(1001CJA102119093)

Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2019 - 2020)

JEE(Advanced)
FULL SYLLABUS
29-06-2020

JEE(Main+Advanced) : ENTHUSIAST COURSE [SCORE-II (PHASE-TEAS, T-AS, TOAS, TNAS, TRAS & TMAS)]

ANSWER KEY — PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	C	B	B	B	A	A	A	B
	Q.	11	12	13	14	15					
	A.	B,C	A,C,D	A,D	A,C,D	A,D					
SECTION-III	Q.	1	2	3	4	5					
	A.	3	8	5	2	6					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	B	B	A	D	D	C	D	B
	Q.	11	12	13	14	15					
	A.	A,B,D	A,B,D	A,B,C,D	A,C,D,	A,B,C,D					
SECTION-III	Q.	1	2	3	4	5					
	A.	8	4	1	9	6					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	B	D	A	B	C	B	A	A
	Q.	11	12	13	14	15					
	A.	A,B,C	A,B,C,D	A,B,C	A,B,D	A,B					
SECTION-III	Q.	1	2	3	4	5					
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ANSWER KEY — PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C,D	B	B,C	A,B,D	B	B	B	C	D	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	4	8	4	7	8	8	4	6	4
	Q.	11	12	13	14						
	A.	8	4	3	5						

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	A,B,C	A,B,C	B,C	B	A	D	C	B	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	7	7	5	8	2	4	7	6	4 or 8
	Q.	11	12	13	14						
	A.	7	3	7	4						

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B	A,C	A,B,C	A,B,C,D	D	C	B	B	D	D
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	8	5	9	7	8	4	8	1	6	4
	Q.	11	12	13	14						
	A.	7	2	4	4						

PAPER-1

PART-1 : PHYSICS SOLUTION

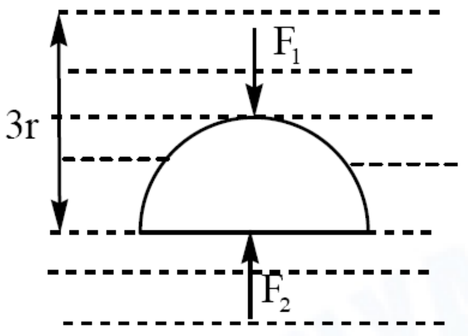
SECTION-I

1. Ans. (D)

Sol. Check for dimensionally correct option.

2. Ans. (A)

Sol. Imagine that lower half of sphere is removed and upper hemisphere is at the given position.



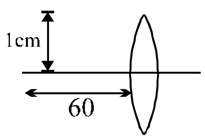
$$F_2 - F_1 = F_b$$

$$[P_0 + 3r\rho g]\pi r^2 - F_1 = \frac{2}{3}\pi r^3 \rho g$$

$$F_1 = \frac{\pi r^3}{3}[3P_0 + 7r\rho g]$$

3. Ans. (C)

Sol. $h_I = mh_0$



$$= \frac{f\,h_0}{u+f} = \frac{40h_0}{-60+40} \quad -2\,h_0$$

$$\frac{dh_I}{dt} = -2\,\frac{dh_0}{dt} = -2 \times 10 = -20 \text{ cm/s}$$

4. Ans. (B)

Sol. $r = (K_1)n^2$

$T \propto r^{3/2} \Rightarrow T \propto n^3 \Rightarrow T = K_2 n^3$

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\cdot\frac{e}{T} \Rightarrow B \propto \frac{1}{n^5}$$

$$M = IA = \frac{e}{T}\pi r^2 \Rightarrow M \propto n$$

$$r \propto n^2, v \propto \frac{1}{n}$$

$\therefore \dfrac{Br^2}{Mv^2}$ will be independent of n.

5. Ans. (B)

Sol. As temperature of second body is more, its average specific heat is more. In the forumla $\Delta Q = ms_{ave}(\Delta T)$, ΔQ and m are same for both. ΔT is less for 2[nd] one.

$(80 - T_f) < (T_f - 20)$

$100 < 2T_f \Rightarrow T_f > 50°C$

6. Ans. (B)

Sol. As the electron approaches the plate, negative potential produced by it at the plate increases gradually. To make its potential zero, earth sends +ve charge to the plate i.e. current is +ve. Reverse thing happens when electron moves away from plate. When the electron is in the middle of plate, potential of plate remains constant due to e^- and hence I = 0

7. Ans. (A)

Sol. Current in resistance R is

$$\left(\frac{E}{r + \frac{50R}{50+R}}\right) \times \frac{50}{(50+R)}$$

$$\text{Power} = \left[\left(\frac{E}{50R + 50r + Rr}\right) \times 50\right]^2 \times R$$

For maximum power

$$\frac{dP}{dR} = 0 \Rightarrow R = \frac{50}{11}\Omega$$

8. Ans. (A)

Sol. $E_i = \frac{k(2Q)}{(2R)^2} - \frac{k(Q)}{(4R)^2} = \frac{7kQ}{16R^2}$

9. Ans. (A)

Sol. From the graph U = 3 sin πx + 3

$\therefore F = -\frac{\partial U}{dx} = -3\pi\cos\pi x$

$\therefore F_{max} = 3\pi$

10. Ans. (B)

Sol. $E_0 = \frac{2\lambda}{2\pi\epsilon_0 d}$

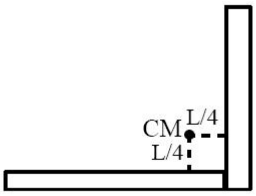
$\lambda = \pi\epsilon_0 dE_0$

For r >> ℓ, charge on the rod can be treated as a point charge.

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ wher $q = \lambda\ell = \pi\epsilon_0 dE_0\ell$

11. Ans. (B,C)

Sol.



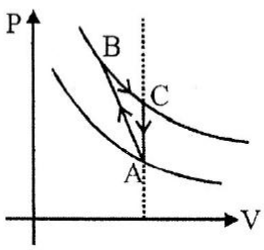
$Mv = 2Mv^1 \rightarrow v^1 = \frac{v}{2}$

$2\left(\frac{ML^2}{12} + M\left(\frac{L}{2\sqrt{2}}\right)^2\right)\omega = Mv\frac{L}{4}$

12. Ans. (A,C,D)

Sol. Let us consider a closed process ABCA. CA is an isochoric process. In CA, T decreases as 'P' decreases.

$\Delta Q_{CA} = nC_V\Delta T$ is negative. Let it is –y.



$Q_{ABCA} = W_{ABCA}$ is +ve. Let it is +z

$\Rightarrow Q_{A\rightarrow B} \Rightarrow Q_{B\rightarrow C} \Rightarrow Q_{C\rightarrow A} = +z$

$\Rightarrow Q_{A\rightarrow B} + 0 + (-y) = +z$

$\Rightarrow Q_{A\rightarrow B} = z + y > 0$

13. Ans. (A,D)

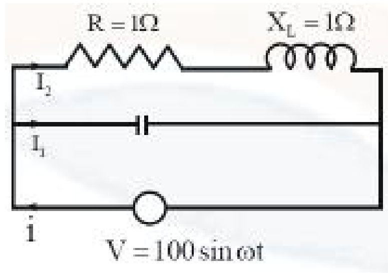
Sol. Conceptual

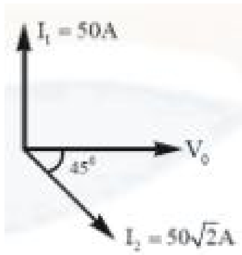
14. Ans. (A,C,D)

Sol. $i_2 = \frac{V_0}{\sqrt{R^2 + X_L^2}} = 50\sqrt{2}A$ and $i_1 = 50$ A

From phasor diagram, resultant of I_1 and I_2 will be along V_0 and $I_{res} = 50$ A

Phase difference between v and i is θ = 0°



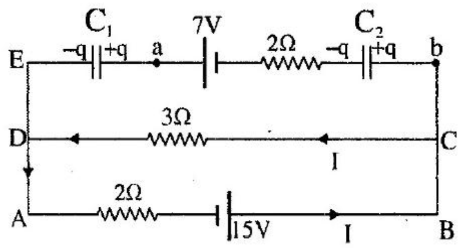


$P_{av} = \frac{i_0 v_0}{2}\cos\theta = 2500W$

$z = \frac{V_0}{i_0} = 2\Omega$

15. Ans. (A,D)

Sol. At steady state I(3) + I(2) = 15 ⇒ I = 3



KVL $C \rightarrow D \rightarrow E \rightarrow a \rightarrow b \rightarrow c$

$$V_C - I(3) + \frac{q}{11} - 7 + \frac{q}{5} = V_C$$

$$\frac{q}{11} + \frac{q}{5} = 7 + 3\times 3 = 16 \Rightarrow q = 55\mu C.$$

Now KVL for a → b

$$V_a - 7 + \frac{q}{5} = V_b$$

$$\Rightarrow V_a - V_b = 7 - \frac{q}{5} = 7 - \frac{55}{5} = \frac{55}{5} = -4V$$

Potential difference across

$$C_1 : \frac{q}{11} = \frac{55}{11} = 5V$$

$\Rightarrow$ P.d. across $C_2 : \frac{q}{5} = 11V$

Potential difference across terminal = 15 – I(2) = 15 – 3 × 2 = 9V

SECTION-III

1. Ans. 3

Sol. Let initial and final speed of stone be u and v.

$\therefore\ v^2 = u^2 - 2gh$

And v cos 30° = u cos 60°

Solving above equations we get

$u = \sqrt{3gh}$

2. Ans. 8

Sol. Conserving angular momentum $m(v_1 \cos 60°)\ 4R = mv_2R \Rightarrow \frac{v_2}{v_1} = 2$.

Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{3}{4}\frac{GM}{R} \Rightarrow v_1^2 = \frac{1}{2}\frac{GM}{R};$$

$$v_1 = \frac{1}{\sqrt{2}}\sqrt{64\times 10^6} = \frac{8000}{\sqrt{2}}\ m/s \Rightarrow X = 8$$

3. Ans. 5

Sol. $f_{\text{observer for source 'A'}}$

$$= f_0\left[\frac{v_{sound} - v_{medium}}{v_{sound} - v_{medium} + v_{source}}\right] = \frac{33}{34}f_0$$

$f_{\text{observer for source 'B'}}$

$$= f_0\left[\frac{v_{sound} + v_{medium}}{v_{sound} + v_{medium} - v_{source}}\right] = \frac{35}{34}f_0$$

$\therefore$ Beat frequency $= f_1 - f_2 = \left(\frac{35-33}{34}\right)f_0 = 5$

4. Ans. 2

Sol. At point O

$\mu_1 \sin i = \mu_2 \sin r$

$$\frac{3}{2}\times \sin\theta_c = \frac{4}{3}\sin r$$

$$\sin r = \frac{3}{4}$$

At point E

$$\frac{4}{3}\times\frac{3}{4} = 1\times\sin\theta$$

$$\sin\theta = \frac{\pi}{2}$$

5. Ans. 6

Sol. For any small change of pressure dp, there will be a change of volume dV and $dp = -B\frac{dV}{V}$. In this change, work is done on the system and the energy stored in the material is

$$dW = -pdV\left(\frac{V}{B}\right)pdp$$

In the change mentioned in the question, the total work done is

$$W = -\int_V^V pdV = \int_V^V \frac{V}{B}pdp$$

The change in volume is negligible and volume can be treated as constant.

$$W = \frac{V_0}{B}\int_{p_0}^{p} pdp = \frac{V_0}{B}\left(\frac{p^2 - p_0^2}{2}\right)$$

Extra energy stored per unit volume is

$$\frac{W}{V_0} = \frac{1}{2B}\left(p^2 - p_0^2\right)$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (A)
2. Ans. (B)
3. Ans. (B)
4. Ans. (B)

$$5 = pK_B + LOG\frac{[NH_4^+]}{0.25}$$

$[NH_4^+] = 0.5$

$[NH_4^+] = 0.25$

m moles of KOH added = 50

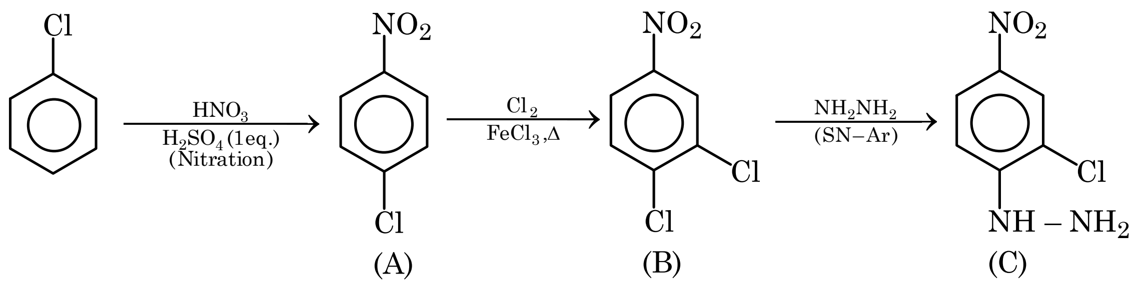
$NH_4^+ + OH^- \rightarrow NH_3 + H_2O$

100	50	50
50	0	100

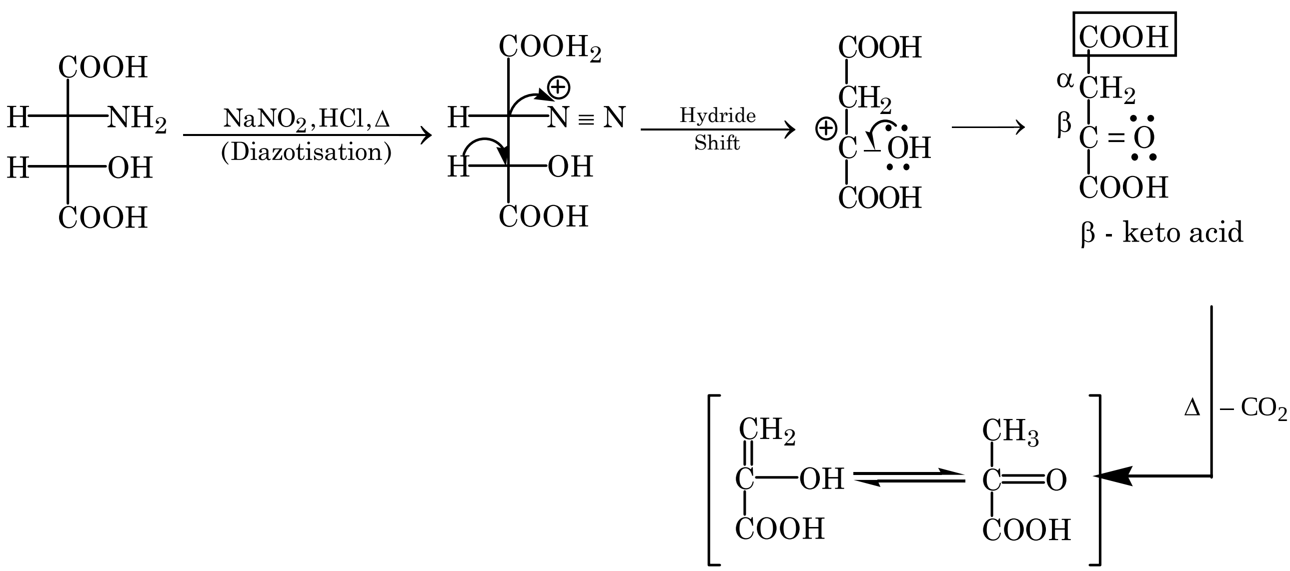
$$pOH = pK_b + \log\frac{50}{100}$$

pOH = 4.7 – 0.3 ⇒4.4

5. Ans.(A)

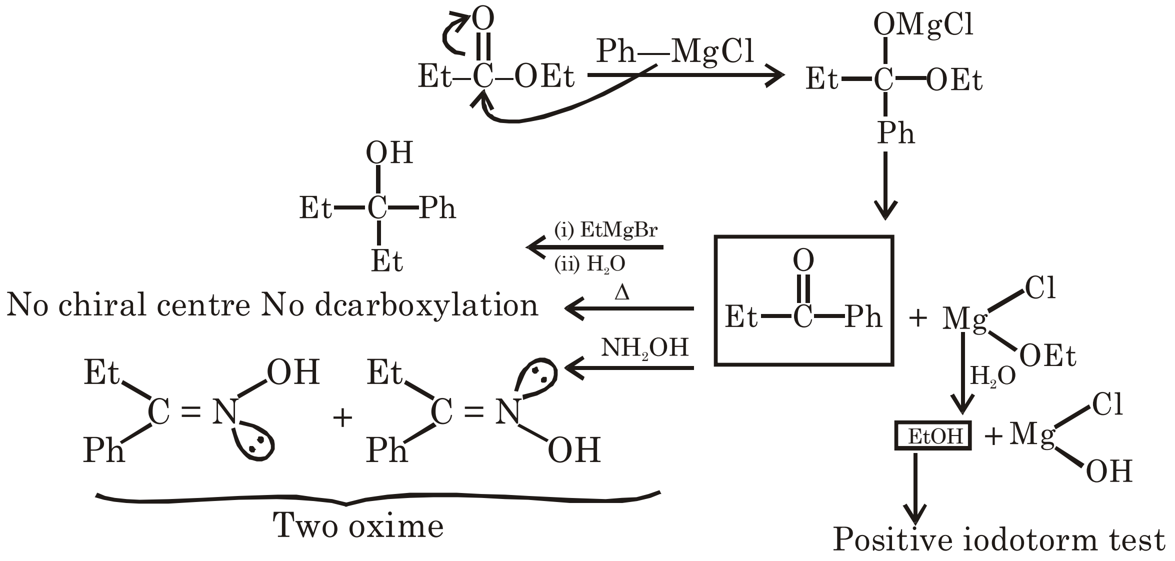


6. Ans.(D)



7. **Ans.(D)**

Sol.



8. **Ans.(C)**

Fake diamonds can be identified by using thermal conductivity.

9. **Ans. (D)**

10. **Ans. (B)**

Since calcium and magnesium salts are soluble, the detergent ion formed will give lather.

11. **Ans.(A,B,D)**

12. **Ans. (A,B,D)**

13. **Ans.(A,B,C,D)**

14. **Ans.(A,C,D)**

With increase in % s-character the size of hybrid orbital decreases. So bond length decreases, bond energy increases electronegativity increases. Bond angle increases from sp^3 to sp^2 to q dx also geometry changes Tetrahedral to planar triangular or linear.

15. **Ans. (A,B,C,D)**

(A) CN^- is stronger ligand than NH_3

(B) en is bidentate ligand cannot stretch to 180° to form chelate complex

(C) Due to formation of $AgCl_2^-$ complex.

(D) Solubility product of AgI is less than AgCl.

SECTION-III

1. **Ans. (8)**

2. **Ans. (4)**

$E = E° - 0.06\ pH$

$0.22 = 0.46 - 0.06\ pH$

$0.06\ pH = 0.46 - 0.22$

3. **Ans. (1)**

4. **Ans. (9)**

Apply $16(X_A - X_B) + 3.5(X_A - X_B)^2$

5. **Ans. (6)**

$6PCl_5 + P_4O_{10} \rightarrow 10POCl_3$

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (D)

Sol. The given expression can be written as

$(1+x)^n(1+y)^n\left(1+\frac{1}{xy}\right)^n$ and constant term

is $C_0^{\ 3} + C_1^{\ 3} + C_2^{\ 3} + ... + C_n^{\ 3}$

2. Ans. (D)

Sol. $\frac{dy}{dx}=\frac{x^2}{4+x^2}$ and $\cot\left(\frac{y-x}{2}\right)=\frac{x}{2}$

$$I=\int\frac{\left(\frac{x^2}{4+x^2}\right)dx}{\left(\frac{x^2-4}{x^2+4}\right)\sin x+\frac{4x}{x^2+4}\cos x}$$

$$=\int\frac{dy}{\cos(y-x)\sin x+\sin(y-x)\cos x}=\int\frac{dy}{\sin y}$$

3. Ans. (B)

Sol. $\cos\theta=\frac{OA^2+OB^2-AB^2}{2OA.OB}$

$$=\frac{OA^2+OB^2-\left(\frac{OA+OB}{2}\right)^2}{2OA.OB}$$

$$=\frac{3}{8}\frac{(OA^2+OB^2)}{OA.OB}-\frac{1}{4}$$

For maximum $\cos\theta$,

$$\frac{3}{8}\left(\frac{OA^2+OB^2}{OA.OB}\right)-\frac{1}{2}\geq\frac{3}{8}\times\frac{2.OA.OB}{OA.OB}-\frac{1}{4}=\frac{1}{2}$$

$$\Rightarrow\theta=\frac{\pi}{3}$$

4. Ans. (D)

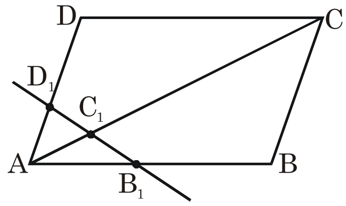
Sol. Let no. of rows in a triangle = k

$1 + 2 + 3 + ... + k = n$

$$\Rightarrow n=\frac{k(k+1)}{2}$$

Also $\frac{k(k+1)}{2}+49=(k+3)^2 \Rightarrow k = 5$

5. Ans. (A)



Sol. $\overrightarrow{AB_1}=\lambda_1\overrightarrow{AB}$

$\overrightarrow{AD_1}=\lambda_2\overrightarrow{AD}$

$\overrightarrow{AC_1}=\lambda_3\overrightarrow{AC}$

As, ABCD is a parallelogram

$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$

$\Rightarrow \frac{\overrightarrow{AB_1}}{\lambda_1}+\frac{\overrightarrow{AD_1}}{\lambda_2}=\frac{\overrightarrow{AC_1}}{\lambda_3}$

