frac{x}{2} + \frac{1}{2}\ell n\left(e^x + 5\sin x + 3\cos x + 6\right) + C$$

2. **Ans. (C)**
We will manipulate our conditions to show that if x is a root, x + 10 and x + 4 are also roots.
$f(2+x) = f(2-x)$ ...(1)
Put $x = z - 2$
$\therefore f(z) = f(4-z)$
& $f(7+x) = f(7-x)$ ...(2)
Put $x = -3 - z$

$f(4 - z) = f(7 + 10)$

$\Rightarrow f(z) = f(z + 10)$

$\therefore$ If 0 is a root , then ±10, ±20,......, ±1000 are also roots.

So there are 201 roots. Also if 0 is a root then 4 is also a root. Hence –996, –986,....., 4,14,.....,984,994 are also roots so there are 200 roots.

$\therefore$ Total roots are 401.

3. Ans. (B)

$$\frac{n^4+3n^2+10n+10}{2^n\left(n^4+4\right)}$$

$$=\frac{1}{2^n}+\frac{3n^2+10n+6}{2^n\left(n^2+2n+2\right)\left(n^2-2n+2\right)}$$

$$=\frac{1}{2^n}+\frac{4n^2+8n+8-n^2+2n-2}{2^n\left(n^2+2n+2\right)\left(n^2-2n+2\right)}$$

$$=\frac{1}{2^n}+\frac{1}{2^{n-1}\left(n^2-2n+2\right)}-\frac{1}{2^n\left(n^2+2n+2\right)}$$

$$\therefore \sum_{n=2}^{\infty}\frac{n^4+3n^2+10n+10}{2^n.\left(n^4+4\right)}$$

$$=\sum_{n=2}^{\infty}\frac{1}{2^n}+\sum_{n=2}^{\infty}\left(\frac{1}{2^{n-2}\left(n^2-2n+2\right)}-\frac{1}{2^n\left(n^2+2n+2\right)}\right)$$

$$=\frac{1}{2}+\frac{6}{10}=\frac{11}{10}$$

4. Ans. (D)

For the equation to have fewer than 2 real solutions

$D \le 0$

$\therefore 4\,(a + b - 7)^2 - 4.2b.a \le 0$

$\Rightarrow a^2 + b^2 - 14a - 14b + 49 \le 0$

$\Rightarrow (a - 7)^2 + (b - 7)^2 \le 7^2$

Which is interior and periphery of circle with centre (7,7)

and radius 7

Area = 49π

5. Ans. (C)

$$\int_{-\pi}^{\pi}\frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}dx$$

$$\int_0^{\pi}\left(\frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}+\frac{x^2}{1-\sin x+\sqrt{1+\sin^2 x}}\right)dx$$

$$=\int_0^{\pi}x^2\left(\frac{2+2\sqrt{1+\sin^2 x}}{1+1+\sin^2 x+2\sqrt{1+\sin^2 x}-\sin^2 x}\right)dx$$

$$=\int_0^{\pi}x^2dx=\frac{\pi^3}{3}$$

6. Ans. (A)

$R = P^TQ^8P$

$= P^TPAP^TPAP^T....PAP^TP = A^8$

$\therefore \quad r_{11} = (\sqrt{3})^8 = 81$

7. Ans. (A)

Let P(A) & P(B) are probabilities of two independent events

given $P(A) + P(B) - 2P(A)P(B) = \frac{26}{49}$

$$P(A)+P(B)-P(A)P(B)=\frac{34}{49}$$

$\therefore \quad P(A)=\frac{2}{7}, P(B)=\frac{4}{7}$

or $\quad P(A)=\frac{4}{7}, P(B)=\frac{2}{7}$

$\therefore$ Probability of least probable of two events is $\frac{2}{7}$

8. Ans. (C)

$f'(x) + f(x)\cot x - 2\cos x = 0$

by using linear differential equation we get

$f(x) = \sin x$

$\therefore \quad f\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

Paragraph for Question 9 & 10

9. Ans. (B)

$$g(x)=\int_0^x f(t)dt$$

$$g(x)=\int_0^{-x} f(t)dt \qquad \text{(put } t=-y)$$

$$=\int_0^x f(y)(-dy)$$

$$=\int_0^x f(y)dy \qquad \because \ f(x) \text{ is odd function.}$$

Given $f(x)$ is periodic function with 2

$\Rightarrow \quad f(x) = f(x+2) \ \forall \ x \in R$

$$g(x+2)=\int_0^{x+2} f(t)dt=\int_0^2 f(t)dt+\int_0^x f(t)dt$$

Let $\int_0^2 f(t)dt = c$

$g(x + 2) = c + g(x)$

$g(x + 2) - g(x) = c \ \forall \ x \in R$

$g(x + 4) - g(x + 2) = c \ \forall \ x \in R$

$\therefore \quad g(x + 4) = g(x)$

$\therefore \quad g(x)$ is an even and periodic function

10. Ans. (A)

Paragraph for Question 11 & 12

11. Ans. (B)

12. Ans. (D)

If a,b,c,d are four terms in G.P then loga, logb, logc and logd are in A.P.

$\therefore \log_{10}x, \log^2_{10}x, \log^2_{10}y, \log^2_{10}(xy)$ in A.P.