$\Rightarrow \frac{\overrightarrow{AB_1}}{\lambda_1}+\frac{1}{\lambda_2}\overrightarrow{AD_1}+\left(\frac{-1}{\lambda_3}\right)\overrightarrow{AC_1}=\vec{0}$

Here points B, C and D are collinear

$\Rightarrow \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{-1}{\lambda_3}=0 \Rightarrow \frac{1}{\lambda_3}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$

6. Ans. (B)

Sol. If $[x^3 + x^2 + x + 1] = [x^3 + x^2 + 1] + x$, then x is integer

$\Rightarrow \log|[x]| = 2 - |[x]|$ has same solutions as $\log|x| = 2 - |x|$, x is integer

$\Rightarrow$ No integral solution $\Rightarrow 0$

7. Ans. (C)

Sol. Let hyperbola is $xy = c^2$ then $\left(ct, \frac{c}{t}\right)$ lies on circle

$\therefore c^2t^4 - 2ct^3 - 20t^2 - 4ct + c^2 = 0$

then $\sum t_1=\frac{2}{c}$, $\sum t_1t_2=\frac{-20}{c^2}$, $\sum t_1t_2t_3=\frac{4}{c}$,

$t_1t_2t_3t_4 = 1$

$\therefore \ell = 2, m = 44, n = 56$

8. Ans. (B)

Sol. $Q=\sum_{r=0}^{n}\frac{\sin 3^r\theta}{\cos 3^{r+1}\theta}$

$$=\frac{\sin\theta}{\cos3\theta}+\frac{\sin3\theta}{\cos9\theta}+\frac{\sin9\theta}{\cos27\theta}+.....+\frac{\sin3^n\theta}{\cos3^{n+1}\theta}$$

Now as $\frac{\sin\theta}{\cos3\theta}-\frac{2\sin\theta\cos\theta}{2\cos\theta\cos3\theta}=\frac{\sin2\theta}{2\cos\theta\cos3\theta}$

$$=\frac{1}{2}\left[\frac{\sin(3\theta-\theta)}{\cos\theta\cos3\theta}\right]$$

$$=\frac{1}{2}\left[\frac{\sin3\theta\cos\theta-\cos3\theta\sin\theta}{\cos\theta\cos3\theta}\right]$$

$$=\frac{1}{2}[\tan3\theta-\tan\theta]$$

$\therefore\ Q=\frac{1}{2}\left[\tan 3\theta-\tan\theta+\tan 9\theta-\tan 3\theta+\right.$

$\left.\ldots..+\tan 3^{n+1}\theta-\tan 3^{n}\theta\right]$

$=\frac{1}{2}\left[\tan^{3n+1}\theta-\tan\theta\right]$

$\theta=\frac{P}{2}$ $\qquad \therefore\ P = 2Q$

9. Ans. (A)

Sol. P = (–4, 0) $\qquad$ Q = (0, 6)

Let A = (x, y)

$\frac{PA}{QA}=\frac{2}{3} \Rightarrow 9PA^2 = 4QA^2$

$9[(x+4)^2+y^2] = 4[x^2+(y-6)^2]$

$5x^2+5y^2+72x+48y=0$

is equation of circum circle of ΔABC

Circumcentre of ΔABC = S = $\left(-\frac{36}{5},-\frac{24}{5}\right)$

and A is orthocenter

SG : GA = 1 : 2

$\Rightarrow SG=\frac{1}{3}SA=\frac{1}{3}\sqrt{\left(\frac{36}{5}\right)^2+\left(\frac{24}{5}\right)^2}$

$=\frac{4\sqrt{13}}{5}$ units

10. Ans. (A)

Sol. Locus of 'Q' is the line of intersection of the plane x + 2y + 3z = 4 and

$1(x-1)+1(y-1)+1(z-1)=0$

$\Rightarrow$ the line is $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$

11. Ans. (A,B,C)

Sol. $\angle BOC = 2A$

In ΔABE, $\angle ABE=\frac{\pi}{2}-A$ $\quad \angle AOM=\pi-2A$

$\frac{c}{b}=e^{i2A}$ $\qquad \frac{a}{m}=e^{i(\pi-2A)}$

$\frac{ac}{bm}=e^{i\pi}=-1$

$\therefore\ m=\frac{-ac}{b}$,

Similarly

$\ell=\frac{-bc}{a}$ $\qquad n=\frac{-ab}{c}$

12. Ans. (A,B,C,D)

Sol. Putting y = x in given functional equation

$f(x)+f(x)=\frac{1}{x}+\frac{1}{x} \Rightarrow f(x)=\frac{1}{x}$

$$\therefore \int_2^3 \frac{3(f(x))^5-f(x)}{1-(f(x))^4}dx=\int_2^3\frac{\left(\frac{3}{x^5}-\frac{1}{x}\right)}{\left(1-\frac{1}{x^4}\right)}dx$$

$$=\int_2^3\left(\frac{\frac{3}{x^7}-\frac{1}{x^3}}{\frac{1}{x^2}-\frac{1}{x^6}}\right)dx$$

Let $\frac{1}{x^2}-\frac{1}{x^6}=t \Rightarrow \left(\frac{6}{x^7}-\frac{2}{x^3}\right)dx=dt$

When x = 2, $t=\frac{1}{4}-\frac{1}{64}=\frac{15}{64}$

x = 3, $t=\frac{1}{9}-\frac{1}{729}=\frac{80}{729}$

$\therefore$Integral $\frac{1}{2}\int_{15/64}^{80/729}\frac{dt}{t}=\frac{1}{2}(\ln t)_{15/64}^{80/729}$

$=\frac{1}{2}\ln\left(\frac{2^{10}}{3^7}\right)=\frac{1}{2}\log\frac{2^{\alpha}}{3^{\beta}}$

$\alpha = 10, \beta = 7$

13. Ans. (A,B,C)

Sol. $P(A)=\frac{3}{10},\ P(B)=\frac{1}{2},\ P(C)=\frac{2}{5}$

P(exactly one)

$= P(A) + P(B) + P(C) - 2P(A\cap B) - 2P(B\cap C) - 2P(C\cap A) + 3P(A\cap B\cap C)$

$=\frac{3}{10}+\frac{1}{2}+\frac{2}{5}-2\times\frac{3}{10}\times\frac{1}{2}-2\times\frac{1}{2}\times\frac{2}{5}-2\times\frac{2}{5}\times\frac{3}{10}+3\times\frac{3}{10}\times\frac{1}{2}\times\frac{2}{5}$

$=\frac{3}{10}+\frac{5}{10}+\frac{4}{10}-\frac{3}{10}-\frac{4}{10}-\frac{12}{50}+\frac{9}{50}=\frac{5}{10}-\frac{3}{10}+3\times\frac{3}{10}\times\frac{1}{2}\times\frac{2}{5}$

P(only A passing the exam)

$=P(A)P(\bar{B})P(\bar{C})=\frac{3}{10}\times\frac{1}{2}\times\frac{3}{5}=\frac{9}{100}$

$\therefore$ Required probability $=\frac{\frac{9}{100}}{\frac{44}{100}}=\frac{9}{44}=\frac{m}{n}$

14. Ans. (A,B,D)

Sol. Let two perpendicular chords through $A(x_1, y_1)$ be PQ and RS

Equation of PQ is $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$

Where $\tan\theta$ = slope of PQ

An point on this line may be taken as

$(x_1 + r\cos\theta,\ y_1 + r\sin\theta)$

As the point lies on $y^2 = 4ax$,

$\therefore (y_1 + r\sin\theta)^2 = 4a(x_1 + r\cos\theta)$

$\Rightarrow r^2\sin^2\theta + 2y_1\sin\theta r - 4a\cos\theta r + y_1^2 - 4ax_1 = 0$

$\therefore\ r_1r_2=\frac{\left(y_1^2-4ax_1\right)}{\sin^2\theta}$

Similarly $r_3r_4=\frac{\left(y_1^2-4ax_1\right)}{\sin^2\alpha}$

where $\tan\alpha$ = slope of RS

Since PQ is perpendicular to RS,

$\therefore \alpha = 90 + \theta$ or $\theta - 90$

In either case ; $\sin^2\alpha = \cos^2\theta$

Now

$$\frac{1}{r_1r_2}+\frac{1}{r_3r_4}=\frac{\sin^2\theta+\cos^2\theta}{\left(y_1^2-4ax_1\right)}=\frac{1}{\left(y_1^2-4ax_1\right)}$$

which is constant

15. Ans. (A,B)

Sol. $f(x)$ is non differentiability at $x = 1, 4, 9, \ldots, 2401, 2500$

$\therefore m = 50$

F(x) has minimum value at all x in [625, 676]

$\therefore$ Number of integral values of x i.e. n = 52

Hence, m + n = 50 + 52 = 102

SECTION-III

1. Ans. 8

Sol. Let $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$,

$\vec{b}=(\cos x\cos y)\hat{i}+(\sin y)\hat{j}+(\sin x\cos y)\hat{k}$

here $|\vec{a}|=3$ and $|\vec{b}|=1$

$\therefore\ \vec{a}$ and $\vec{b}$ are collinear.

$\Rightarrow \tan x = 2$ and $\tan y = 2\cos x$

$\Rightarrow\ \tan^2 y=\frac{4}{\sec^2 x}+\frac{4}{1+4}=\frac{4}{5}$

2. Ans. 3

Sol. Given $f(3x) = f(x)$

$\therefore\ f(3x)=f(x)=f\left(\frac{x}{3}\right)=\ldots..=f\left(\frac{x}{3^n}\right)$

as $n\to\infty;\ 3^n\to\infty$

$\therefore f(3x) = f(0) \Rightarrow f(x)$ is a constant function

$\therefore f(x) = 5\ \ \forall\ x \in R$

$\text{sgn}(f(x)) = 1$

3. Ans. 5

Sol. $175 = 5^2.7.245 = 5.7^2.875 = 5^37.1715 = 5.7^3$

Let $\alpha = \log 5$, $\beta = \log 7$

$a=\frac{\log 175}{\log 245}=\frac{2\alpha+\beta}{\alpha+2\beta}$

$b=\frac{\log 875}{\log 1715}=\frac{3\alpha+\beta}{\alpha+3\beta}$

$\frac{1-ab}{a-b}=\frac{(\alpha+2\beta)(\alpha+3\beta)-(2\alpha+\beta)(3\alpha+\beta)}{(2\alpha+\beta)(\alpha+3\beta)-(\alpha+2\beta)(3\alpha+\beta)}$

$=\frac{5\left(\beta^2-\alpha^2\right)}{\beta^2-\alpha^2}=5$

4. Ans. 3

Sol. $\lim\limits_{y\to\infty}\left[(8y+3)(y+1)(y+2)(y+3)(2y+3)(4y+5)(2y+7)\right]^{\frac{1}{7}}-2y$

$=\lim\limits_{y\to\infty}2y\left[\left(1+\frac{3}{8y}\right)\left(1+\frac{1}{y}\right)\left(1+\frac{2}{y}\right)\left(1+\frac{3}{y}\right)\left(1+\frac{3}{2y}\right)\left(1+\frac{5}{4y}\right)\left(1+\frac{7}{2y}\right)\right]^{\frac{1}{7}}-2y$

$=2\times\left(\frac{3}{8}+1+2+3+\frac{3}{2}+\frac{5}{4}+\frac{7}{2}\right)\times\frac{1}{7}$

$=2\times\frac{97}{56}=\frac{194}{56}\ \Rightarrow\left[\frac{A}{B}\right]=3$

5. Ans. 6

Sol. $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ A(1, 0, 0), B(0, 2, 0), C(0, 0, 3)

Foat of perpendicular from (0, 0, 0) the plane of triangle ABC is orthocenter.

Length of perpendicular from origin to the plane

$6x + 3y + 2z - 6 = 0$

$=\frac{6}{7}=k\ \Rightarrow 7k=6$

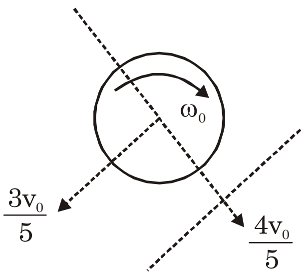
PAPER-2

PART-1 : PHYSICS SOLUTION

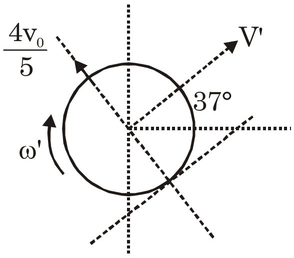
SECTION-I

1. Ans. (A,C,D)

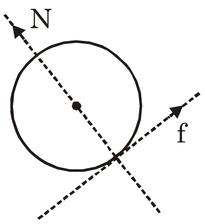
Sol.



Before



After collision



We can neglect impulse of mg during collision.

Impulse perpendicular to surface

$$\text{Ndt} = 2m\left(\frac{4V_0}{5}\right) \quad \dots \text{(i)}$$

Impulse along the surface

$$\mu\text{Ndt} = mv' - \left(-m\frac{3v_0}{5}\right)$$

$$\mu\text{Ndt} = mv' + \frac{3mv_0}{5} \quad \dots \text{(ii)}$$

$$\mu \times 8\frac{mv_0}{5} = mv' + \frac{3mv_0}{5};$$

$$\frac{8\mu}{5}v_0 = v' + \frac{3v_0}{5} \quad \dots. (3)$$

After collision ball will move vertically upwards if

$$v'\cos 37° = \frac{4v_0}{5}\sin 37°;\ v' = \frac{3}{5}v_0$$

From Eq. (3)

$$\frac{8\mu v_0}{5} = \frac{3}{5}v_0 + \frac{3v_0}{5}; \qquad \mu = \frac{6}{8} = \frac{3}{4}$$

Velocity after collision

$$= v'\sin 37° + \frac{4v_0}{5}\cos 37°$$

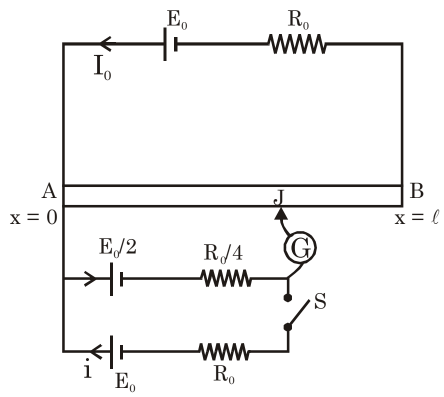
$= v_0$ upwards

$-\mu\text{Ndt} \times R = I\omega' - I\omega_0$

$$\Rightarrow \omega' = \omega_0 - \frac{3v_0}{R}$$

2. Ans. (B)

Sol.



When S is open :

$$R_{AB} = \int_0^{\ell}\frac{3R_0 x dx}{\ell^2} = \frac{3R_0}{2}$$

$$I_0 = \frac{E_0}{R_0 + \frac{3R_0}{2}} = \frac{2E_0}{5R_0}$$

$$\frac{E_0}{2} = I_0 \times R_{AJ} = \frac{2E_0}{5R_0} \times \frac{3R_0}{\ell^2}\int_0^x x\,dx$$

$$x = \ell\sqrt{\frac{5}{6}}$$

When s is closed :

Current in secondary circuit

$$i = \frac{\frac{E_0}{2}}{R_0 + \frac{R_0}{4}} = \frac{2E_0}{5R_0}$$

Terminal voltage of battery =

$$\left[\varepsilon_0 - \frac{2\varepsilon_0}{5}\right] = \frac{3\varepsilon_0}{5}$$

$$\frac{3\varepsilon_0}{5} = I_0 \times R_{AJ} = \frac{2\varepsilon_0}{5R_0} \times \frac{3R_0}{\ell^2} \times \frac{x^2}{2}$$

$\Rightarrow\ x = \ell$

3. Ans. (B,C)

Sol. Negative zero error = –1.25

thickness = 18 + 0.34 – (–1.25)

= 19.59 mm

4. Ans. (A,B,D)

Sol. Displacement of centre of mass is zero. So take centre of mass as origin then find displacement of all points.

5. Ans. (B)

Sol. ↑+ve $\quad V_{AL} = +2$ m/s

$V_L = +6$m/s

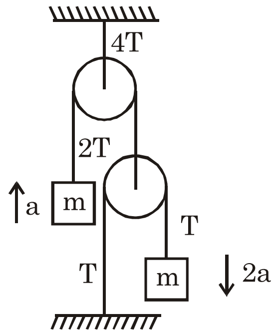
$V_A = V_{AL} + V_L = +8$m/s

$V_{BL} = -2V_{AL} = -4$m/s

$V_B = V_{BL} + V_L = -4 + 6 = +2$m/s

6. Ans. (B)

Sol.



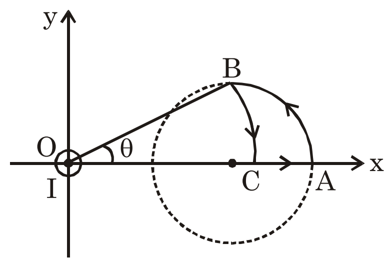
$$mg - T = 2ma \Rightarrow a = \frac{g}{5}$$

$$2T - mg = ma \qquad T = mg - \frac{2mg}{5}$$

$$= \frac{3mg}{5}$$

7. Ans. (B)

Sol. Let segment OB = OC and arc BC is a circular arc with centre at origin. Since the shown closed path ABCA encloses no current, the path integral of magnetic field over this path is zero.



Hence $\int_A^B \vec{B}\cdot\overrightarrow{d\ell} + \int_B^C \vec{B}\cdot\overrightarrow{d\ell} + \int_C^A \vec{B}\cdot\overrightarrow{d\ell} = 0$

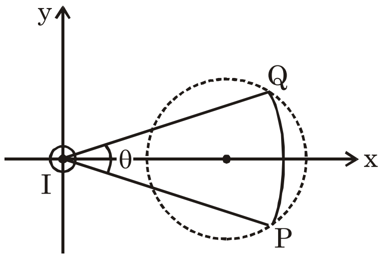
Because is perpendicular to segment AC at all points, therefore $\int_C^A \vec{B}\cdot\overrightarrow{d\ell} = 0$

Hence

$$\int_A^B \vec{B}\cdot\overrightarrow{d\ell} = \int_C^B \vec{B}\cdot\overrightarrow{d\ell} = \frac{\mu_0 I}{2\pi}\frac{OB(\theta)}{OB} = \frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$$

8. Ans. (C)

Sol. Consider two points P and Q lying on dotted circle and equidistant from origin O. We draw a circular arc QP with centre at origin O. The path integral of magnetic field, that is $\int \vec{B}\cdot d\vec{\ell}$, along the dotted circle between two points P and Q is also is equal to path integral $\int \vec{B}\cdot d\vec{\ell}$ along the arc QP whose centre is at origin.



Therefore the path integral of magnetic field $\int \vec{B}\cdot d\vec{\ell}$ along the dotted circle between two points P and Q

$$=\frac{\mu_0 I}{2\pi}\frac{OP(\theta)}{OP}=\frac{\mu_0 I}{2\pi}\theta.$$

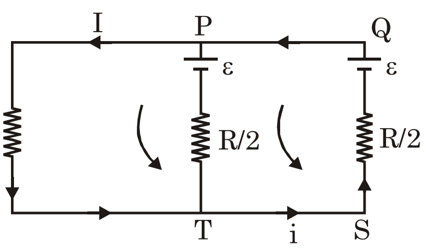
The value of θ will be maximum when chord OQ and chord OP will be tangent to the dotted circle, that is, $\theta=\frac{\pi}{3}$.

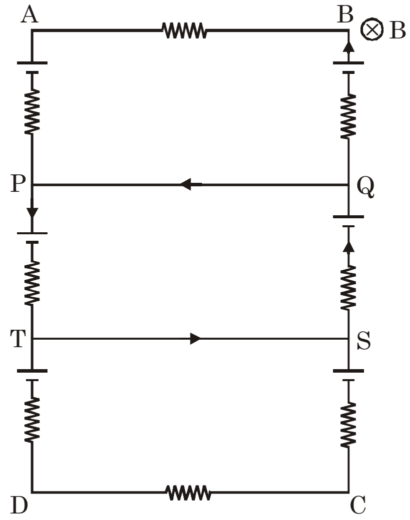
Hence the required maximum value = $\frac{\mu_0 I}{6}$.

9. Ans. (D)

10. Ans. (B)

Sol.