$\Rightarrow \log^2_{10}(xy) - \log^2_{10}y = \log^2_{10}y - \log^2_{10}x$ ...(1)

Also; $\log^2_{10}y - \log^2_{10}x = \log^2_{10}x - \log_{10}x$ ...(2)

$\therefore$ from (1)

$(\log_{10}xy + \log_{10}y)(\log_{10}xy - \log_{10}y)$
$= \log^2_{10}y - \log^2_{10}x$

$(\log_{10}x + 2\log_{10}y)(\log_{10}x) = \log^2_{10}y - \log^2_{10}x$

$\Rightarrow \log^2_{10}y - 2\log_{10}y.\log_{10}x - 2\log^2_{10}x = 0$ ...(3)

from (2)

$$\log^2_{10}y - 2\log^2_{10}x + \log_{10}x = 0 \qquad ....(4)$$

(3)- (4) $\Rightarrow -2\log_{10}y.\log_{10}x - \log_{10}x = 0$

$\Rightarrow x = 1$ or $y = \frac{1}{\sqrt{10}}$

Now, if $x = 1 \Rightarrow y = 1$

Also, if $x = 1 \Rightarrow y = 1$

Also, if $y=\frac{1}{\sqrt{10}} \Rightarrow 2\log^2_{10}x - \log^x_{10} - \frac{1}{4} = 0$

$$\log^x_{10} = \frac{1\pm\sqrt{1+2}}{4} = \frac{1\pm\sqrt{3}}{4}$$

$$\Rightarrow x = 10^{\frac{1\pm\sqrt{3}}{4}}$$

$$\Rightarrow (1,1),\left(10^{\frac{1+\sqrt{3}}{4}},\frac{1}{\sqrt{10}}\right)\left(10^{\frac{1-\sqrt{3}}{4}},\frac{1}{\sqrt{10}}\right)$$

are three ordered pair

Paragraph for Question 13 & 14

13. Ans. (B)

$y = \frac{8}{27}x^3$ & $y = (x + a)^2$

Let slope of common tangent m

$$\therefore \quad y - y_1 = \frac{8}{9}x_1^2(x - x_1)$$

$$\Rightarrow \quad y = \frac{8}{9}x_1^2 x + y_1 - \frac{8}{9}x_1^3 \qquad ...(1)$$

$$\& \quad y = m(x+a) - \frac{m^2}{4}$$

$$\downarrow \text{put } m = \frac{8}{9}x_1^2$$

$$y = \frac{8}{9}x_1^2 x + \frac{8}{9}x_1^2 a - \frac{16}{81}x_1^4 \qquad ...(2)$$

(1) & (2) are same

$$\therefore \quad y_1 - \frac{8}{9}x_1^3 = \frac{8}{9}x_1^2 a - \frac{16}{81}x_1^4 \quad \left(\text{Use } y_1 = \frac{8}{27}x_1^3\right)$$

$$\Rightarrow \quad 2x_1^2 - 6x_1 - 9a = 0$$

$\because \quad x_1 \in R - \{0\} \ \Rightarrow \ D > 0 \ \& \ a \neq 0$
($\because$ we get two tangents)

$$\Rightarrow \quad a > -\frac{1}{2} \ \& \ a \neq 0$$

$$a \in \left(-\frac{1}{2}, \infty\right) - \{0\}$$

14. Ans. (B)

If a = 4, then $C_2 : y = (x + 4)^2$

common tangents y = 8x + 16 at x = 0

& y = 32x – 128 at x = 12

intersection point of these tangents (6, 64)

required area

$$= \int_0^{12} (x+4)^2 dx - \int_0^6 (8x+16)dx - \int_6^{12} (32x-128)dx$$

= 144.

Paragraph for Question 15 & 16

$$\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t))dt = \int_0^1 f^{-1}(t)dt$$

on differentation $f^{-1}(f(x))f'(x) - \cos x + f(x) = 0$

$d(xf(x)) = \cos x$

$\therefore \quad xf(x) = \sin x + c$

$$\because \quad f(0) = 1 \Rightarrow f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \in \left(0, \dfrac{\pi}{2}\right) \\ 1, & x = 0 \end{cases}$$

15. Ans. (B)

$2f(2x) - \sin x f(x) = 0$

$$2.\frac{\sin 2x}{2x} - \sin x.\frac{\sin x}{x} = 0$$

$\sin x(2\cos x - \sin x) = 0$

$\sin x = 0$ or $\tan x = 2$

$\therefore$ no solution in $\left(0, \dfrac{\pi}{4}\right)$

16. Ans. (B)

$$\frac{2}{\pi} < \frac{\sin x}{x} < 1 \ \forall \ x \in \left(0, \frac{\pi}{2}\right)$$

$$\int_0^{\pi/2} \frac{2}{\pi} dx < \int_0^{\pi/2} \frac{\sin x}{x} dx < \int_0^{\pi/2} 1 dx$$

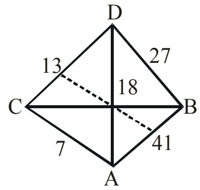
$$1 < I < \frac{\pi}{2}$$

$$\therefore \quad \int_0^{\pi/2} \frac{\sin x}{x} dx \in \left(1, \frac{\pi}{2}\right)$$

SECTION – IV

1. Ans. 5

Let 'x' be the median from A to CD and y be the median from B to CD.



$\therefore \ 2(7^2 + 18^2) = 4x^2 + 13^2$

& $2(27^2 + 36^2) = 4y^2 + 13^2$

& $2(x^2 + y^2) = 41^2 + 4d^2$

$= 7^2 + 18^2 + 27^2 + 36^2 - 13^2$

$\therefore \ d^2 = 137$

2. Ans. 1

$$56x + 33y = -\frac{y}{x^2+y^2} \quad ...(1)$$

$$33x - 56y = \frac{x}{x^2+y^2} \quad ...(2)$$

Multiply equation (1) by i and add to equation (2) then $56iz + 33z = \dfrac{1}{z}$ where z = x + iy

$$z^2 = \frac{1}{33+56i} \Rightarrow z = \pm\frac{1}{7+4i} = \pm\frac{(7-4i)}{65}$$

$$\therefore \quad |x| + |y| = \frac{11}{65}$$

3. Ans. 7

$$2.\frac{2\tan\beta}{1+\tan^2\beta} = 3.\frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\Rightarrow \frac{6\tan\beta}{1+9\tan^2\beta} = \frac{3\tan\alpha}{1+\tan^2\alpha}$$

$\Rightarrow \ 1 = 7\tan^2\alpha$

$$\therefore \quad \cos^2\alpha = \frac{7}{8}$$

4. Ans. 9

$$f(x) = \frac{(x+a)^2}{(a-b)(a-c)} + \frac{(x+b)^2}{(b-a)(b-c)} + \frac{(x+c)^2}{(c-a)(c-b)}$$

on solving we get

$f(x) = 1$

$\therefore \ (P + 8)^{2015} = (9)^{2015} = (10 - 1)^{2015}$

$= 10\lambda + 9$

$\therefore$ unit digit = 9.