(7) (D) Current in side AB and CD = 0

$\varepsilon = BV_0\ell$

Apply KVL

$$I\times R+(I-i)\times\frac{R}{2}-\varepsilon=0 \qquad ...(1)$$

$$I\times\frac{R}{2}-\varepsilon+\varepsilon-(I-i)\times\frac{R}{2}=0 \qquad ...(2)$$

$$\Rightarrow I=\frac{4\varepsilon}{5R}, i=\frac{2\varepsilon}{5R}$$

(A) Current in R = $\frac{4Bv_0I}{5R}$

(B) Current in AB = 0

(C) Current in CD = 0

(D) Current in ST = $i=\frac{2\varepsilon}{5R}=\frac{2BV_0\ell}{5R}$

(8) (D) Magnetic force on wire

Force = force on QS + force on PT

= BiI + B(I – i)ℓ

$$=\frac{4B^2\ell^2V_0}{5R}$$

Let potential at S is zero

$V_S = 0$

$$V_C=-\frac{\varepsilon}{2}$$

$$V_Q=\varepsilon-i\times\frac{R}{2}=\frac{4\varepsilon}{5R}$$

$$V_B=V_Q+\frac{\varepsilon}{2}$$

$$V_{BC}=V_B-V_C=V_Q+\frac{\varepsilon}{2}+\frac{\varepsilon}{2}$$

$$V_{BC}=\frac{9\varepsilon}{5}=\frac{9BV_0I}{5}$$

SECTION-II

1. Ans. 3

Sol. $P = \frac{(2)^2 \times 4}{(4-2)^3} = 2$;

$$\frac{\Delta P}{P} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + 3\frac{\Delta t}{(4-t)};$$

$$= \frac{2\times 0.01}{2} + \frac{0.02}{4} + \frac{3\times 0.01}{2}$$

$$= \frac{\Delta P}{P} = 0.01 + \frac{0.01}{2} + \frac{0.03}{2}$$

$\Delta P = 0.02 + 0.01 + 0.03$

$\Delta P = 0.06$

$P \pm \Delta P = 2 \pm 0.06$

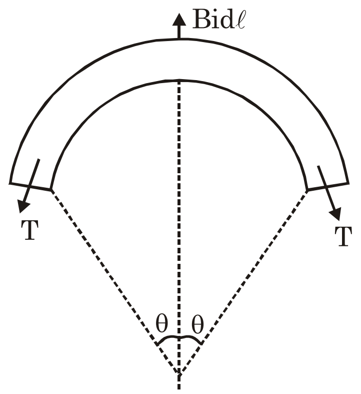
2. Ans. 4

Sol. $2T \sin\theta = Bid\ell$

$2T\theta = Bi(R \times 2\theta)$

$T = BiR$

$$v = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{BiR}{\rho S}}$$



$$B = \frac{v^2 \rho S}{iR} = \frac{10\times 10\times 2\times 10^3 \times 0.2\times 10^{-4}}{1\times 1}$$

$B = 4T$

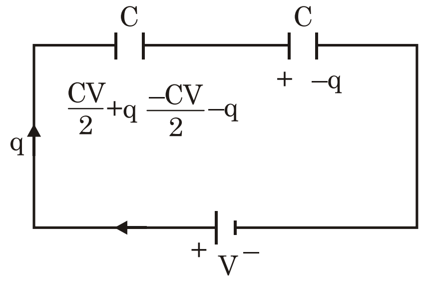
3. Ans. 8

Sol. Let charge supplied by battery is q.

Apply KVL

$$\frac{\frac{CV}{2}+q}{C} + \frac{q}{C} = V$$

$$\frac{CV}{2} + 2q = CV$$



$$2q = \frac{CV}{2};\ q = \frac{CV}{4}$$

Energy supplied by battery $= q \times V = \frac{CV^2}{4}$;

Energy consumed by capacitors = $U_f - U_i$

$$= \left[\frac{1}{2}C\left(\frac{3V}{4}\right)^2 + \frac{1}{2}C\left(\frac{V}{4}\right)^2\right] - \left[\frac{1}{2}C\left(\frac{V}{2}\right)^2 + 0\right]$$

$$= \frac{9CV^2}{32} + \frac{CV^2}{32} - \frac{CV^2}{8} = \frac{3CV^2}{16}$$

Heat loss = $\frac{CV^2}{4} - \frac{3CV^2}{16} = \frac{CV^2}{16} = \frac{CV^2}{2\times 8}$

4. Ans. 4

Sol. The resulted electric field at every point of line C_1C_2 is zero

$\Rightarrow$ No force on any charge particle

$\Rightarrow$ No change in velocity

5. Ans. 7

Sol. $d_m = \rho_0 r 4\pi r^2 dr$

Inside $g_1 = \frac{Gm}{x^2} = \frac{G\int_0^x \rho_0 \times 4\pi r^2 dr}{x^2}$

$g_1 = \frac{G\rho_0 \pi x^4}{x^2}$

$g_1 = G\rho_0\pi x^2$

Outside-

$g_2 = \frac{Gm}{y^2} = \frac{G\int_0^R \rho_0 \times 4\pi r^2 dr}{y^2} = \frac{G\rho_0\pi R^4}{y^2}$

Given $g_1 = g_2 = \frac{1}{2} g_{surface}$

$g_{surface} = G\rho_0\pi R^2$

$G\rho_0\pi x^2 = \frac{G\rho_0\pi R^4}{y^2} = \frac{1}{2}G\rho_0\pi R^2$

$\Rightarrow x = \frac{R}{\sqrt{2}}, y = \sqrt{2}R$

$x + y = \frac{R}{\sqrt{2}} + \sqrt{2}R = \frac{3R}{\sqrt{2}}$

$\Rightarrow x + y = R\sqrt{\frac{9}{2}}$

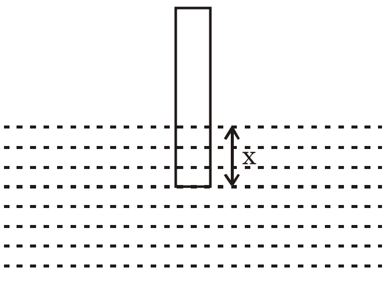
$\alpha = 9$

$\beta = 2$

6. Ans. 8

Sol. $A\ell\sigma g - Ax\rho g = A\ell\sigma a$

$a = g - \frac{\rho g x}{\sigma \ell}$



$\int_0^v v dv = \int_0^x \left(g - \frac{\rho g x}{\sigma\ell}\right) dx$

$\frac{V^2}{2} = gx - \frac{\rho g x^2}{2\sigma\ell}$

At maximum displacement

$v = 0$

$x = \frac{2\sigma\ell}{\rho} = 8m$

7. Ans. 8

Sol. By the conservation of energy

$\frac{1}{2}mV_0^2 = mgh + \frac{1}{2}mv^2$ ... (1)

By the conservation of angular momentum

$\frac{mV_0R}{2} = mVR$ ... (2)

From (1) & (2)

$V_0 = \sqrt{\frac{8gh}{3}}$

K = 8

8. Ans. 4

Sol. $\upsilon = \frac{C^2}{\sqrt{t}} \Rightarrow \upsilon^2 t = C^4$

Differentiate w.r.t. 't'

$\upsilon^2 \times 1 + t \times 2\upsilon\frac{d\upsilon}{dt} = 0$

$\Rightarrow \frac{d\upsilon}{dt} = -\frac{\upsilon}{2t}$ ... (1)

$\frac{1}{slope} = \frac{d\upsilon^2}{dt} = 2\upsilon\frac{d\upsilon}{dt}$

$\Rightarrow \frac{d\upsilon}{dt} = \frac{1}{slope \times 2\upsilon}$ ... (2)

From (1) and (2) $-\frac{\upsilon}{2t} = \frac{1}{slope \times 2v}$

$-\frac{4}{2t} = \frac{1}{-1 \times 2 \times 4}$

$\Rightarrow t = 16$

$C^2 = \upsilon\sqrt{t} = 4\sqrt{16} = 4 \times 4$

$\Rightarrow C = 4$

9. Ans. 6

Sol. $N_1 = 2N_2$

$N_0 e^{-\lambda_1 t} = 2N_0 e^{-\lambda_2 t}$

$\Rightarrow \frac{1}{t} = \frac{1}{T_2} - \frac{1}{T_1}$

10. Ans. 4

Sol. $\frac{\Delta Q}{\Delta t} = K_A A \frac{dT}{dx}$

$\frac{dT}{dx}$ in conductor A = slope of graph = $\frac{4}{3}$

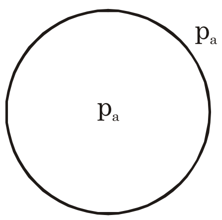
$\frac{dT}{dx}$ in conductor B = slope of graph = 1

Since both conductors are connected in series, same heat current will flow in A and B.

$120 \times \frac{4}{3} = 160$

11. Ans. 8

Sol. Inside pressure must be $\frac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble.



$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$

$\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0} \ldots\ldots \left[\sigma = \frac{Q}{4\pi r^2}\right]$

$Q = 8\pi r\sqrt{2rT\varepsilon_0}$

12. Ans. 4

Sol. $t = \frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{I}{pE}}$

$I = \frac{4t^2}{\pi^2} pE$

13. Ans. 3

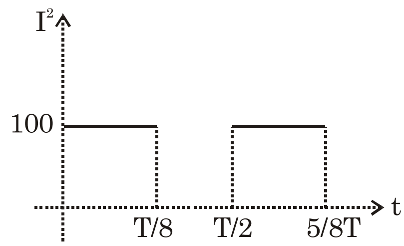
Sol. CA is adiabatic, BC is isothermal as per slope.

Hence $V_1 = \frac{P_0 V_0}{3P_0} = \frac{V_0}{3}$

Using PV^{γ} = const $\Rightarrow \gamma = \frac{\ell n 6}{\ell n 3}$

14. Ans. 5

Sol.



$I_{rms} = \left[\frac{\int I^2 dt}{\int dt}\right]^{\frac{1}{2}}$

$= \left[\frac{100\frac{T}{8} + 100\frac{T}{8}}{T}\right]^{\frac{1}{2}}$

$=$ 5A

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans.(A,B,C)**

2. **Ans.(A,B,C)**

Phase (A) and Phase (B) are at equilibrium, thus

$\Delta G_{P.T} = G_B - G_A = 0.$

So , $G_A = G_B$

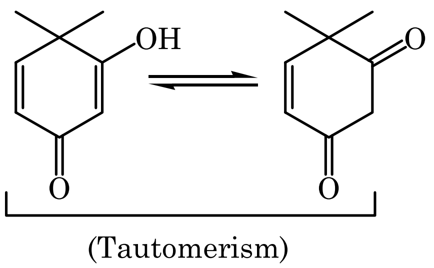
3. **Ans.(A,B,C)**

4. **Ans.(B,C)**

(A) Incorrect

In abbreviated form of dipeptides, amino acid having free amino group is written first so

Abb. From : ALA-GLY not GLY-ALA

(B) 

(C) Correct

(D) Ph – COOH + $NaHCO_3 \rightleftharpoons$ Ph-COONa + $H_2CO_3 \rightarrow H_2O + CO_2 \rightarrow$ Ph–OH + $NaHCO_3$

$\rightarrow$ No reaction.

5. **Ans.(B)**

6. **Ans.(A)**

$H_2O + 2AX + BY \longrightarrow 2HA + YO + BX_2$

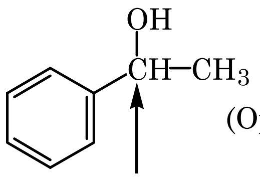
7. **Ans.(D)**

8. **Ans.(C)**

$2CuFeS_2 + 4O_2 \xrightarrow{\Delta} Cu_2S + 2FeO + 3SO_2$

$Cu_2S + FeO + SiO_2 \xrightarrow{\Delta} FeSiO_3$ (fusible slag) + Cu_2S (matte)

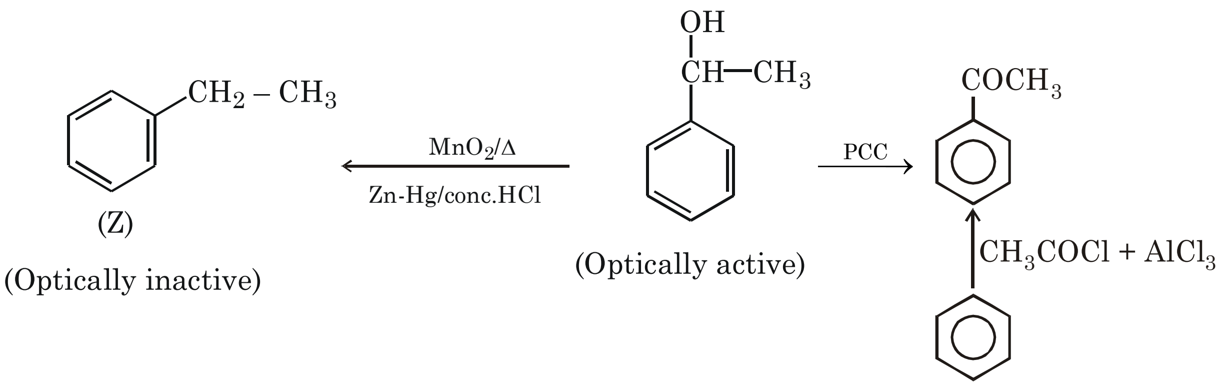
9. Ans.(B)

The compound (X) is  (Optically active)

(Chiral carbon atom)

Observations (i), (ii), (iii), (iv), (vi) and (vii) support it.

10. Ans.(B)



Hence choice (B) is correct while (A), (C) & (D) are incorrect.

SECTION-II

1. Ans.(3)

d,f,c

2. Ans.(7)

(vi) $O_2F_2 < H_2O_2$ Bond length of O–O bond

3. Ans.(7)

PT = constant $\therefore$ $PV^{1/2}$ = constant

$\therefore$ $C = C_v + R/1\text{–}n = 3R/2 + 2R = 7R/2$

$\therefore$ $q = nC\Delta T = 2 \times 7R/2\ (300 - 600) = -2100R$

4. Ans.(5)

$$\frac{112}{22400} \times 6 \times 10^{23} \times 16 \times 10^{-22} = 4.8\ m^2$$

5. Ans.(8)

x = 6
y = 1
z = 3

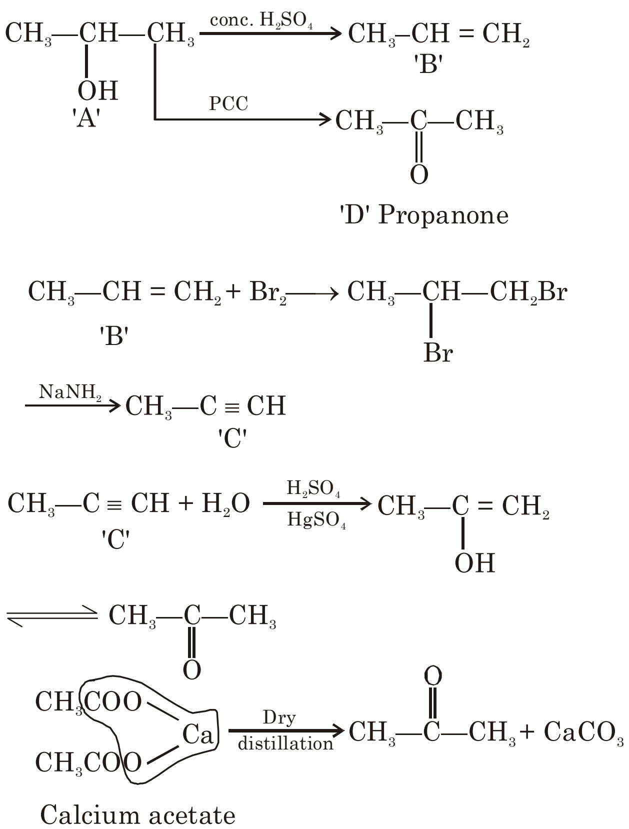
6. Ans.(2)

$2 + 6 + 3n - 12 - 2 = 0$ $\quad [Si_4O_{11}]_n^{6n-}$

$3n - 6 = 0$

$n = 2$

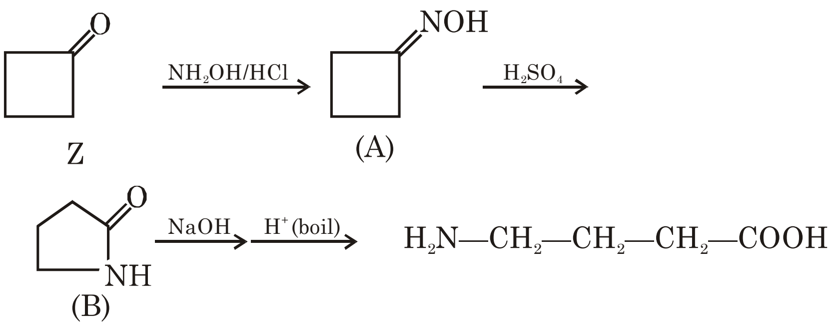
7. Ans.(4)



The molecular weight of 'C' (C_3H_4) is 36 + 4 = 40 g mol^{-1}

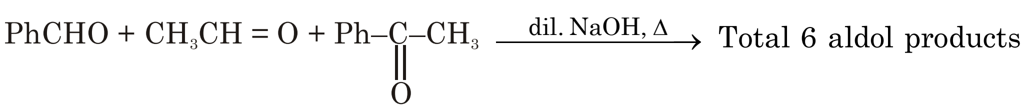
= 4 × 10 hence Y = 4

8. Ans.(7)

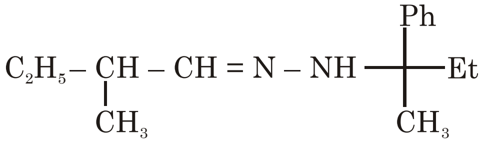


Molar mass of compound Z is 70. So x = 7

9. Ans.(6)



10. Ans.(4 or 8)



(I) (–, +) (Z)

(II) (+, +) (Z)

(III) (–, +) E

(IV) (+, +) E

11. Ans.(7)

Formula will be = X_3Y_4

12. Ans.(3)

$P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$

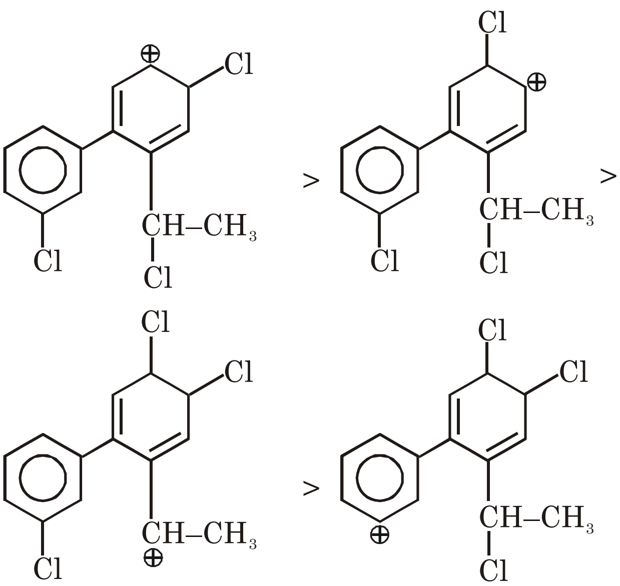
13. Ans.(7)

(a) $CH_3—C(=O)—OC_2H_5$ (b) $H—C(=O)—Cl$ (e) $COCl_2$ (f) $C_2H_5—O—C(=O)—OC_2H_5$

(g) $Cl—C(=O)—OC_2H_5$ (h) $H—C(=O)—OC_2H_5$ (i) C_6H_5COCl

14. Ans.(4)

Rate of S_N1 reaction ∝ Stability of carbocation
Stability of carbocation

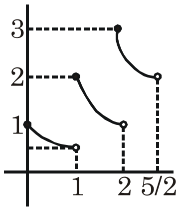


PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. **Ans. (A,B)**

Sol.



$$f(x)=\begin{cases}\dfrac{1}{x+1}, & 0\le x<1\\ \dfrac{2}{x}, & 1\le x<2\\ \dfrac{3}{x-1}, & 2\le x<5/2\end{cases}$$

Clearly $f(x)$ is discontinuous bijective function

2. **Ans. (A,C)**

Sol. Let A' = (x_2, y_2, z_2) be image of (2, 1, 6) about mirror x + y – 2z = 3, then

$$\frac{x_2-2}{1}=\frac{y_2-1}{1}=\frac{z_2-6}{-2}$$

$$=-2\left[\frac{2+1-12-3}{1^2+1^2+2^2}\right]=4$$

$\Rightarrow A' \equiv (6, 5, -2)$

$$\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}=\lambda$$

$x = 2 + 3\lambda$, $y = 4\lambda + 1$, $z = 6 + 5\lambda$ lies on plane than B is point where line cuts plane then $B \equiv (-10, -15, -14)$

The equation of reflected ray will be

$$\frac{x+10}{6+10}=\frac{y+15}{5+15}=\frac{z+14}{-2+14}$$

$$\Rightarrow \frac{x+10}{16}=\frac{y+15}{20}=\frac{z+14}{12}$$

$$\Rightarrow \frac{x+10}{4}=\frac{y+15}{5}=\frac{z+14}{3}$$

3. **Ans. (A,B,C)**

Sol. Given $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$ ...(1)

Take dot product with $\vec{a}$

$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=p+q\cos\theta+r\cos\theta$ ...(2)

Since $\vec{a}.\vec{a}=|\vec{a}|^2=1$, $\vec{a}.\vec{b}=\cos\theta$

and $\vec{a}.\vec{c}=\cos\theta$

taking (.) product with $\vec{b}$ & $\vec{c}$

$0 = p\cos\theta + q + r\cos\theta$ ...(3)

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = p\cos\theta + q\cos\theta + r$ ...(4)

from (2) and (4) we get

p = r

Adding (2), (3) and (4)

$2\left[\vec{a}\ \vec{b}\ \vec{c}\right] = (2\cos\theta + 1)(p + q + r)$

$\Rightarrow p+q+r=\dfrac{2\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{2\cos\theta+1}$ ...(5)

multiplying equation (5) by cosθ and subtracting it from (2)

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]-\frac{2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\cos\theta}{2\cos\theta+1}=p(1-\cos\theta)$$

$$p=\frac{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{(2\cos\theta+1)(1-\cos\theta)}$$

similarly $q=\dfrac{-2\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{(2\cos\theta+1)(1-\cos\theta)}$

Now $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}$

$$=\begin{vmatrix}1 & \cos\theta & \cos\theta\\ \cos\theta & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix}$$

$= (1 + 2\cos\theta)(1 - \cos\theta)^2$

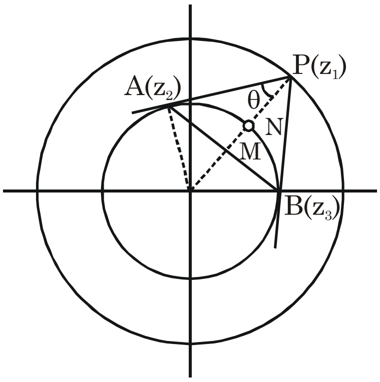
$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\sqrt{1+2\cos\theta}\,(1-\cos\theta)$

Since θ is acute cosθ > 0

Hence $p=r=\dfrac{1}{\sqrt{1+2\cos\theta}}$, $q=\dfrac{-2\cos\theta}{\sqrt{1+2\cos\theta}}$

4. Ans. (A,B,C,D)

Sol.