CLASSROOM CONTACT PROGRAMME
(ACADEMIC SESSION 2014-2015)

ENTHUSIAST & LEADER COURSE	ALL INDIA OPEN TEST # 01
TEST TYPE : MAJOR	PATTERN : JEE (Advanced)
TARGET : JEE (Advanced) 2015	Date : 08 - 02 - 2015

PAPER-2

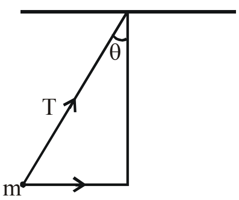
PART-1 : PHYSICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	A,C	A,C	A,C,D	B,D	A,B,C	A,D	A,D	A	B
	Q.	11	12								
	A.	C	A								
SECTION-III	Q.	1	2	3	4						
	A.	074	124	030	060						
SECTION-IV	Q.	1	2	3	4						
	A.	3	5	6	8						

SOLUTION

SECTION-I

1. Ans. (A,D)

Sol. 

$T\sin\theta + F = m\omega^2(L\sin\theta)$

$T\sin\theta = mg$

$$\frac{mg}{\cos\theta}\sin\theta + q(\omega L\sin\theta)B = m\omega^2 L\sin\theta$$

$$B = \frac{m\omega^2 L\sin\theta}{(\omega L\sin\theta)q} - \frac{\frac{mg\sin\theta}{\cos\theta}}{(\omega L\sin\theta)q}$$

$$= \frac{m\omega}{q} - \frac{mg}{q\omega L\cos\theta} = \frac{1}{\beta}\left[\omega - \frac{g}{\omega L\cos\theta}\right]$$

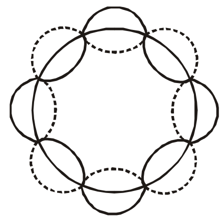
2. Ans. (A,C)

Sol. $$\frac{P^2}{2m_\alpha} + \frac{P^2}{2m_{th}} = Q \Rightarrow P = \left(\frac{2m_\alpha m_{th} Q}{m_\alpha + m_{th}}\right)^{1/2}$$

and $$\frac{K.E_\alpha}{K.E_{th}} = \frac{m_{th}}{m_\alpha} = \frac{234}{4}$$

3. Ans. (A,C)

4. Ans. (A,C,D)

Sol. 

$$\frac{8\lambda}{2} = 2\pi R$$

$4\lambda = 2\pi R \Rightarrow n\lambda$

$n = 4$

so given excite state = n = 4

(1) change in angular momentum $= \frac{(n_2 - n_1)h}{2\pi}$

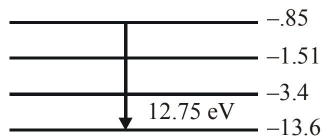
$$= \frac{(4-1)\pi}{2\pi} = \frac{3h}{2\pi}$$

(B) $n\lambda = 2\pi R_n$

$\lambda = 2\pi R_1$

$4\lambda = 2\pi R_4$

$$\frac{4\lambda_4}{\lambda_1} = \frac{R_4}{R_1} = \frac{(4)^2}{(1)^2} \Rightarrow \frac{\lambda_4}{\lambda_1} = \frac{4}{1}$$

(C) 

(D) $\frac{T_4}{T_1} = (4)^3$

$T \propto n^3$

$T = \frac{2\pi r}{v} \Rightarrow n^2 \times n$

$T \propto n^3$

$T \propto \frac{n^3}{z^2}$

5. Ans. (B,D)

Sol. $\lambda_1 + \lambda_2 + \lambda_3 = \lambda_{eff}$

$T_1 = a$

$\frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} = \frac{1}{T} = \frac{1}{10}$

$T_2 = ar$

$\frac{1}{9}\left[1 + \frac{1}{r} + \frac{1}{r^2}\right] = \frac{1}{10}$

$T_3 = ar^2$

$\frac{r^2}{70}\left[1 + \frac{1}{r} + \frac{1}{r^2}\right] = \frac{1}{10}$

$(r^2 + r + 1) = 7$

$r^2 + r - 6 = 0$

$(r + 3)(r - 2) = 0$

$r = 2$

$a = \frac{70}{r^2}$

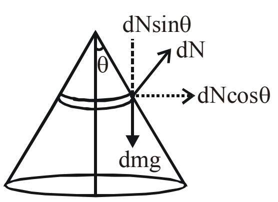
$a = \frac{70}{(2)^2} = \frac{70}{4} = 17.5$

$a = 17.5$

$air = \frac{70 + 2}{4} = \frac{70}{2} = 3.5$

6. Ans. (A,B,C)

Sol.