OA = OB = 1 ; OP = 2

$\sin\angle OPA = \frac{1}{2}$ $\angle OPA = \frac{\pi}{6}$ $\angle APB = \frac{\pi}{3}$

OM ⊥ AB so ΔAPB is equilateral, so (D) is true.

$OM = \frac{1}{2}$; $MN = \frac{1}{2}$ PN = 1 so

centroid lies on |z| = 1 ...(A)

$\left|\frac{z_1+z_2+z_3}{3}\right| = 1$

$|z_1 + z_2 + z_3|^2 = 9$

$(z_1+z_2+z_3)(\bar{z}_1+\bar{z}_2+\bar{z}_3) = 9$

$z_1\bar{z}_1 = 4$ $z_2\bar{z}_2 = 1$ $z_3\bar{z}_3 = 1$

or $\left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right) = 9$ ...(C)

$\angle AOB = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ ...(B)

5. Ans. (D)

Sol. (A) $P\left(\frac{Y}{X_5}\right) = P\left(\frac{Y}{X_6}\right) = \frac{P(Y\cap X_6)}{P(X_6)}$

$P(Y \cap X_6)$ = P(AB AB AB) + P(AB BA AA) + P(BA AB AA) + P(BA BA AA)

$= 4.\frac{4}{81}.\frac{4}{9} = \frac{64}{729}$

$P(X_6)$ = P(ABBA) + P(ABAB) + P(BABA) + $P(BAAB) = 4.\frac{4}{81}$

$\Rightarrow P\left(\frac{Y\cap X_6}{X_6}\right) = \frac{4}{9}$

(B) $P\left(\frac{Z}{X_4}\right) = \frac{P(Z\cap X_4)}{P(X_4)}$

$P(Z \cap X_4)$ = P(AB BB) + P(BA BB) + P(AB AB BB) + P(AB BA BB) + P(BA AB BB) + P(BA BA BB)

$= \frac{4}{81} + 4.\frac{4}{81}.\frac{1}{9} = \frac{52}{729}$

$P(X_4)$ = 1 – P(AA) – P(BB)

$= 1 - \left(\frac{4}{9}+\frac{1}{9}\right) = \frac{4}{9}$

$P\left(\frac{Z}{X_4}\right) = \frac{52}{729}\times\frac{9}{4} = \frac{13}{81}$

(C) Match will end at 2, 4 or 6 sets only so

$P(X_{2k-1}) = P(X_{2k})\ \forall\ k \in \{1, 2, 3\}$

(D) Hence D is incorrect

$\rightarrow P\left(\frac{Z}{X_1}\right) = P(z) = \frac{133}{729}$

6. Ans. (C)

Sol. $P\left(\frac{Y}{W}\right) = \frac{P(Y\cap W)}{P(W)}$

Where W is the event that A has won the third set

$P(Y \cap W)$ = P(AB AA) + P(BA AA) + P(AB AB AA) + P(BA AB AA)

$= 2.\frac{2}{9}\left(\frac{4}{9}\right) + 2.\left(\frac{4}{81}\right)\left(\frac{4}{9}\right)$

$= \frac{6}{81} + \frac{32}{729} = \frac{144+32}{729} = \frac{176}{729}$

P(W) = P(ABA) + P(BAA)

$= 2.\frac{2}{9}.\frac{2}{3} = \frac{8}{27}$

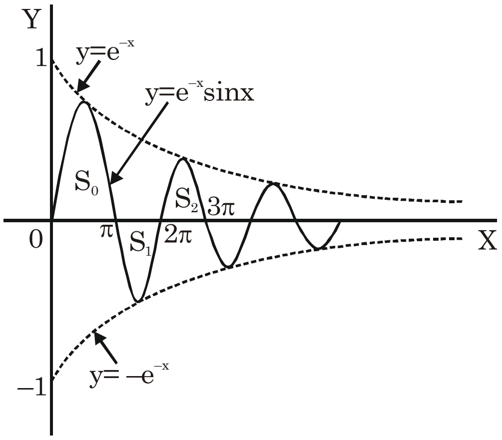
$P\left(\frac{Y}{W}\right) = \frac{P(Y\cap W)}{P(W)} = \frac{\frac{176}{729}}{\frac{8}{27}} = \frac{176}{27}.\frac{27}{8} = \frac{22}{27}$

Paragraph for Question 7 & 8

7. Ans. (B)

8. Ans. (B)

Sol.



$-1 \le \sin x \le 1$ the curve $y = e^{-x}\sin x$ is bounded by $y = e^{-x}$ and $y = -e^{-x}$

Now S_n ; $n\pi \le x \le (n+1)\pi$

Now S_n ; $n\pi \le x \le (n+1)\pi$

$$S_n = (-1)^n \int_{n\pi}^{(n+1)\pi} e^{-x} \sin x \, dx$$

$$= \frac{e^{-n\pi}}{2}\left(1 + e^{-\pi}\right)$$

$$S_0 = \frac{1}{2}\left(1 + e^{-\pi}\right) \text{ and } \frac{S_{n+1}}{S_n} = e^{-\pi}$$

$$\sum_{n=0}^{\infty} S_n = \frac{\frac{1}{2}\left(1 + e^{-\pi}\right)}{1 - e^{-\pi}} \qquad \left[S_\infty = \frac{a}{1-r}\right]$$

Paragraph for Question 9 & 10

Sol. $$D_n = \begin{vmatrix} a & 1 & 0 & 0 & 0 & \ldots & 0 \\ 1 & a & 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & a & 1 & 0 & \ldots\ldots & 0 \\ 0 & 0 & 1 & a & \ldots & \ldots & 0 \\ 0 & 0 & 0 & 1 & a & \ldots & 0 \\ 0 & 0 & 0 & 0 & 1 & \ldots & a \end{vmatrix}$$

expanding it from 1st Row

$$D_n = aD_{n-1} \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & \ldots\ldots \\ 0 & a & 1 & 0 & 0 & \ldots\ldots \\ 0 & 1 & a & 1 & 0 & \ldots\ldots \\ \vdots & 0 & 1 & a & \ldots & \ldots\ldots \\ 0 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$

$$D_n = aD_{n-1} - \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & \ldots\ldots 0 \\ 0 & a & 1 & 0 & 0 & \ldots\ldots 0 \\ 0 & 1 & a & 1 & 0 & \ldots\ldots 0 \\ 0 & 0 & 1 & a & 1 & \ldots\ldots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots\ldots & & 001a \end{vmatrix}$$

$D_n = aD_{n-1} - D_{n-2}$

9. Ans. (D)

Sol. $a = 2$ $\qquad D_n = 2D_{n-1} - D_{n-2}$

$\qquad D_n + D_{n-2} = 2D_{n-1}$

i.e., D_{n-2}, D_{n-1}, D_n are in A.P.

$D_1 = 2 \qquad D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$

$D_3 = 4$ ..... $D_{2017} = 2018$

10. Ans. (D)

Sol. $a = 1$

$D_n = D_{n-1} - D_{n-2}$

$D_1 = 1 \ D_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$D_3 = D_2 - D_1 = -1$

$D_4 = -1$

$D_5 = 0$

$D_6 = 1$

$D_7 = 1$

$D_8 = 0$

$D_9 = -1$

$D_{10} = -1$

$D_{11} = 0$

$D_{12} = 1$

$D_{13} = 1$

i.e., $D_1 = D_6 = D_7 = D_{12} = D_{13} = 1$

$D_2 = D_5 = D_8 = D_{11} = 0$

$D_3 = D_4 = D_9 = D_{10} = -1$

$6\lambda, 6\lambda + 1$ type no. gives 1

$6\lambda + 2, 6\lambda + 5$ gives 0

$6\lambda + 3, 6\lambda + 4$ gives -1

Hence $\sum_{k=1}^{2017} |D_k| = \frac{4}{6}(2016) + 1$

$= 2(672) + 1 = 1344 + 1 = 1345$

SECTION–II

1. Ans. 8

Sol. $\Delta = 0$ & $a_0 + a_1 + a_2 \neq 0$

then $a_0 = a_1 = a_2$

$$\Delta = -\left[\left(a_0^3 + a_1^3 + a_2^3\right) - 3a_0a_1a_2\right]$$

putting x = 0

$a_0 = 1$

coefficient of x = 4a

coefficient of $x^2 = 4b + 6a^2$

$1 = 4a = 6a^2 + 4b$

$$\Rightarrow a = \frac{1}{4}\ ;\ b = \frac{5}{32} \quad 5 \times \left(\frac{1/4}{5/32}\right) = 8$$

2. Ans. 5

Sol. $z_1 = (8\sin\theta + 7\cos\theta) + i(\sin\theta + 4\cos\theta)$

$z_2 = (\sin\theta + 4\cos\theta) + i(8\sin\theta + 7\cos\theta)$

$z_1 = a + ib \quad z_2 = b + ia$

$z_1z_2 = ab - ab + i(a^2 + b^2) = x + iy$

$\Rightarrow x = 0\ ;\ y = a^2 + b^2$

Now $x + y = a^2 + b^2$

$M = (x + y)_{max}$

$\Rightarrow x + y = (8\sin\theta + 7\cos\theta)^2 + (\sin\theta + 4\cos\theta)^2$

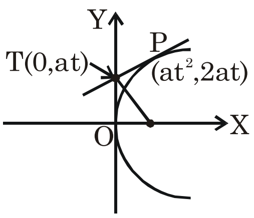
$= 65 + 60\sin2\theta$

$(x + y)_{max} = 125$

$m = 125 \Rightarrow M^{1/3} = 5$

3. Ans. 9

Sol.



(h, k) is mid point of PT then

$$h = \frac{at^2}{2}\ ;\ k = \frac{2at + at}{2}$$

$$\Rightarrow 2h = a.\frac{4k^2}{9a^2}$$

$\Rightarrow 2y^2 = 9ax$

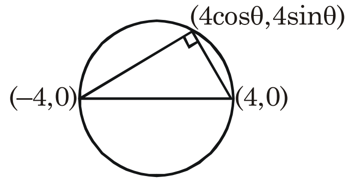
a = 2

$y^2 = 9x$

L.R = 9

4. Ans. 7

Sol.



$$\text{Area} = \frac{1}{2} \times 8 \times 4\sin\theta = |16\sin\theta|$$

If area is integer then $\sin\theta$ can be equal to

$$\frac{1}{16}, \frac{2}{16}, \frac{3}{16} \ldots\ldots \frac{15}{16}$$

15 in all four quadrants and 2 are on aixs

$$\left[\frac{62}{8}\right] = 7$$

5. Ans. 8

Sol. $$f'(x) = \lim_{h\to0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to0}\frac{f(x) + f(h) + 2xh(x+h) - \frac{1}{3} - f(x)}{h}$$

$$= \lim_{h\to0}\frac{f(h) - f(0)}{h} + 2x^2 = f'(0) + 2x^2$$

$$\text{also } \lim_{h\to0}\frac{3f(h) - 1}{6h} = \lim_{h\to0}\frac{f(h) - \frac{1}{3}}{2h} = \frac{f'(0)}{2} = \frac{2}{3}$$

$$\text{so } f'(x) = \frac{4}{3} + 2x^2$$

$$f(x) = \frac{2x^3}{3} + \frac{4}{3}x + \lambda \text{ or } f(0) = \lambda = \frac{1}{3}$$

$$f(2) = \frac{25}{3} \quad [f(2)] = 8$$

6. Ans. 4

Sol. $f(x) = f(6 - x)$ ...(i)

$f'(x) = -f'(6 - x)$ ...(ii)

Putting x = 0, 2, 5

$f'(0) = f'(6) = f'(2) = f'(4) = f'(5) = f'(1) = 0$

and since $f'(3) = -f'(3)$

$\Rightarrow f'(3) = 0$

So $f'(x) = 0$ has minimum seven (7) roots in [0, 6]

Now if $h(x) = f'(x)f''(x)$

$h'(x) = (f''(x))^2 + f'(x)f'''(x)$

h(x) = 0 has minimum 7 + 6 = 13 roots so h'(x) will have minimum 12 zeros is [0, 6]

So $\frac{12}{3} = 4$

7. Ans. 8

Sol. Eliminating t will give

$y^2(x - 1) = 1$

equation of tangent at (2, 1) is x + 2y = 4 solving with curve,

we get x = 5 and $y = -\frac{1}{2}$

$Q = \left(5, -\frac{1}{2}\right)$ or $|PQ| = \frac{3\sqrt{5}}{2}$

8. Ans. 1

Sol. $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ ...(i)

Putting x = y = 0, we get $f(0) = 0$

Putting y = –x, we get

$F(+x) + f(-x) = f(0)$

or, $f(-x) = -f(x)$ ...(ii)

Also, $\lim_{x\to 0} \frac{f(x)}{x} = 2$

Now, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ...(iii)

$= \lim_{h\to 0} \frac{f(x+h) + f(-x)}{h}$ [Using (ii)]

$= \lim_{h\to 0} \frac{f\left(\frac{x+h-x}{1-(x+h)(-x)}\right)}{h}$ [Using (i)]

$= \lim_{h\to 0} \left[\frac{f\left(\frac{h}{1+x(x+h)}\right)}{h}\right]$

$= \lim_{h\to 0} \frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)} \times \left(\frac{1}{1+xh+x^2}\right)$

$= \lim_{h\to 0} \frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)} \times \lim_{h\to 0}\left(\frac{1}{1+xh+x^2}\right)$

(Using $\lim_{x\to 0} \frac{f(x)}{x} = 2$)

$= 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$

Integration both sides, we get

$f(x) = 2\tan^{-1}(x) + c$, where $f(0) = 0 \Rightarrow c = 0$

Thus, $f\left(\frac{1}{\sqrt{3}}\right) = 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 2.\frac{\pi}{6} = \frac{\pi}{3}$

And, $f'(1) = \frac{2}{1+1^2} = \frac{2}{2} = 1$

9. Ans. 6

Sol. OP = OM

$y - x\frac{dy}{dx} = \sqrt{x^2+y^2}$

$\frac{dy}{dx} = \frac{y - \sqrt{x^2+y^2}}{x}$

$\frac{dy}{dx} = \frac{y}{x} - \sqrt{1+\left(\frac{y}{x}\right)^2}$

Putting $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore\ v + x\frac{dv}{dx} = v - \sqrt{1+v^2}$

$\Rightarrow \log\left(v + \sqrt{1+v^2}\right) = \log\frac{c}{x}$

$\Rightarrow \left(v + \sqrt{1+v^2}\right) = \frac{c}{x}$

$\Rightarrow y + \sqrt{x^2+y^2} = c$

Curve is parabola so e = 1

10. Ans. 4

Sol. $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$

or $\frac{x(xdy - ydx)}{dx} = 1 + \cos\left(\frac{y}{x}\right)$

$\Rightarrow \frac{\frac{(xdy - ydx)}{x^2}}{1+\cos\left(\frac{y}{x}\right)} = \frac{dx}{x^3}$

$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{1+\cos\left(\frac{y}{x}\right)} = \int \frac{dx}{x^3}$

$$\Rightarrow \frac{1}{2}\int \frac{d\left(\frac{y}{x}\right)}{\cos^2\left(\frac{y}{2x}\right)} = \int \frac{dx}{x^3}$$

$$\Rightarrow \frac{1}{2}\frac{\tan\left(\frac{y}{2x}\right)}{\frac{1}{2}} = \frac{x^{-2}}{-2} + c$$

$$\Rightarrow \tan\left(\frac{y}{2x}\right) + \frac{1}{2x^2} = c$$

11. Ans. 7

Sol. We have

$a^2 - (2x^2 + 1)a + x^4 + x = 0$

$$\therefore\ a = \frac{(2x^2+1) \pm \sqrt{(2x^2+1)^2 - 4(x^4+x)}}{2}$$

$\therefore$ $2a = (2x^2 + 1) \pm (2x - 1)$

positive sign $a = x^2 + x$

negative sign $2a = 2x^2 - 2x + 2$

$a = x^2 - x + 1$

if $x^2 + x - a = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1+4a}}{2}$

if $x^2 - x + 1 - a$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4+4a}}{2} = \frac{1 \pm \sqrt{4a-3}}{2}$$

for x to be real $a \geq 3/4$ and $a \geq -1/4$

$\Rightarrow a \geq 3/4 \Rightarrow 3 + 4 = 7$

12. Ans. 2

Sol. $P(E) = \dfrac{1}{2}$ $\quad P(F_k) = {}^nC_k \times \dfrac{1}{2^n}$

$$P(E \cap F_k) = {}^{n-1}C_{k-1} \times \left(\frac{1}{2}\right)^n$$

$P(E \cap F_k) = P(E) \times P(F_k)$

$${}^{n-1}C_{k-1} \times \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n \times \frac{1}{2} \times {}^nC_k$$

or $2 \times {}^{n-1}C_{k-1} = {}^nC_k$

$$\frac{{}^nC_k}{{}^{n-1}C_{k-1}} = 2 = \frac{n}{k}$$

13. Ans. 4

Sol. Required no. of ways

$= \text{coeff of } x^{30} \text{ in } (x^2 + x^3 + ... + x^8)^5$

$$= \text{coeff of } x^{30} \text{ in } \left(\frac{x^2(1-x^7)}{1-x}\right)^5$$

$= \text{coeff of } x^{20} \text{ in } (1 - x^7)^5(1 - x)^{-5}$

$= {}^{24}C_{20} - 5 \times {}^{17}C_{13} + 10 \times {}^{10}C_6 = 826$

$8 + 2 - 6 = 4$

14. Ans. 4

Sol. Given $\alpha + \beta = 1$ ...(i)

$2\alpha^2 + 2\beta^2 = 1$ ...(ii)

Solving equation (i) and (ii), we get

$$\alpha = \beta = \frac{1}{2}$$

$$Q = \frac{\alpha}{\beta} = 1 \qquad ...(iii)$$

And given

$f(2 + x) + f(x) = 2 \quad \forall\ x \in [0, 2]$ ...(iv)

Now, $p = \displaystyle\int_0^4 f(x)\,dx - 4$

$$= \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx - 4$$

$$= \int_0^2 f(x)\,dx + \int_0^2 f(t+2)\,dt - 4$$

{Let, x = t + 2 for second integration}

$$= \int_0^2 f(x)\,dx + \int_0^2 \{2 - f(x)\}\,dx - 4$$

$$= \int_0^2 f(x)\,dx + 2\int_0^2 dx - \int_0^2 f(x)\,dx - 4 = 0$$

Then, p = 0, q = 1

Let the roots of equation $ax^2 - bx + c = 0$ be α and β

$\therefore$ $f(x) = ax^2 - bx + c = a(x - \alpha)(x - \beta)$ ...(v)

Since, equation $f(x) = 0$ has both roots between 0 and 1

$\therefore$ $f(0).f(1) > 0$ ...(vi)

But $f(0).f(1) = c(a - b + c)$ = an integer ...(vii)

$\therefore$ least value of $f(0).f(1) = 1$ ...(viii)

Now, from equation (v)

$F(0).f(1) = a\alpha\beta\ a(1 - \alpha)(1 - \beta)$

$= a^2\alpha\beta(1 - \alpha)(1 - \beta)$ ...(ix)

As we know,

$\alpha(1 - \alpha)$ as greatest value $\frac{1}{4}$ at $\alpha = \frac{1}{2}$ and

$\beta(1 - \beta)$ has greatest value $\frac{1}{4}$ at $\beta = \frac{1}{2}$.

But $\alpha \neq \beta$

Thus from (viii) greatest value of

$f(0).f(1) < \frac{a^2}{16}$ ...(x)

$\therefore$ from (viii) and (x)

$1 < \frac{a^2}{16}$

$\Rightarrow a^2 - 16 > 0$

$\Rightarrow a < -4$ or $a > 4$ $(\because a \in N)$

$\Rightarrow$ least value of $a = 5$ (as $a \in$ Natural number)