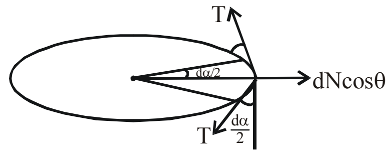


$dN\sin\theta = (dm)g$

$N\sin\theta = mg$

$\frac{N}{\sqrt{2}} = mg$

$N = \sqrt{2}mg$ ....(i)



$2T\sin\left(\frac{d\alpha}{2}\right) = dN\cos\theta$

$2T\left(\frac{d\alpha}{2}\right) = dN\cos\theta$

(1) $T(2\pi B) = \sqrt{2}mg \times \frac{1}{\sqrt{2}}$

$T = \frac{mg}{(2\pi)}$

(2) $\frac{T}{A} = Y \times \frac{\Delta\ell}{\ell}$ $\qquad \frac{mg}{2\pi A} = \frac{Y\Delta\ell}{2\pi R}$

$\Delta\ell = \frac{mgR}{Ay}$

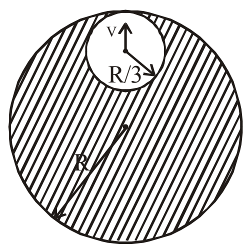
7. Ans. (A,D)

Sol. Superposition of waves (i) & (iii) will give travelling wave having amplitude of $a\sqrt{2}$ {waves are along x-axis but particle displacements are along y & z-axis respectively}

$z_1 + z_2 = a\left[\sin\omega\left(t - \frac{x}{v}\right) + \sin\left\{\omega\left(t + \frac{x}{v}\right) + \frac{\pi}{2}\right\}\right]$

8. Ans. (A,D)

Sol.



Total potential energy at Pt 'P'

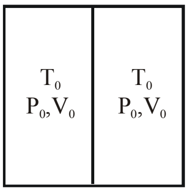
$\left(\frac{GMm}{R}\left[\frac{3}{2} - \frac{(2/3R)^2}{2R^2}\right] - \frac{3GMm}{2 \times R/3}\right)$

$= \frac{20GMm}{18R} = \frac{1}{2}mv_p^2$

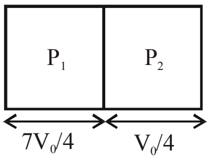
$v_P = \sqrt{\frac{20GM}{9R}}$

9. Ans. (A)

Sol.



(A)



$$\frac{P_0V_0}{T_0} = \frac{(10P_0)\times\frac{V_0}{4}}{T_2}$$

(B) $T_2 = \frac{10}{4}T_0 = \frac{5}{2}T_0$

$$P_0 \times V_0^{\,r} = P_2 \times \left(\frac{V_0}{4}\right)^r$$

$P_r = (4)^r(P_0)$

$= (4)^{5/3}\, P_0[2^{4/3}]$

$P_0/10$

$$\Delta w_2 \Rightarrow \frac{n\Delta R\Delta I_2}{1-r}$$

$$\Rightarrow \frac{10P_0 \times \frac{V_0}{4} - P_0V_0}{\left(1-\frac{5}{3}\right)} \Rightarrow \frac{1.5P_0V_0}{-2} = -\frac{9}{4}P_0V_0$$

(C) $$\frac{P_0V_0}{T_0} = \frac{10P_0 \times \frac{7}{4}V_0}{T_1}$$

$T_1 = \frac{35}{2}T_0$

(D) $\Delta Q_1 = \Delta U_1 + \Delta W_1$

$= \frac{f}{2}nR\Delta T + \frac{9}{4}nRT_0$

$= \frac{3}{2}nR\Delta T + \frac{9}{4}nRT_0$

$= \frac{3}{2}nR\left[\frac{35}{2}-1\right]T_0 + \frac{9}{4}nRT_0$

$= \left(\frac{3}{2}\times\frac{33}{2}+\frac{9}{4}\right)nRT_0 = \left(\frac{108}{4}\right)nRT_0 = 27nRT_0$

10. Ans. (B)

11. Ans. (C)

Sol. If a body is rolling by applying a force at the centre parallel to the surface then an instant firction (if present) will be acting opposite to this applied force and parallel to the surface.

12. Ans. (A)

Sol. Path difference remains same on a circle for case D

Shape of fringe pattern for pin hole is hyperbolic

Shape of fringe pattern for slit is straight line

Δx_{max} can't be greater than 'd' distance between the source in A, B & C & Δx_{min} can't be less than d-distance between the source in D.

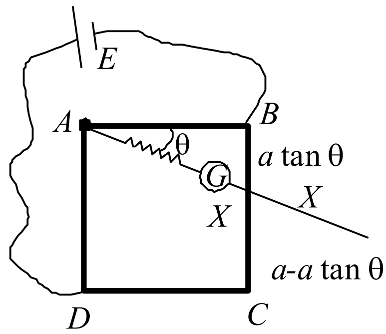
SECTION-III

1. Ans. 074

Sol. a be the side length of square and θ be the position where galvanometer gives zero deflection. To have zero deflection bridge is to be balanced.

$$\Rightarrow \frac{R_{AB}}{R_{AD}} = \frac{R_{BX}}{R_{DC}+R_{CX}}$$

[R_{DC} and R_{CX} is in series]



$$\frac{100}{200} = \frac{\frac{400}{a}a\tan\theta}{500+\frac{400}{a}(a-a\tan\theta)}$$

$$\Rightarrow \frac{1}{2} = \frac{400\tan\theta}{500+400(1-\tan\theta)}$$

Solving tan θ = 3/4, θ = 37^0

t be the time taken from start, θ = ωt [θ is radian]

$$\frac{\pi}{180}\times 37 = \frac{\pi}{360}\times t$$

t = 74 sec

2. Ans. 124

Sol. Let ice absorbs heat at a rate q & mass of water be x.

Total heat absorbed after tenth reading

$= Q(9t) = 80\,x + 640 \times .5 \times .5$...(i)

total heat absorbed after element reading

$= \theta(10t) = 80\,x + 640 \times .5 \times 4$...(ii)

dividing (i) & (ii)

$$\frac{9}{10} = \frac{80x + 160}{80x + 1280}$$

solving we get x = 124 g

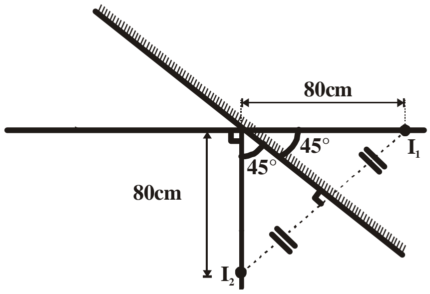
3. Ans. 030

4. Ans. 060

Sol. Image formation due to convex lens

$$\frac{1}{v} - \frac{1}{-36} = \frac{1}{30} \Rightarrow v = \frac{30 \times 36}{6} = 180 \text{ cm}$$

This image will act like a virtual object for mirror and after reflection from mirror its image (shown by I_2) will be formed at 80 cm below optical axis of convex lens.



For concave lens, this image will be object at a position of 15 cm below the lens.

For final image formed by concave lens.

$$\frac{1}{20} - \frac{1}{15} = \frac{1}{f} \Rightarrow \frac{1}{f} = -\frac{5}{300}$$

Also, $\frac{1}{f} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{R}\right)$

or $-\frac{5}{300} = \left(\frac{3}{2} - 1\right)\left(-\frac{2}{R}\right)$

$\Rightarrow R = \frac{300}{5}$; R = 60 cm

SECTION-IV

1. Ans. 3

Sol. $Y = \frac{F}{A}\frac{\ell}{\Delta \ell}$

$$\therefore \frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta \ell}{\ell} + \frac{\Delta(\Delta \ell)}{\Delta \ell} = \frac{2\Delta d}{d} + \frac{\Delta \ell}{\ell} + \frac{\Delta(\Delta \ell)}{\Delta \ell}$$

2. Ans. 5

Sol. Work done by gravity = work done by electric force

$$n = \int_R^{2R} \frac{2k\lambda q}{r} dr = mgR$$

$2kq\lambda n2 = mgR$

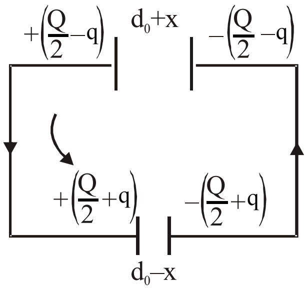
$$\frac{nq^2}{L2\pi \epsilon_0} \ell n2 = mgR$$

$$n = \frac{mgRL \cdot 2\pi \epsilon_0}{q^2 \ell n2} = 1 \times 10^4$$

3. Ans. 6

Sol. Let each plate moves a distance 'x' from its initial position. Let q charge flows in the loop. using KVL

$$\frac{\left(\frac{Q}{2} - q\right)(d + x)}{\varepsilon_0 A} - \frac{\left(\frac{Q}{2} + q\right)(d - x)}{\varepsilon_0 A} = 0$$



$$\therefore q = \frac{Qx}{2d_0} ; I = \frac{dq}{dt} = \frac{Q}{2d_0}\left(\frac{dx}{dt}\right) ;$$

Ans, $I = \frac{Qv_0}{2d_0}$

4. Ans. 8

Sol. $\Delta\phi = L\Delta I$

$$\frac{\left(\pi \cdot a^2\right)B}{L} = I - 0$$

$$W = \frac{1}{2}LI^2$$

$$= \frac{1}{2}L \cdot \left(\frac{\pi a^2 B}{L}\right)^2 = \frac{1}{2}\frac{\pi^2 a^4 B^2}{L} = 8 \text{ J}$$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,D	A,B,C	B,C	A,B,C,D	A,B,C,D	A,B	A,B,C,D	A,B,C	D	C
	Q.	11	12								
	A.	A	C								
SECTION-III	Q.	1	2	3	4						
	A.	120	040	001	004						
SECTION-IV	Q.	1	2	3	4						
	A.	3	6	5	3						

SOLUTION

SECTION-I

1. **Ans (B, D)**

$mvr = \frac{nh}{2\pi} \quad \Rightarrow \quad r = \frac{n^2h^2}{4\pi^2 kZe^2 \times m}$

$\frac{kZe^2}{r} = mV^2 \qquad V = \frac{kZe^2}{nh} \times 2\pi$

$E = -\frac{1}{2}\frac{kZe^2}{r} \alpha - m$

2. **Ans. (A,B,C)**

$[OH^-] = \sqrt{k_b . C} = 10^{-3}$

pOH = 3 ; pH = 11

3. **Ans. (B,C)**
4. **Ans. (A, B, C, D)**
5. **Ans. (A, B, C, D)**
6. **Ans. (A, B)**
7. **Ans. (A,B,C,D)**
8. **Ans. (A,B,C)**
9. **Ans.(D)**
10. **Ans. (C)**
11. **Ans. (A)**
12. **Ans. (C)**

SECTION-III

1. **Ans.120**

$\frac{P_C V_C}{RT_C} = \frac{3}{8}$

$\frac{75 \times V_C}{0.08 \times 300} = \frac{3}{8}$

$V_C = \frac{3}{8} \times \frac{100 \times 0.08}{25} = 0.12L = 120$ ml.

2. **Ans.040**

A + B + C

10 + 22 + 8 = 40

3. **Ans.001**

4. **Ans.004**

SECTION-IV

1. **Ans.(30)**
OMR Ans. 3

$A + 2B \longrightarrow 3C \; ; K_C = 2 \times 10^{-3}$

2	4	–
2-x	4–2x	3x

$K_C = \frac{(3x)^3}{(2-x)(4-2x)^2} = 2 \times 10^{-3}$

$\left(\frac{3x}{2-x}\right)^3 = 8 \times 10^{-3} \Rightarrow \frac{3x}{2-x} = 2 \times 10^{-1}$

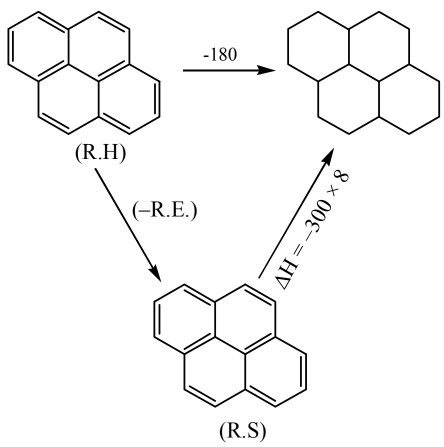
3x = 0.4 – 0.2 x

3.2 x = 0.4

$x = \frac{0.4}{3.2} = \frac{1}{8}$

$[B] = \left(\frac{4-2x}{1/8}\right) = 30 mol/L$

2. **Ans. (60)**
OMR Ans.6



–180 = – R.E. – 240

R.E. = 180 –240 = – 60kcal / mol

3. **Ans. 5**
4. **Ans. (336)**
OMR Ans.3

PART-3 : MATHEMATICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,D	B,C	A,B,C	D	A,B,D	B,D	A,C,D	A,B,C	C	B
	Q.	11	12								
	A.	A	A								
SECTION-III	Q.	1	2	3	4						
	A.	100	199	041	010						
SECTION-IV	Q.	1	2	3	4						
	A.	2	4	4	1						

SOLUTION

SECTION-I

1. Ans. (B,D)

$$T_r = \frac{x^{2^r}}{1-x^{2^{r+1}}} = \frac{x^{2^r}+1-1}{1-x^{2^{r+1}}}$$

$$\Rightarrow \quad T_r = \frac{1}{1-x^{2^r}} - \frac{1}{1-x^{2^{r+1}}}$$

For n terms, $\sum_{r=0}^{\infty} T_r = \frac{1}{1-x} - \frac{1}{1-x^{2^{n+1}}}$

For infinite terms and $x \in (0, 1)$:

$$f(x) = \sum_{r=0}^{\infty} T_r = \frac{1}{1-x} - \frac{1}{1-0} = \frac{x}{1-x}$$

Range of $f(x)$ is not R

$\therefore \quad f(x)$ is non invertible.

$$\int_0^1 f(x)dx = \int_0^1 \frac{x}{1-x}dx = \int_0^1 x + x^2 + x^3 + ...\infty\, dx$$

$$= \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty \Big|_0^1$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...\infty$$

Also $\int_0^{1/2} f(x)dx = -\left[\int_0^{1/2} \frac{1-x}{1-x}dx - \int_0^{1/2} \frac{1}{1-x}dx\right]$

$$= -(x + \ell n(1-x))\Big|_0^{1/2} = \ell n 2 - \frac{1}{2}$$

2. Ans. (B,C)

$$2\cos^2 2x - 2\cos 2x \cos\left(\frac{2014\pi^2}{x}\right) = -2\sin^2 2x$$

$$\Rightarrow \quad \cos 2x \cos \frac{2014\pi^2}{x} = 1$$

$\cos 2x$, $\cos\left(\frac{2014\pi^2}{x}\right)$ should both be simultaneously 1 or –1.

positive solutions : π, 19π, 53π, 1007π

sum = 1080π.

3. Ans. (A,B,C)

(B) $P(n = 2) = {}^6C_1 . \frac{1}{6} . \frac{1}{6} = \frac{1}{6}$

(C) $P(n = 4) = 1 - \left(4!\ {}^6C_4 \left(\frac{1}{6}\right)^4\right) = \frac{13}{18}$

(D) $P(n = 6) = 1 - \left({}^6C_6 . 6! \left(\frac{1}{6}\right)^6\right) = \frac{319}{324}$

$P(n \geq 7) = 1$ (Numbers will be repeated)

4. Ans. (D)

Function satisfying the given rule and for which $f'(1) = 3$ will be $f(x) = x\ell nx + 2x$.

No point of inflection.

No asymptote.

5. Ans. (A,B,D)

$$f(\lambda) = \int_{-\pi}^{\pi} \frac{-\sin x}{\pi + x + \lambda}dx = \int_0^{\pi}\left(\frac{-\sin x}{\pi + x + \lambda} + \frac{\sin x}{\pi - x + \lambda}\right)dx = \int_0^{\pi} \frac{2x\sin x}{(\pi+\lambda)^2 - x^2}dx$$

Now,

$$\forall x \in [0, \pi]; \frac{2x\sin x}{(\pi+\lambda)^2} \leq \frac{2x\sin x}{(\pi+\lambda)^2 - x^2} \leq \frac{2x\sin x}{(\pi+\lambda)^2 - \pi^2}$$

Also $\int_0^{\pi} x\sin x dx = \pi$

$$\therefore \int_0^{\pi} \frac{2x\sin x}{(\pi+\lambda)^2}dx \leq \int_0^{\pi} \frac{2x\sin x}{(\pi+\lambda)^2 - x^2}dx \leq \int_0^{\pi} \frac{2x\sin x}{(\pi+\lambda)^2 - \pi^2}dx$$

$$\Rightarrow \frac{2\pi}{(\pi+\lambda)^2} \leq f(\lambda) \leq \frac{2\pi}{(\pi+\lambda)^2 - \pi^2} \Rightarrow f(\lambda) > 0$$

$$\lim_{\lambda\to\infty} f(\lambda) = 0$$

& $\lim_{\lambda\to\infty} \lambda^2 f(\lambda) = 2\pi$ (using sandwich theorem)

6. Ans. (B,D)

$$\alpha + \beta = -\frac{b}{a}$$

$\alpha\beta = \frac{c}{a} = 3 \quad \Rightarrow \quad c = 3a$

$2b = a + c \quad \Rightarrow \quad b = 2a$

$\therefore \quad \alpha + \beta = -2$

$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 2(3) = -2$

7. Ans. (A,C,D)

For continuous : $3 = k + 4 \Rightarrow k = -1$.

Derivability depends on continuity and value of k.

For local minima at $x = 2$: $k + 4 > 3$

$\Rightarrow k > -1$

$$\int_2^3 f(x)dx = \left.\frac{(x-2)^3}{3} + 3x\right|_2^3 = \frac{10}{3}$$

8. Ans. (A,B,C)

Let mid point be P(h,k). Chord with mid point P for $x^2 - y^2 = 9 : hx - ky - 9 = h^2 - k^2 - 9$

$$\Rightarrow \quad y = \frac{kx}{x} - \frac{(h^2-k^2)}{k}$$

for tangent to $y^2 = 12x$: $y = mx + \frac{3}{m}$

$$m = \frac{h}{k}; \frac{3}{m} = -\frac{(h^2-k^2)}{k}$$

$$\therefore \quad \frac{3k}{h} = -\frac{(h^2-k^2)}{k} \Rightarrow x^3 + 3y^2 - xy^2 = 0$$

$\therefore \quad \lambda_1 = -1; \lambda_2 = 3$

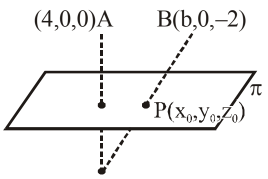
9. Ans. (C)

Let the plane be $\alpha x + \beta z + 1 = 0$ pass through (1,0,1); (3,2,–1)

$$\therefore \quad \alpha = -\frac{1}{2}; \beta = -\frac{1}{2}$$

(P) $\pi : x + z = 2$

(Q)



Both A and B are on same side of π.

Reflection of A in plane is A'(2, 0, –2)

Equation of line A'B : $\vec{r} = 6\hat{i} - 2\hat{k} + \lambda(4\hat{i})$

for P : $6 + 4\lambda + 0 - 2 = 2 \Rightarrow \lambda = \frac{1}{2}$

$\therefore$ P(4, 0, –2)

$\therefore$ $|4x_0 + y_0 + 2z_0| = 12$

(R) Now $|PA - PB|_{min} = 0$

$|PA - PB|_{max}$ will approach

$AB = \sqrt{4+0+4} = \sqrt{8}$

$\therefore$ $|PA - PB| \in \left[0, \sqrt{8}\right)$.

(S) Also, A' will lie on $\frac{x-2}{1} = \frac{y-\alpha}{0} = \frac{z+\beta}{-1}$.

$$\Rightarrow \quad \frac{2-2}{1} = \frac{0-\alpha}{0} = -\frac{2+\beta}{-1}$$

$\Rightarrow$ $\alpha = 0, \beta = 2$

$\therefore$ $\alpha^4 + \beta^4 = 16$

10. Ans. (B)

$|2z + 3i| = |z^2|$

$\because$ $|2z + 3i| \le 2|z| + 3$

$\Rightarrow$ $|z|^2 \le 2|z| + 3$

$\Rightarrow$ $|z|^2 - 2|z| - 3 \le 0$

$\Rightarrow$ $0 \le |z| \le 3$

again $|2z + 3i| \ge |2|z| - 3|$.

$\Rightarrow$ $|z|^2 \ge |2|z| - 3|$

Solving which, we get $|z| \ge 1$

$\therefore$ $1 \le |z| \le 3$

(P) $|z|_{max} = 3$

(Q) $|z|_{min} = 1$

(R) $\because$ $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow$ O, z_1, z_2 are collinear with z_1, z_2 on same side of O.

$\therefore$ $|2z + 3i| = 2|z| + |3i|$ at O, 2z, 3i collinear

$\Rightarrow$ for $|z|_{max}$: $z = 3i$

$\therefore$ $\alpha = 0, \beta = 3 \Rightarrow \alpha^3 + \beta^3 = 27$

(S) $\because$ $|z_1 + z_2| = ||z_1| - |z_2||$

$\Rightarrow$ z_1, z_2, O collinear

with z_1, z_2 on opposite side of O.

$\therefore$ $|2z + 3i| = ||2z| - |3i||$

for $|z|_{min}$: $z = -i$

$\therefore$ $x = 0, y = -1 \Rightarrow x^2 + 2y^2 = 2$

11. Ans. (A)

$$f(x) = \int_0^1 t|t-x|dt$$

for $x \ge 1$: $f(x) = \int_0^1 t(x-t)dt = \frac{x}{2} - \frac{1}{3}$

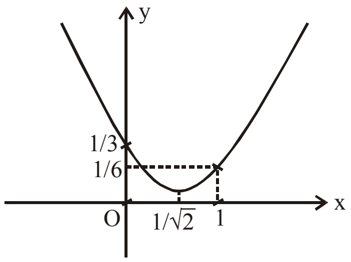
for $x \le 0$: $f(x) = \int_0^1 t(t-x)dt = \frac{1}{3} - \frac{x}{2}$

for $0 < x < 1$:

$$f(x) = -\int_0^1 t(t-x)dt + \int_x^1 t(t-x)dt$$

$\Rightarrow \quad f(x) = \frac{1}{3} - \frac{x}{2} + \frac{x^3}{3}$; $F'(x) = 0 \Rightarrow x = \frac{1}{\sqrt{2}}$

Graph :



$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{2-\sqrt{2}}{6}$$

$$\min_{x \to R} f(x) = \frac{2-\sqrt{2}}{6};\ \max_{x \to [0,1]} f(x) = \frac{1}{3}$$

$$\min_{x \ge 1} f(x) = \frac{1}{6}$$

$f(x)$ is derivable $\forall\ x \in R$

12. Ans. (A)

(P) Arrange A's in gaps

$\therefore \quad \frac{6!}{2!}.\ {}^7C_2 = 7560$

(Q) Arrange $\boxed{EE}, B, R, K, G$ and then two A's in gaps.

$\therefore \quad 5!\,{}^6C_2 = 1800$

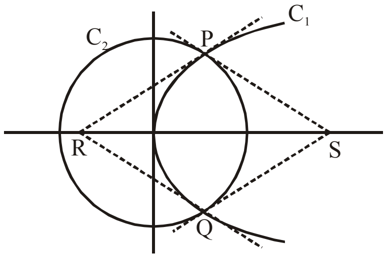
(R) Ans.(P) – Ans.(Q) = 7560 – 1800 = 5760

(S) Arrange B, R, K, G and vowels in gaps

$\therefore \quad 4!.\,{}^5C_4.\frac{4!}{2!2!} = 720$

SECTION – III

1. Ans. 100



Put z = x + iy

$C_1 : y^2 = 8x$

$C_2 : x^2 + y^2 = 9$

On solving C_1 & C_2

$P(1, 2\sqrt{2})$

Tangent to C_1 at P : $y(2\sqrt{2}) = 8\frac{(x+1)}{2}$

at y = 0 : x = –1 $\Rightarrow$ R(–1, 0)

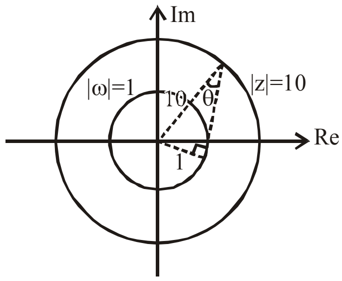
Tangent to C_2 at P : $x(1) + y(2\sqrt{2}) = 9$

at y = 0 : x = 9 $\Rightarrow$ S(9, 0).

$\therefore$ Area $\Delta PRS : \frac{1}{2} \times (9-(-1)) \times 2\sqrt{2} = 10\sqrt{2}$

$\therefore \quad \lambda = 10 \Rightarrow \lambda^2 = 100$

2. Ans. 199



$$\theta = \arg\left(\frac{w-z}{z}\right)$$

$$\sin\theta = \frac{1}{10}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{99}}$$

$$\Rightarrow \quad \tan^2\theta = \frac{1}{99}$$

$\therefore$ p + 2q = 199.

3. Ans. 041

$x^4 + ax^3 + bx^2 + ax + 1 = 0$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} + b + a\left(x + \frac{1}{x}\right) = 0$$

put $x + \frac{1}{x} = t \Rightarrow t^2 + at + b - 2 = 0$

$\Rightarrow$ at + b + t^2 – 2 = 0; $t^2 \in [4, \infty)$

This represents equation of a line in a-b plane and $a^2 + b^2$ represents square of distance of a point on this line from O.

$$d = \frac{t^2-2}{\sqrt{1+t^2}}$$

$\therefore$ For $t^2 \in [4, \infty)$: $d_{min} = \frac{2}{\sqrt{5}}$ at $t^2 = 4$

$\therefore \quad d^2_{min} = \frac{4}{5} = \frac{p}{q} \Rightarrow p^2 + q^2 = 41$

4. Ans. 010

$P(\text{tail}) = \frac{n}{2n+1}.1 + \frac{n+1}{2n+1}.\frac{1}{2} = \frac{31}{42} \Rightarrow n = 10$

SECTION – IV

1. Ans. 2

Let R(a$\sec\theta$, b$\tan\theta$)

Tangent at $R : \frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

pass through (0, – b)

$\Rightarrow \quad \theta = \frac{\pi}{4}$ (only)

Normal at R : $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

Pass through $\left(2\sqrt{2}a, 0\right)$ and $\theta = \frac{\pi}{4}$

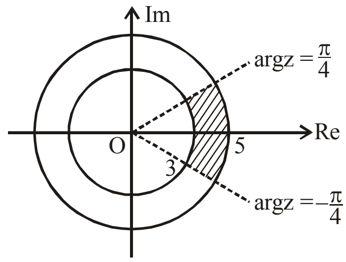
$\therefore \quad 2a^2 = a^2 + b^2 \Rightarrow \quad a = b$

$\therefore \quad e = \sqrt{2}$

2. Ans. 4

z lies in ring shown.

Probability that P lies in shaded region as shown is required.

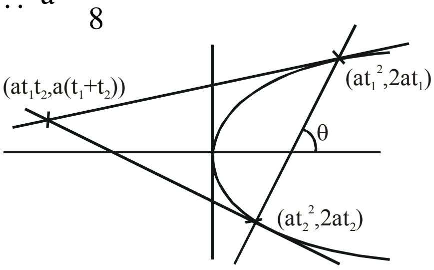


$\therefore$ Probability $= \frac{1}{4}$

3. Ans. 4

Here parabola is $y^2 = \frac{x}{2}$

$\therefore \ a = \frac{1}{8}$



Also, slope of normal at $t_1 = -t_1 = 2 - \sqrt{3}$

$\Rightarrow t_1 = \sqrt{3} - 2$

Now, $t_2 = -t_1 - \frac{2}{t_1} = 2 - \sqrt{3} + \frac{2}{2-\sqrt{3}}$

$= 2 - \sqrt{3} + 2\left(2+\sqrt{3}\right) = 6 + \sqrt{3}$

$\therefore \ at_1t_2 = \frac{1}{8}\left(\sqrt{3}-2\right)\left(6+\sqrt{3}\right) = \frac{1}{8}\left(-9+4\sqrt{3}\right)$

& $a\left(t_1+t_2\right) = \frac{1}{8}\left(4+2\sqrt{3}\right) = \frac{1}{4}\left(2+\sqrt{3}\right)$

Hence Area of triangle

$$= \frac{\left(y_1^2 - 4ax_1\right)^{3/2}}{2a} = \frac{\left(\frac{1}{16}\left(2+\sqrt{3}\right)^2 - 4.\frac{1}{8}.\frac{1}{8}\left(-9+4\sqrt{3}\right)\right)^{3/2}}{\left(\frac{1}{4}\right)}$$

$$= 4\left(\frac{1}{16}\left(7+4\sqrt{3}\right) - \frac{1}{16}\left(-9+4\sqrt{3}\right)\right)^{3/2} = 4$$

4. Ans. 1

$$I = \frac{1}{2}\int_0^{\pi/2}\left(f\left(2x\right) + f''\left(2x\right)\right)\sin 2x\,dx$$

put 2x = t $\Rightarrow \ I = \frac{1}{4}\int_0^{\pi}\left(f\left(t\right) + f''\left(t\right)\right)\sin t\,dt$

$$\Rightarrow \quad I = \frac{1}{4}\left[\int_0^{\pi} f(t)\sin t\,dt + \int_0^{\pi} f''(t)\sin t\,dt\right] = 0$$

$\Rightarrow \quad 0! = 1$